Example 1. Let $n = 13941$.

We want to show that if you know that

- $n - 1$ is not odd, and
- if n is even then $n - 1$ is odd

then you can conclude that n is not even for the following reason.

Proof. Suppose n is even. Then $n - 1$ is odd (since you know "if n is even then $n - 1$ is odd"). But you also know that "$n - 1$ is not odd". This is a contradiction.

Thus, assuming that n is even leads to a contraction.

Since assuming n is even leads to a contradiction, you can conclude that n is not even. $\square$

Note that in this proof we did not make any use of the properties of the number n. For example, we did not make use of the fact that a number cannot be both even and odd. In fact, we did not even make use of the definition of the words *even* and *odd*!

Definition 1. n is an *odd* number if $n = 2k + 1$ for some integer k. n is an *even* number if $n = 2k$ for some integer k.

Note that we will not define what an integer is because this is something which is actually (relatively) difficult to do. So we will take for granted that we know that we are all using the same definition of "integer".

Example 2. The examples given here motivate the definition of $\rightarrow$ (by its truth table).

Theorem 1. *If* 3 *is odd then* $3 - 1$ *is even.*

Proof. Suppose 3 is odd. Then $3 = 2k + 1$ for some integer k. $3 - 1 = 2k + 1 - 1 = 2k$. Thus $3 - 1$ is even (since k is an integer). $\square$

Theorem 2. *If* 3 *is even then* $3 - 1$ *is odd*

Proof. Suppose 3 is even. Then $3 = 2k$ for some integer k. $3 - 1 = 2k - 1 = 2(k - 1) - 1$. $k - 1$ is an integer since k is an integer. Thus $3 - 1$ is odd. $\square$

Theorem 3. *If* 3 *is even then* 2 *is even.*

Proof. Suppose 3 is even. $2 = 2 * 1$ and 1 is an integer so 2 is even by definition. $\square$