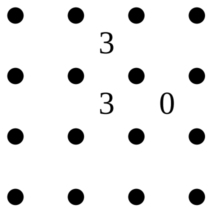
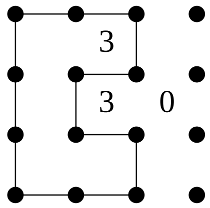
Slitherlink

We have seen the rules of inference for propositional logic. But rules of inference can occur in other systems. For example, we have the following logic puzzle call *Slitherlink* [1].

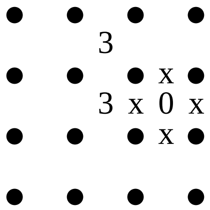


In this puzzle we are asked to draw a single loop through some of the points where each line of the loop link two adjacent points. Furthermore, the number in the grid indicates how many lines are around that cell. So, for example, the above puzzle has the following solution.



We can use rules of inference to solve these puzzles.

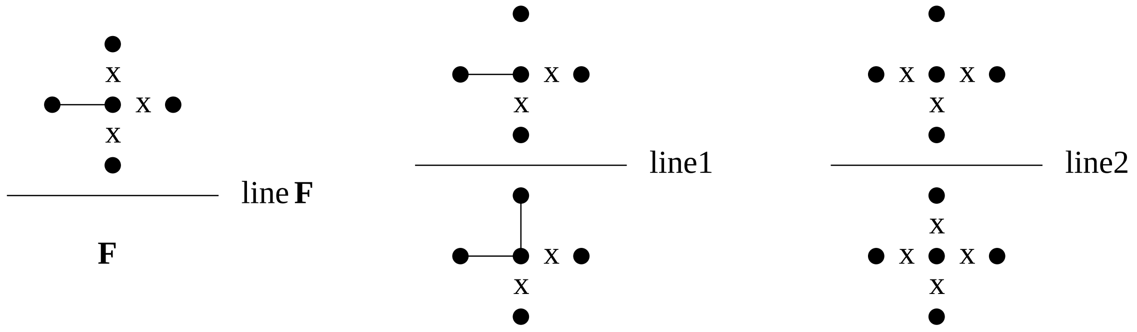
Note that for each pair of adjacent point, in the final solution, we know that there either is a line between them or there is not. If we are sure that there is no line between a pair of points then we can mark this information on our puzzle by an "X" between those two points. For example,

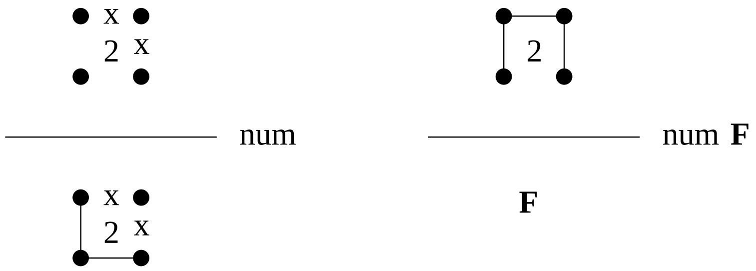


in the above puzzle.

Note that in the final solution, every point has exactly 0 or 2 lines going to it (the loop is required to not have any "branching" or "loose ends"). We can translate this into the following rules of inference.

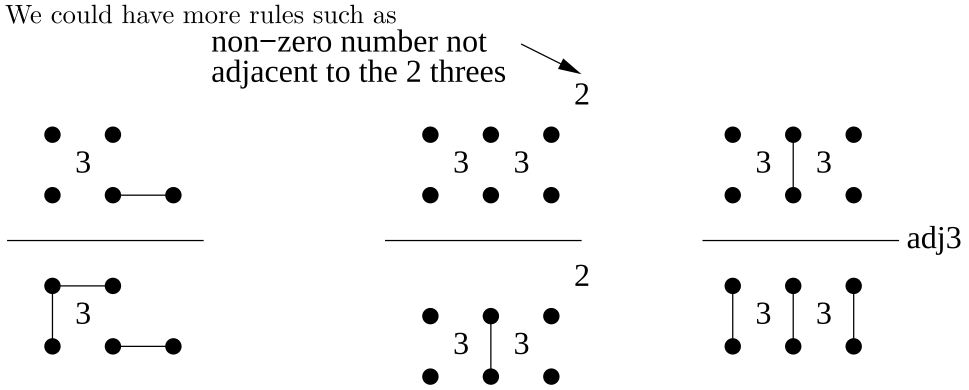
[1] Slitherlink was first introduced by Nikoli in (see `http://www.nikoli.co.jp/en/puzzles/slitherlink/`)





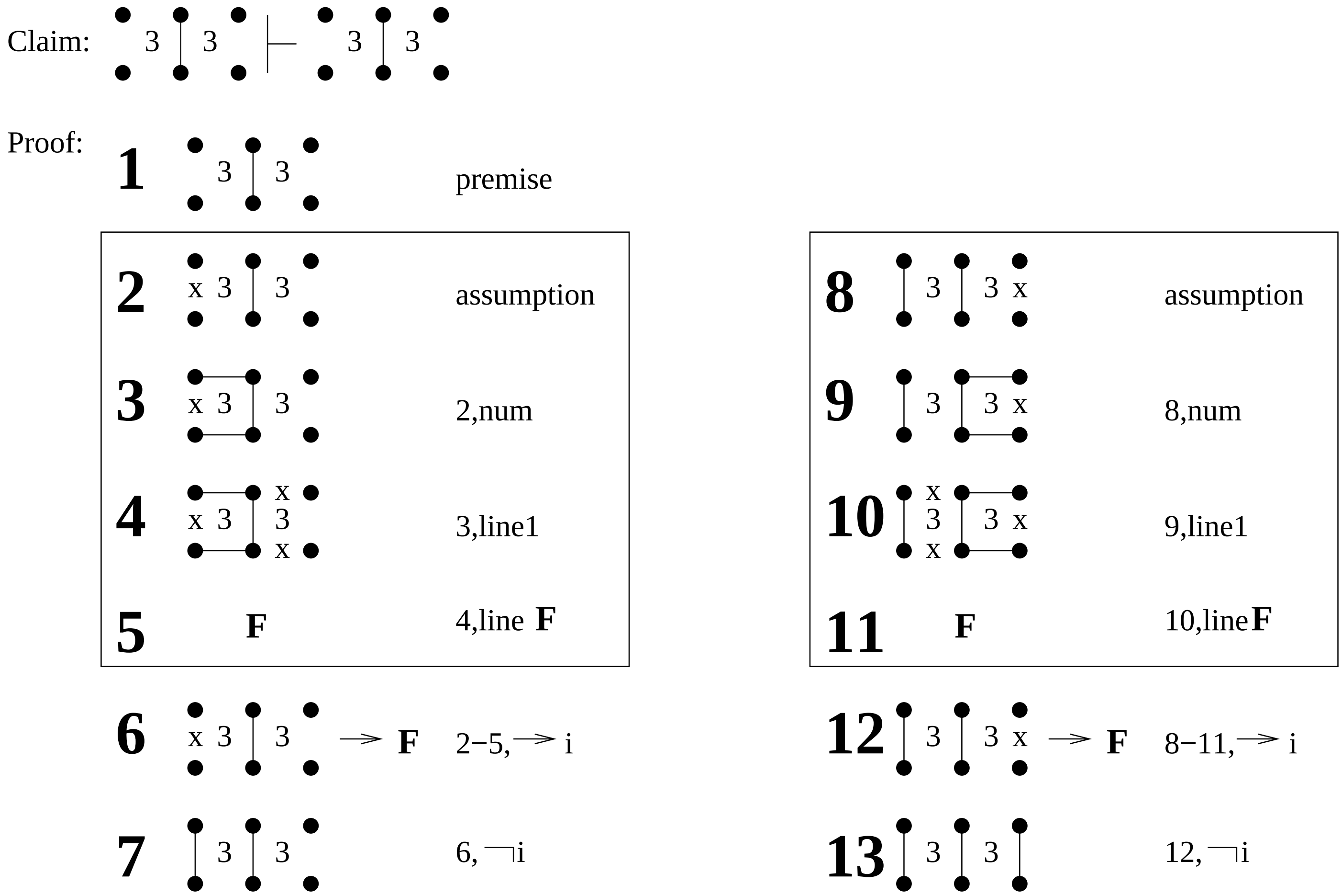
The diagrams above give a "representative" for each rule. In reality, we should draw many more diagrams per rule. Also implicit in the diagrams is the fact that if the "hypothesis configuration" occurs anywhere in the puzzle then we can apply our rule.

We could have more rules such as



but in fact, some of these rules could be derived from our previous rules.

Example 1. For example, we can infer adj3 as follows.



Two column were used in this proof in order to illustrate the symmetry in the proof (our existing rules of inference does not allow us to argue by symmetry directly).

We have chosen a rather arbitrary set of rules and we could have chosen a different set. This is, in fact, also true for propositional logic. But for the purpose of this course, we will keep using the rules given in lecture 2.