

# **Maximum Satisfiability in Software Analysis: Applications & Techniques**

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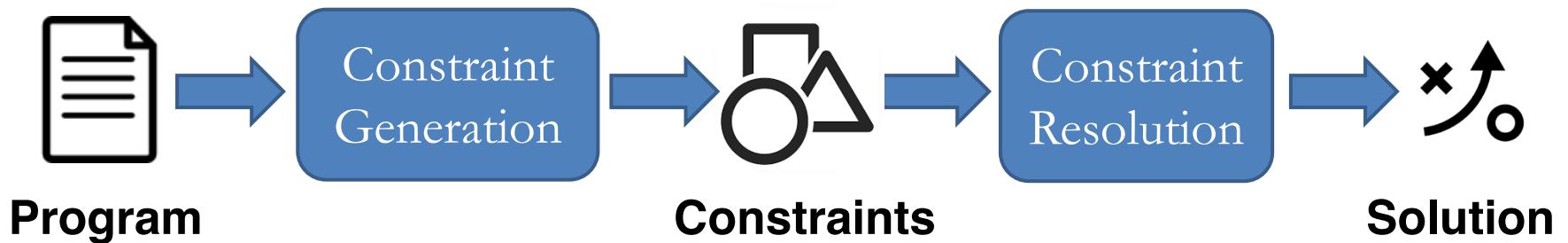
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University of Kent

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Microsoft Research  
Hongseok Yang  
University of Oxford

# Constraint-Based Software Analysis



- ▶ Long history
  - ▶ Type constraints, set constraints, SAT/SMT constraints
- ▶ Many benefits
  - ▶ Separates analysis specification from implementation
  - ▶ Allows to leverage sophisticated off-the-shelf solvers
  - ▶ Yields natural program specifications
  - ▶ ...

# Challenges in Software Analysis

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But constraint-based approach is not well-suited for ...

- ▶ Balancing trade-offs
  - ▶ e.g. precision vs. scalability
- ▶ Handling uncertainty
  - ▶ e.g. incorrect specifications
- ▶ Modeling missing information
  - ▶ e.g. incomplete programs

# An Emerging Approach

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Constraint **Satisfaction** ➡ Constraint **Optimization**

- ▶ Balancing trade-offs
  - ▶ e.g. precision vs. scalability
- ▶ Handling uncertainty
  - ▶ e.g. incorrect specifications
- ▶ Modeling missing information
  - ▶ e.g. incomplete programs



**Objectives**

# The Maximum Satisfiability Problem

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SAT:

$a \wedge$	(C1)
$\neg a \vee b \wedge$	(C2)
$\neg b \vee c \wedge$	(C3)
$\neg c \vee d \wedge$	(C4)
$\neg d$	(C5)

# The Maximum Satisfiability Problem

## MaxSAT:

	$a \wedge$	(C1)
	$\neg a \vee b \wedge$	(C2)
4:	$\neg b \vee c \wedge$	(C3)
2:	$\neg c \vee d \wedge$	(C4)
7:	$\neg d$	(C5)

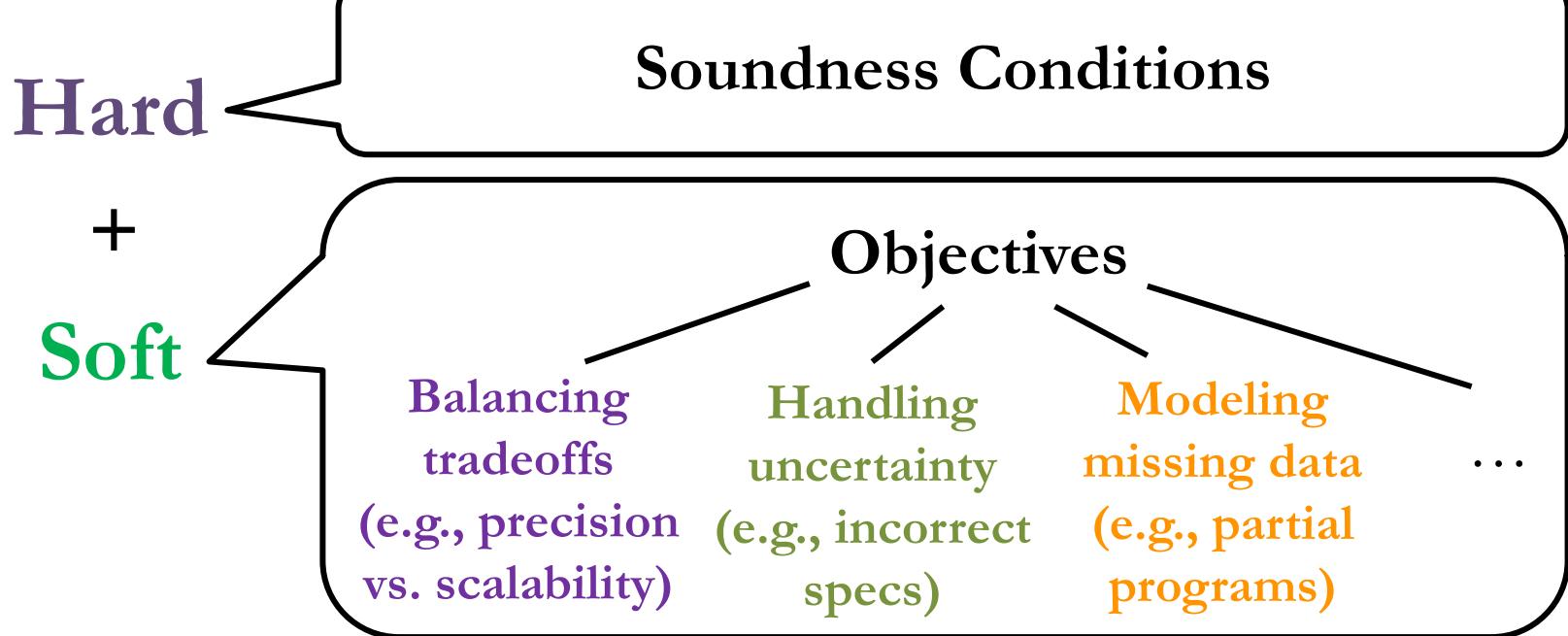
=

Subject to	C1
	C2
Maximize	$4 \times C3 + 2 \times C4 + 7 \times C5$

**Solution:**  $a = \text{true}$ ,  $b = \text{true}$ ,  $c = \text{true}$ ,  $d = \text{false}$   
**(Objective = 11)**

# The Maximum Satisfiability Problem

+ Expressive  
for our problems

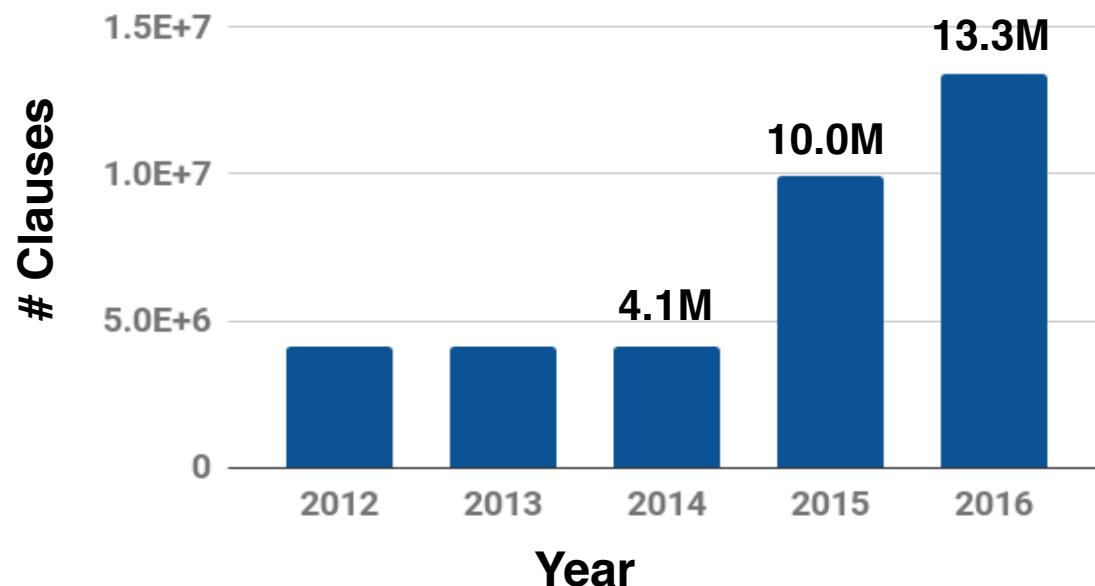


# The Maximum Satisfiability Problem

+ Expressive  
for our problems

+ Efficient  
(and improving) solvers

Largest instance solved in MaxSAT competition:



# The Maximum Satisfiability Problem

+ Expressive  
for our problems

+ Efficient  
(and improving) solvers

$$\begin{array}{l} \forall x. \text{path}(x, x) \\ \wedge \quad \forall x, y, z. \text{path}(x, y) \wedge \text{edge}(y, z) \Rightarrow \text{path}(x, z) \\ \wedge \quad \textbf{1.5: } \forall x, y. \neg \text{path}(x, y) \end{array}$$

- Cannot concisely  
specify our problems  
(lacks quantifiers)

- Loses high-level  
structure that solvers  
could exploit

# The Maximum Satisfiability Problem

+ Expressive

+ Efficient

**How to overcome limitations of MaxSAT  
while retaining its benefits?**

$$\begin{aligned} & \forall x. \text{path}(x, x) \\ \wedge \quad & \forall x, y, z. \text{path}(x, y) \wedge \text{edge}(y, z) \Rightarrow \text{path}(x, z) \\ \wedge \quad & \textbf{1.5: } \forall x, y. \neg \text{path}(x, y) \end{aligned}$$

**A solution: Markov Logic Network (MLN)  
[Richardson & Domingos, 2006]**

specify our problems  
(lacks quantifiers)

structure that solvers  
could exploit

# Markov Logic Network: Syntax

---

(constraints)	$C ::= (H, S)$
(hard constraints)	$H ::= \{ h_1, \dots, h_n \}, \quad h ::= \bigwedge_{i=1}^n t_i \Rightarrow \bigvee_{i=1}^m t_i'$
(soft constraints)	$S ::= \{ s_1, \dots, s_n \}, \quad s ::= w : h$
(fact)	$t ::= r(a_1, \dots, a_n)$
(argument)	$a ::= v \mid c$
(ground fact)	$g ::= r(c_1, \dots, c_n)$
(input, output)	$P, Q \subseteq \mathbf{G}$

## Example:

$$\begin{aligned} & \forall x. \text{path}(x, x) \\ \wedge \quad & \forall x, y, z. \text{path}(x, y) \wedge \text{edge}(y, z) \Rightarrow \text{path}(x, z) \\ \wedge \quad & \textcolor{red}{1.5}: \forall x, y. \neg \text{path}(x, y) \end{aligned}$$

# Datalog-like Notation

## Input relations:

$\text{edge}(x, y)$

## Hard constraints:

$\text{path}(x, x).$

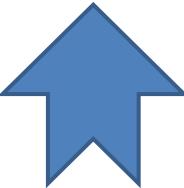
$\text{path}(x, z) :- \text{path}(x, y), \text{edge}(y, z).$

## Output relations:

$\text{path}(x, y)$

## Soft constraints:

$\neg \text{path}(x, y).$  weight 1.5



## Example:

$\forall x. \text{path}(x, x)$

$\wedge$

$\forall x, y, z. \text{path}(x, y) \wedge \text{edge}(y, z) \Rightarrow \text{path}(x, z)$

$\wedge$

1.5:  $\forall x, y. \neg \text{path}(x, y)$

# Markov Logic Network: Grounding

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(valuation)  $\sigma \in V \rightarrow C$

$$\begin{aligned}\llbracket (H, S) \rrbracket &= (\llbracket H \rrbracket, \llbracket S \rrbracket) \\ \llbracket \{ h_1, \dots, h_n \} \rrbracket &= \bigwedge_{i=1}^n \llbracket h_i \rrbracket \\ \llbracket \{ s_1, \dots, s_n \} \rrbracket &= \bigwedge_{i=1}^n \llbracket s_i \rrbracket \\ \llbracket h \rrbracket &= \bigwedge_{\sigma} \llbracket h \rrbracket_{\sigma} \\ \llbracket w : h \rrbracket &= \bigwedge_{\sigma} (w, \llbracket h \rrbracket_{\sigma}) \\ \llbracket \bigwedge_{i=1}^n t_i \Rightarrow \bigvee_{i=1}^m t_i' \rrbracket_{\sigma} &= \bigvee_{i=1}^n \neg \llbracket t_i \rrbracket_{\sigma} \vee \bigvee_{i=1}^m \llbracket t_i' \rrbracket_{\sigma} \\ \llbracket r(a_1, \dots, a_n) \rrbracket_{\sigma} &= r(\sigma(a_1), \dots, \sigma(a_n)) \\ \llbracket v \rrbracket_{\sigma} &= \sigma(v) \\ \llbracket c \rrbracket_{\sigma} &= c\end{aligned}$$

# Markov Logic Network: Semantics

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- ▶ Conceptually, two steps:
  - ▶ Step 1: Ground the MLN instance
    - ▶ Substitute quantifiers by all possible valuations to constants, to yield a MaxSAT instance
  - ▶ Step 2: Solve the MaxSAT instance
    - ▶ Using off-the-shelf MaxSAT solver
- ▶ Challenge: both steps intractable for our problems
  - ▶ MaxSAT instances can comprise upto  $10^{30}$  clauses!
- ▶ Solution: Iterative lazy refinement

# Markov Logic Network: Example

**Input relations:**

$\text{edge}(x, y)$

**Output relations:**

$\text{path}(x, y)$

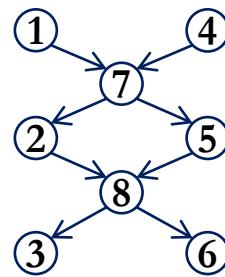
**Hard constraints:**

$\text{path}(x, x)$ .

$\text{path}(x, z) :- \text{path}(x, y), \text{edge}(y, z)$ .

**Soft constraints:**

$\neg \text{path}(x, y)$ . **weight 1.5**



Grounding



**Input:**

$\text{edge}(1, 7), \text{edge}(4, 7), \dots$

**Hard clauses:**

$\text{edge}(1, 7) \wedge \text{edge}(4, 7) \wedge \dots \wedge$   
 $\text{path}(1, 1) \wedge \text{path}(2, 2) \wedge \dots \wedge$

$(\text{path}(1, 1) \vee \neg \text{path}(1, 1) \vee \neg \text{edge}(1, 1)) \wedge$   
 $(\text{path}(1, 2) \vee \neg \text{path}(1, 1) \vee \neg \text{edge}(1, 2)) \wedge$   
 $(\text{path}(1, 2) \vee \neg \text{path}(1, 2) \vee \neg \text{edge}(2, 2)) \wedge$

...

**Soft clauses:**

$(\neg \text{path}(1, 1) \text{ weight } 1.5) \wedge$   
 $(\neg \text{path}(1, 2) \text{ weight } 1.5) \wedge$

...

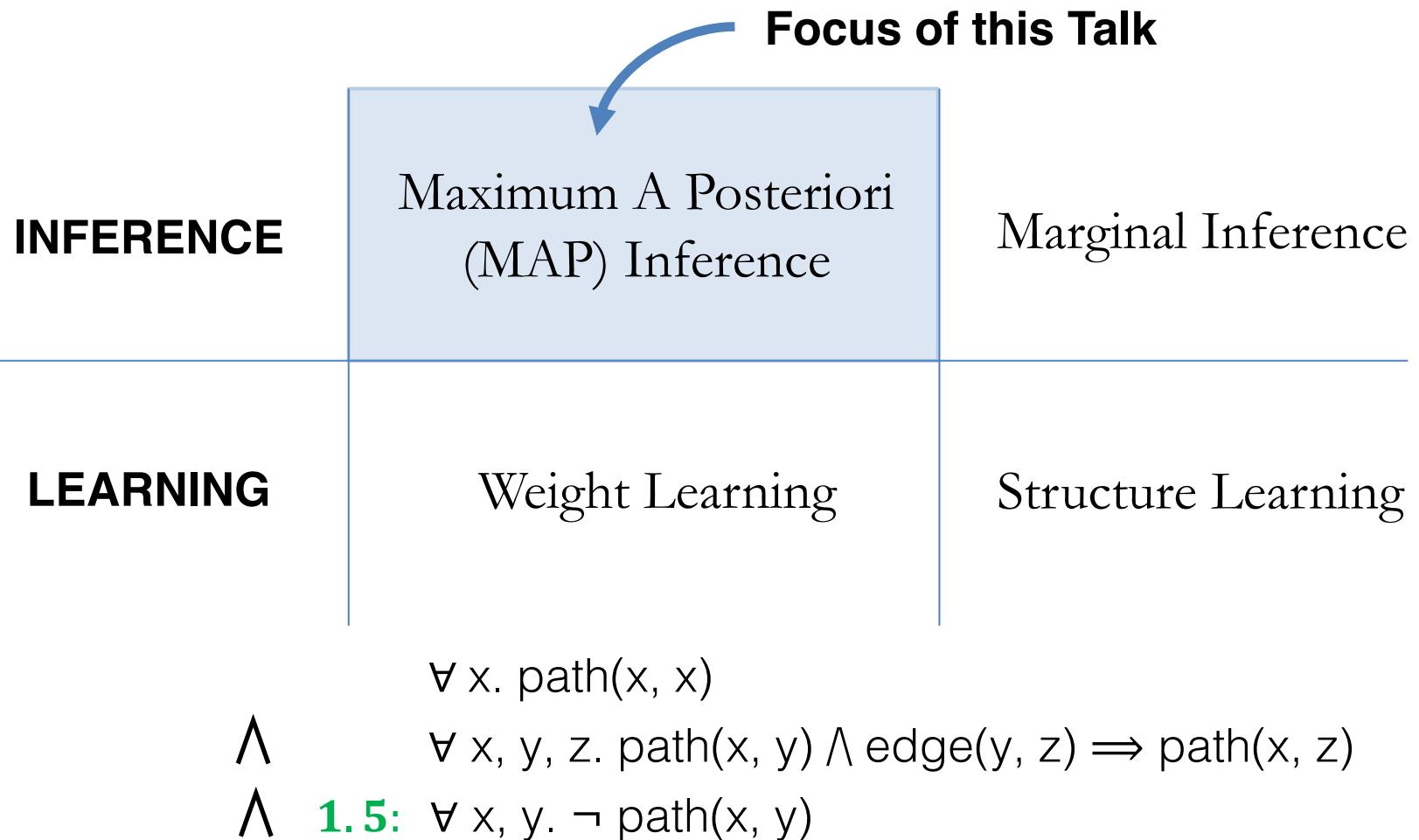
**Output:**

$\text{path}(1, 1) = T, \text{path}(2, 2) = T,$   
 $\text{path}(1, 2) = T, \text{path}(2, 1) = F,$   
...

Solving



# Landscape of Problems



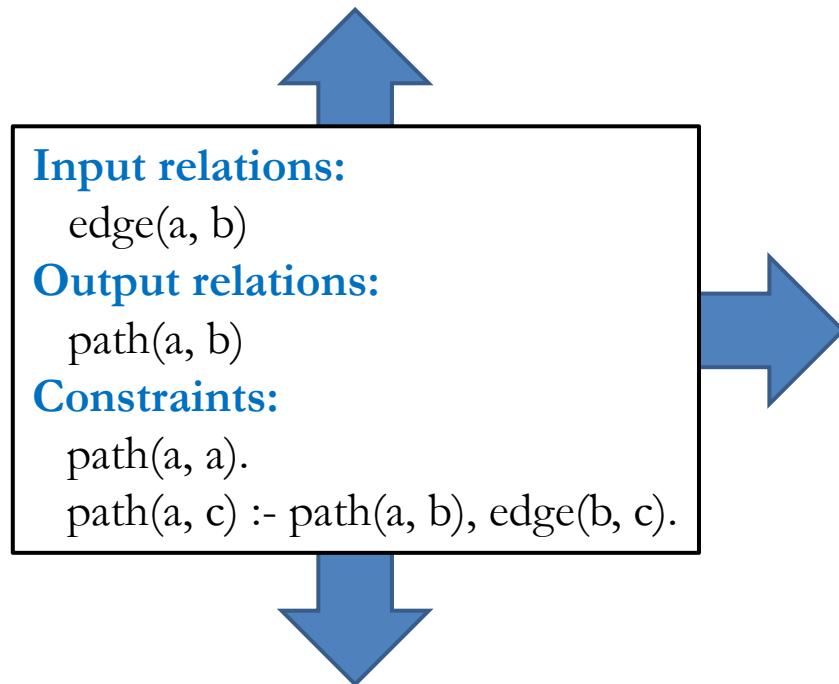
# Talk Outline

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- ▶ Background
- ▶ Part I: Applications in Software Analysis
- ▶ Part II: Techniques for MaxSAT Solving
- ▶ Conclusion

# Applications in Software Analysis

Abstraction Selection in Automated  
Verification [PLDI 2014]



Alarm Classification in Static  
Bug Detection [FSE 2015]

Alarm Resolution in Interactive Verification  
[OOPSLA 2017]

# Overview of Applications

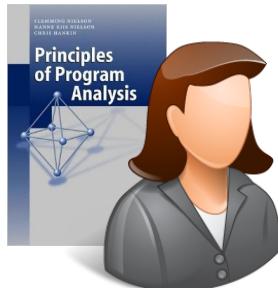
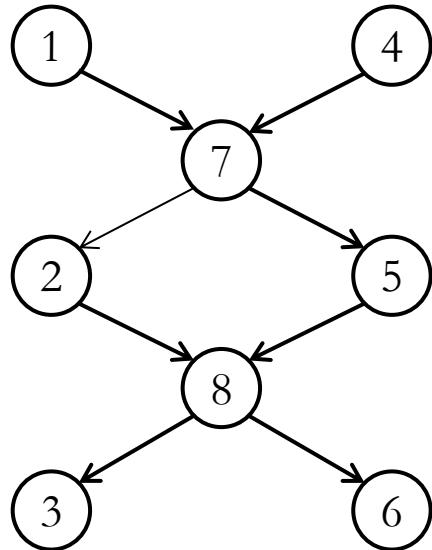
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- ▶ Abstraction Selection [PLDI 2014]
- ▶ Alarm Classification [FSE 2015]
- ▶ Alarm Resolution [OOSPLA 2017]

# An Example: Pointer Analysis

```
f () {                                g () {  
    v1 = new ...                      v4 = new ...  
    v2 = id1(v1)                      v5 = id1(v4)  
    v3 = id2(v2)                      v6 = id2(v5)  
    ✗ assert(v3 != v1) q1           ✓ assert(v6 != v1) q2  
}  
  
id1(v) { return v }                  id2(v) { return v }
```

# Pointer Analysis via Graph Reachability



Analysis Writer  
(Alice)

```
f () {  
    v1 = new ...  
    v2 = id1(v1)  
    v3 = id2(v2)  
    assert(v3 != v1) q1  
}  
id1(v) { return v }
```

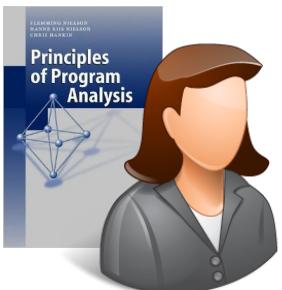
```
g () {  
    v4 = new ...  
    v5 = id1(v4)  
    v6 = id2(v5)  
    assert(v6 != v1) q2  
}  
id2(v) { return v }
```

**assert( $v6 \neq v1$ )** holds if there is no path from **1** to **6**.

**assert( $v3 \neq v1$ )** holds if there is no path from **1** to **3**.

# Analysis Specification in Datalog

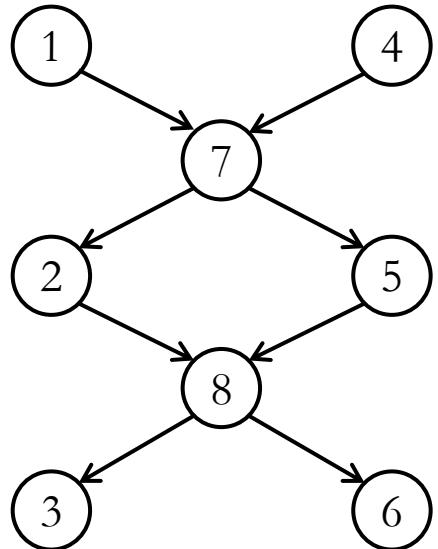
<b>Input relations:</b> edge(a, b)	<b>Output relations:</b> path(a, b)
<b>Constraints:</b> path(a, a). path(a, c) :- path(a, b), edge(b, c).	



**Analysis Writer  
(Alice)**

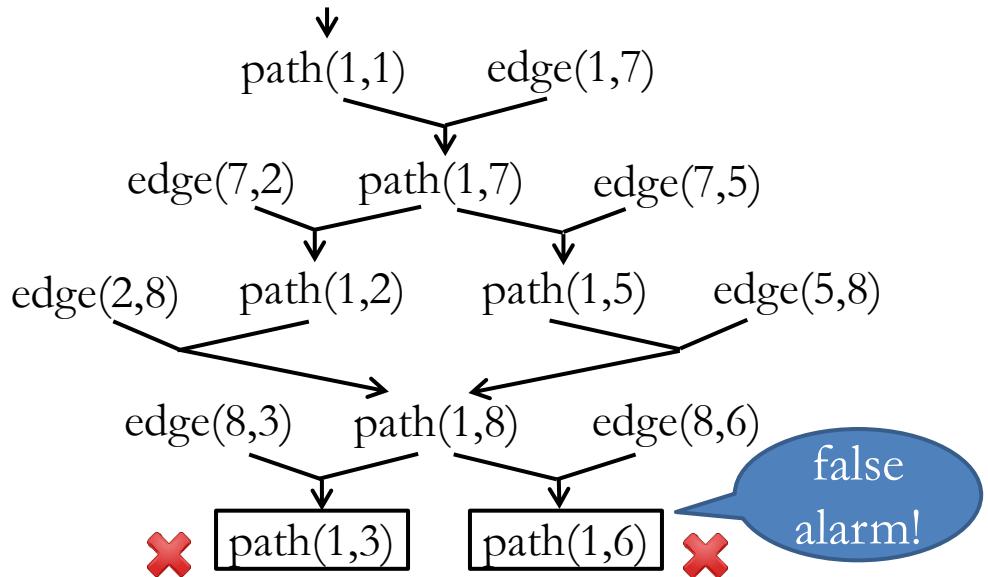
If  
there is a path from **a** to **b**, and  
there is an edge from **b** to **c**,  
then  
there is a path from **a** to **c**.

# Analysis Evaluation in Datalog

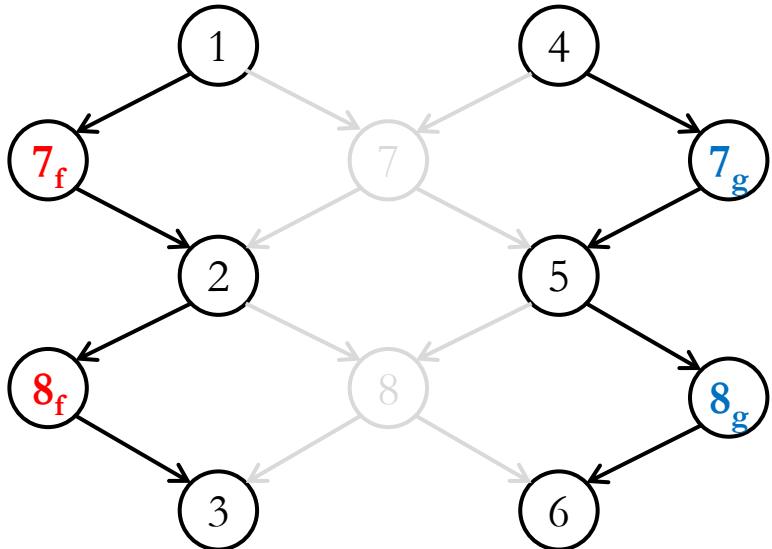


Analysis User  
(Bob)

```
f () {  
    v1 = new ...  
    v2 = id1(v1)  
    v3 = id2(v2)  
    assert(v3 != v1) q1  
}  
id1(v) { return v }  
  
g () {  
    v4 = new ...  
    v5 = id1(v4)  
    v6 = id2(v5)  
    assert(v6 != v1) q2  
}  
id2(v) { return v }
```

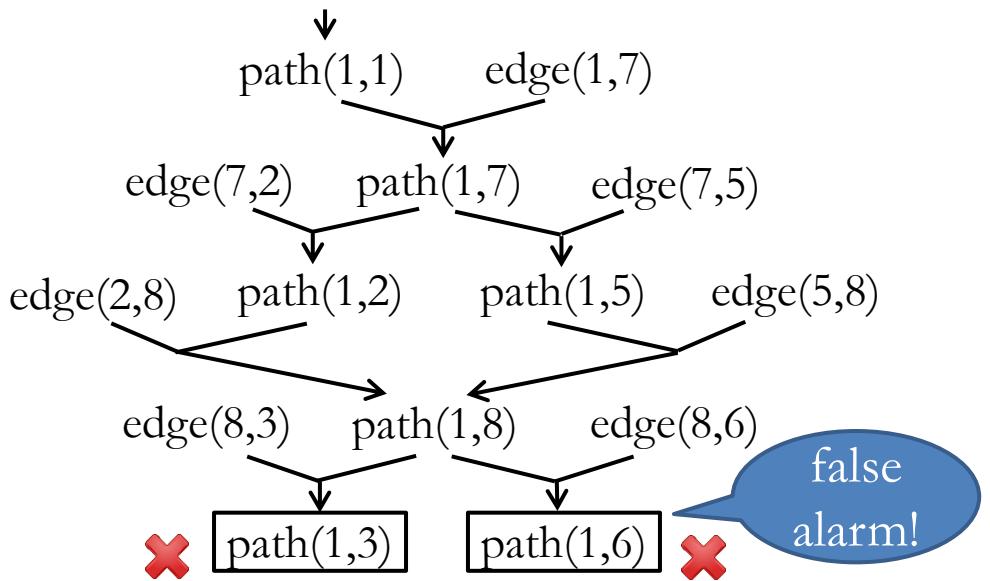


# A More Precise Abstraction

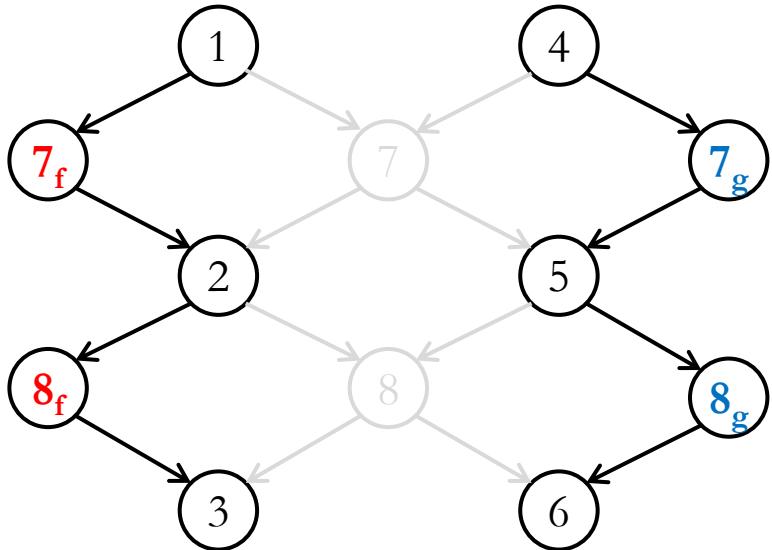


Analysis User  
(Bob)

```
f () {  
    v1 = new ...  
    v2 = id1(v1)  
    v3 = id2(v2)  
    assert(v3 != v1) q1  
}  
id1(v) { return v }  
  
g () {  
    v4 = new ...  
    v5 = id1(v4)  
    v6 = id2(v5)  
    assert(v6 != v1) q2  
}  
id2(v) { return v }
```

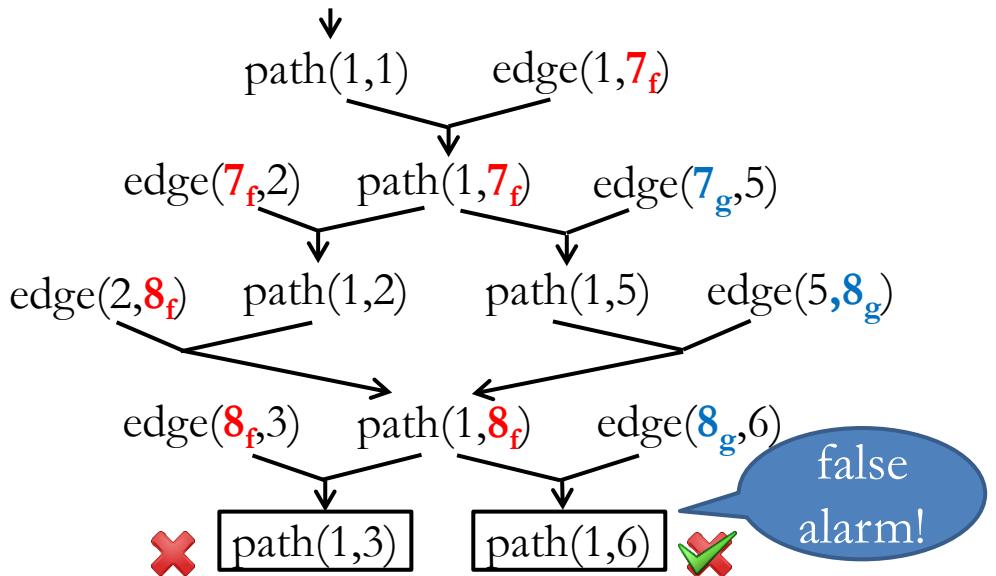


# A More Precise Abstraction

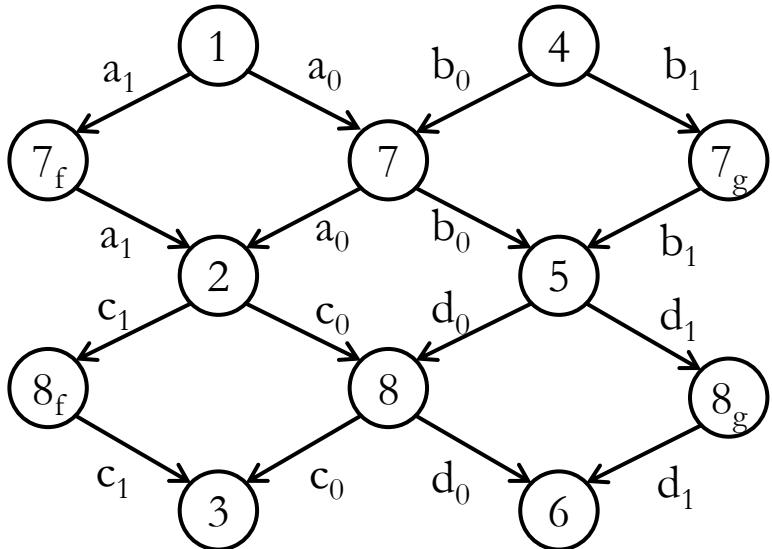


Analysis User  
(Bob)

```
f () {  
    v1 = new ...  
    v2 = id1(v1)  
    v3 = id2(v2)  
    assert(v3 != v1) q1  
}  
id1(v) { return v }  
  
g () {  
    v4 = new ...  
    v5 = id1(v4)  
    v6 = id2(v5)  
    assert(v6 != v1) q2  
}  
id2(v) { return v }
```



# Abstraction Refinement



**Input relations:**

$\text{edge}(a, b)$ ,  $\text{abs}(n)$

**Output relations:**

$\text{path}(a, b)$

**Constraints:**

$\text{path}(a, a).$

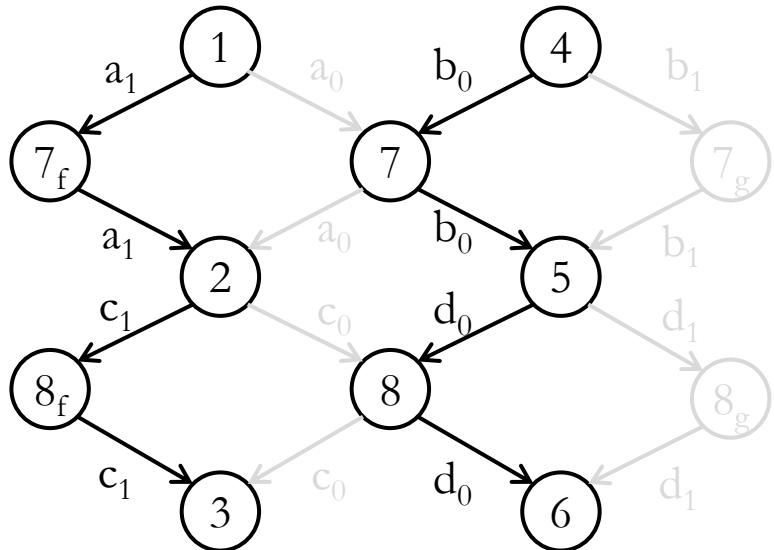
$\text{path}(a, c) :- \text{path}(a, b), \text{edge}(b, c), \text{abs}(n).$

$\text{abs}(a_0) \oplus \text{abs}(a_1), \text{abs}(b_0) \oplus \text{abs}(b_1),$   
 $\text{abs}(c_0) \oplus \text{abs}(c_1), \text{abs}(d_0) \oplus \text{abs}(d_1).$

16 possible abstractions in total

Query Tuple	Original Query
$q_1: \text{path}(1, 3)$	<b>assert</b> ( $v3 \neq v1$ )
$q_2: \text{path}(1, 6)$	<b>assert</b> ( $v6 \neq v1$ )

# Desired Result



**Input relations:**

$\text{edge}(a, b), \text{abs}(n)$

**Output relations:**

$\text{path}(a, b)$

**Constraints:**

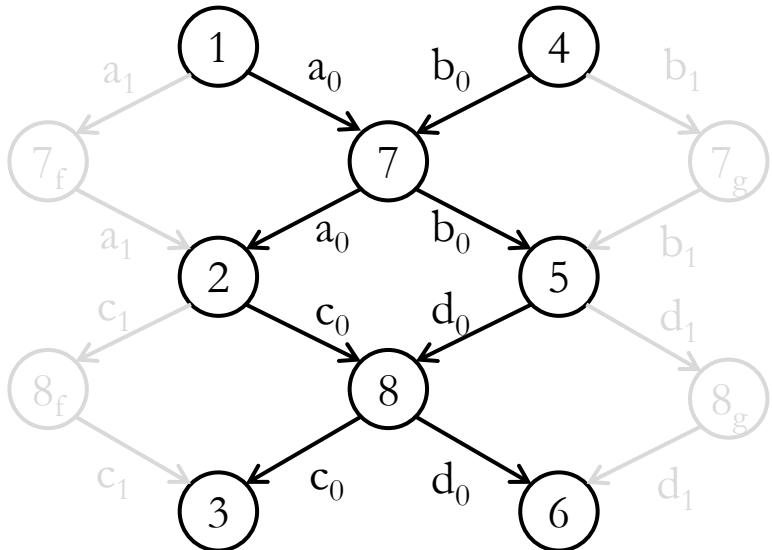
$\text{path}(a, a).$

$\text{path}(a, c) :- \text{path}(a, b), \text{edge}(b, c, n), \text{abs}(n).$

$\text{abs}(a_0) \oplus \text{abs}(a_1), \text{abs}(b_0) \oplus \text{abs}(b_1),$   
 $\text{abs}(c_0) \oplus \text{abs}(c_1), \text{abs}(d_0) \oplus \text{abs}(d_1).$

Query	Answer
$q_1: \text{path}(1, 3)$	✗ Impossibility
$q_2: \text{path}(1, 6)$	✓ $a_1 b_0 c_1 d_0$

# Iteration 1



**Input relations:**

$\text{edge}(a, b), \text{abs}(n)$

**Output relations:**

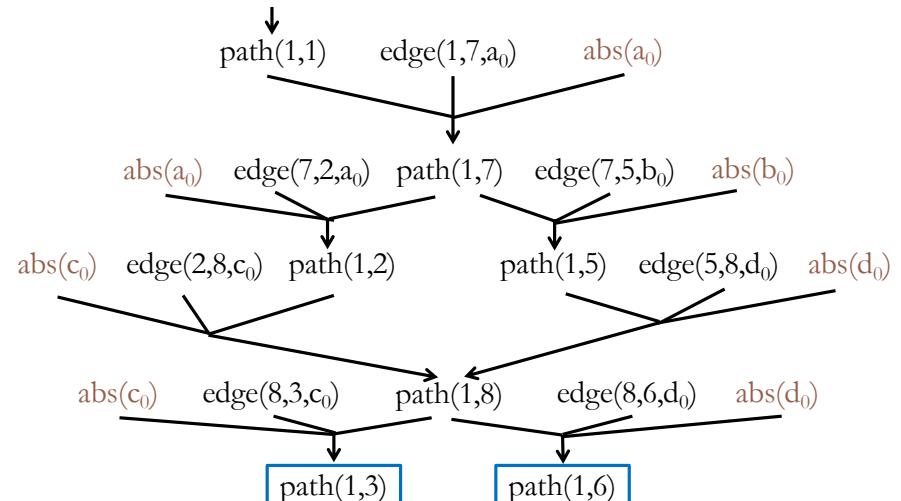
$\text{path}(a, b)$

**Constraints:**

$\text{path}(a, a).$

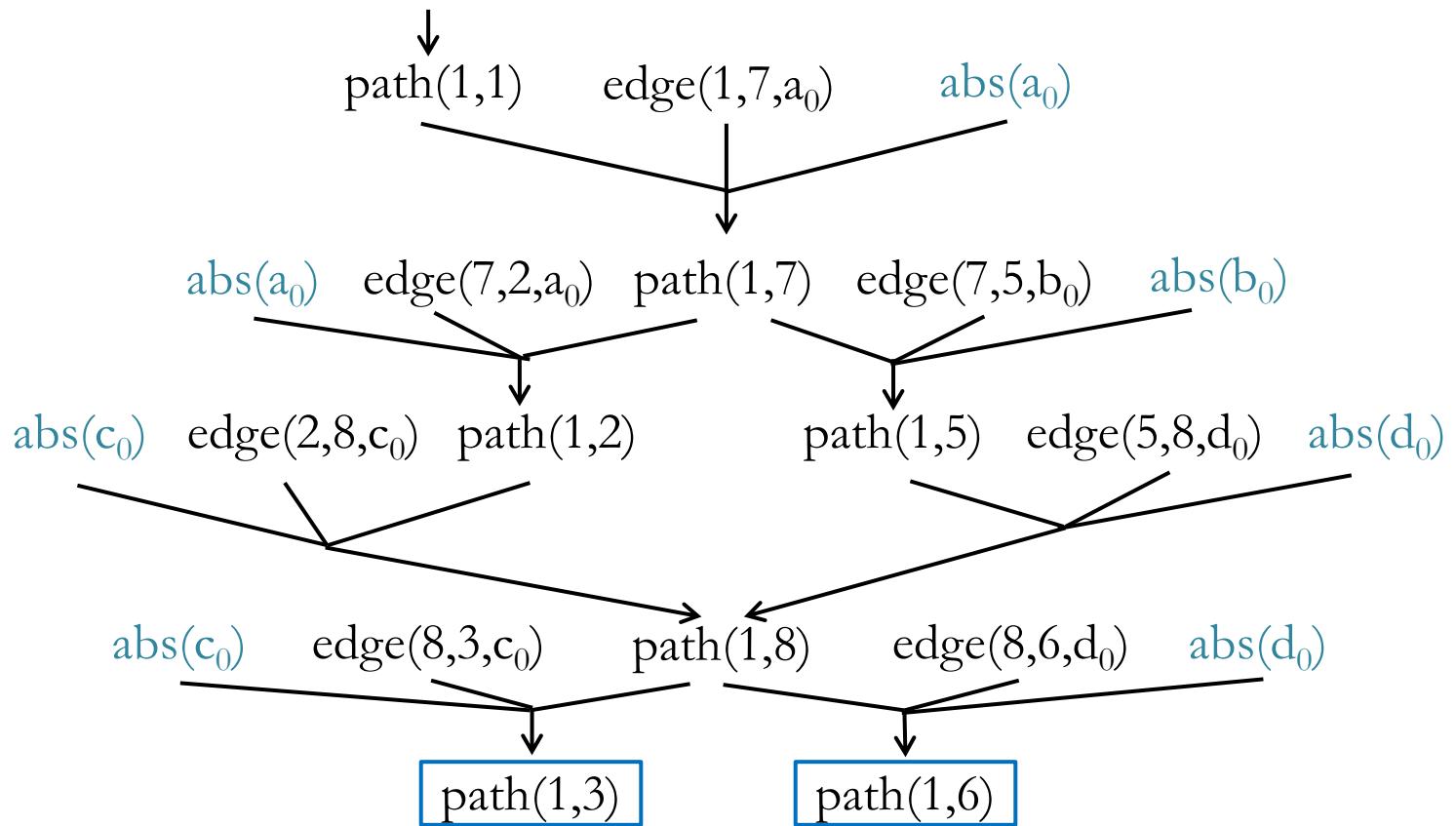
$\text{path}(a, c) :- \text{path}(a, b), \text{edge}(b, c, n), \text{abs}(n).$

$\text{abs}(a_0) \oplus \text{abs}(a_1), \text{abs}(b_0) \oplus \text{abs}(b_1),$   
 $\text{abs}(c_0) \oplus \text{abs}(c_1), \text{abs}(d_0) \oplus \text{abs}(d_1).$

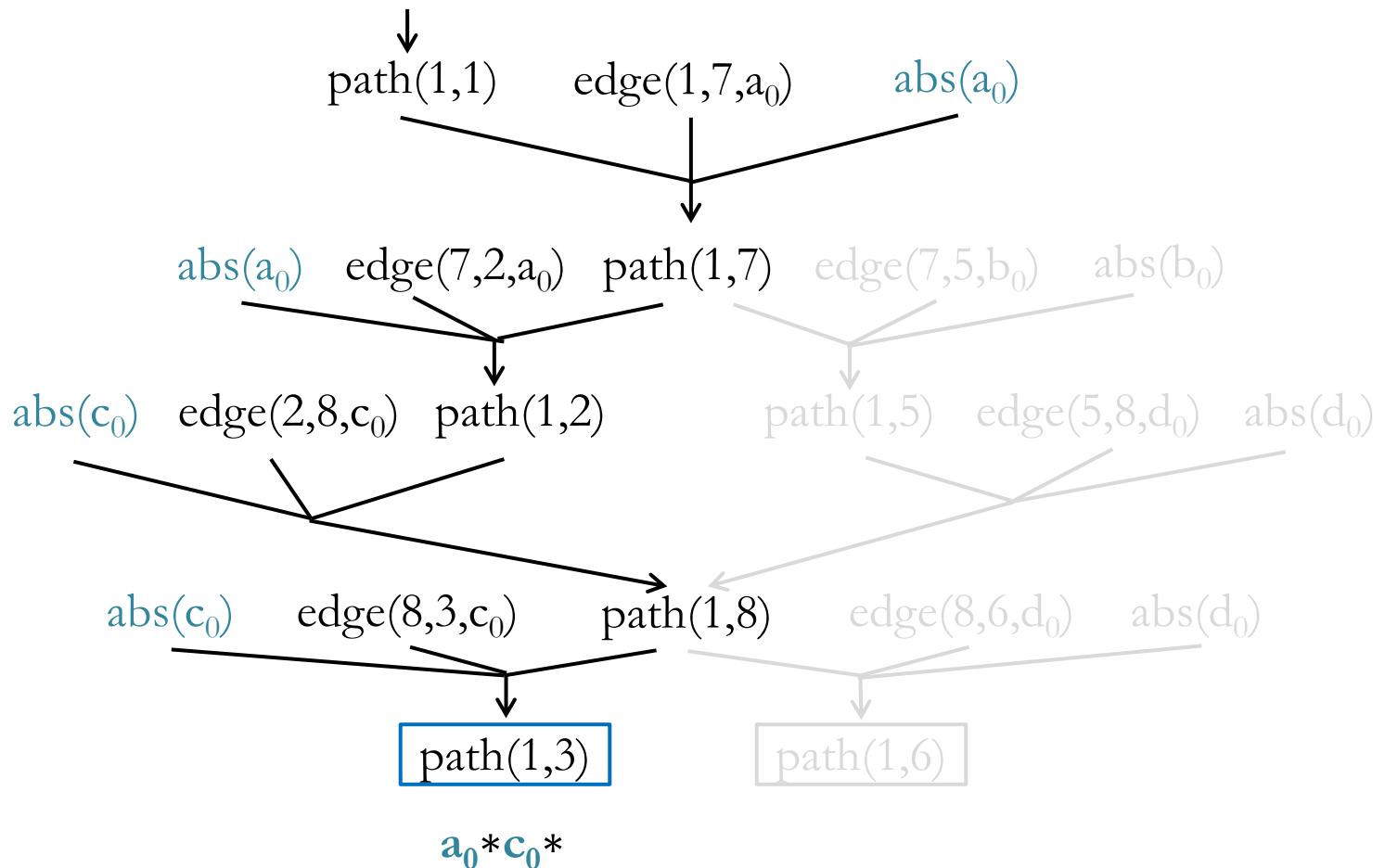


Query	Eliminated Abstractions
$q_1: \text{path}(1, 3)$	
$q_2: \text{path}(1, 6)$	

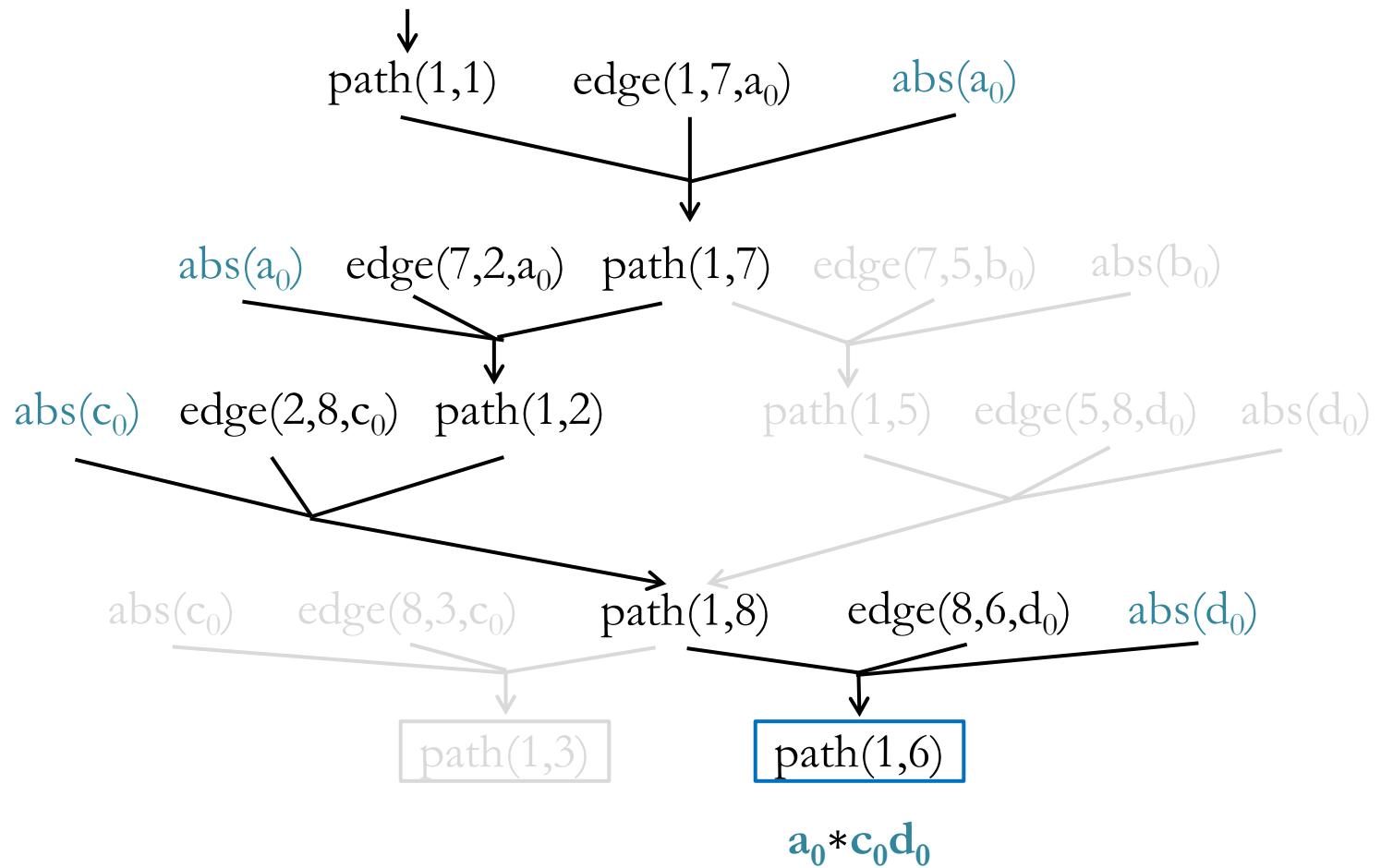
# Iteration 1 - Derivation Graph



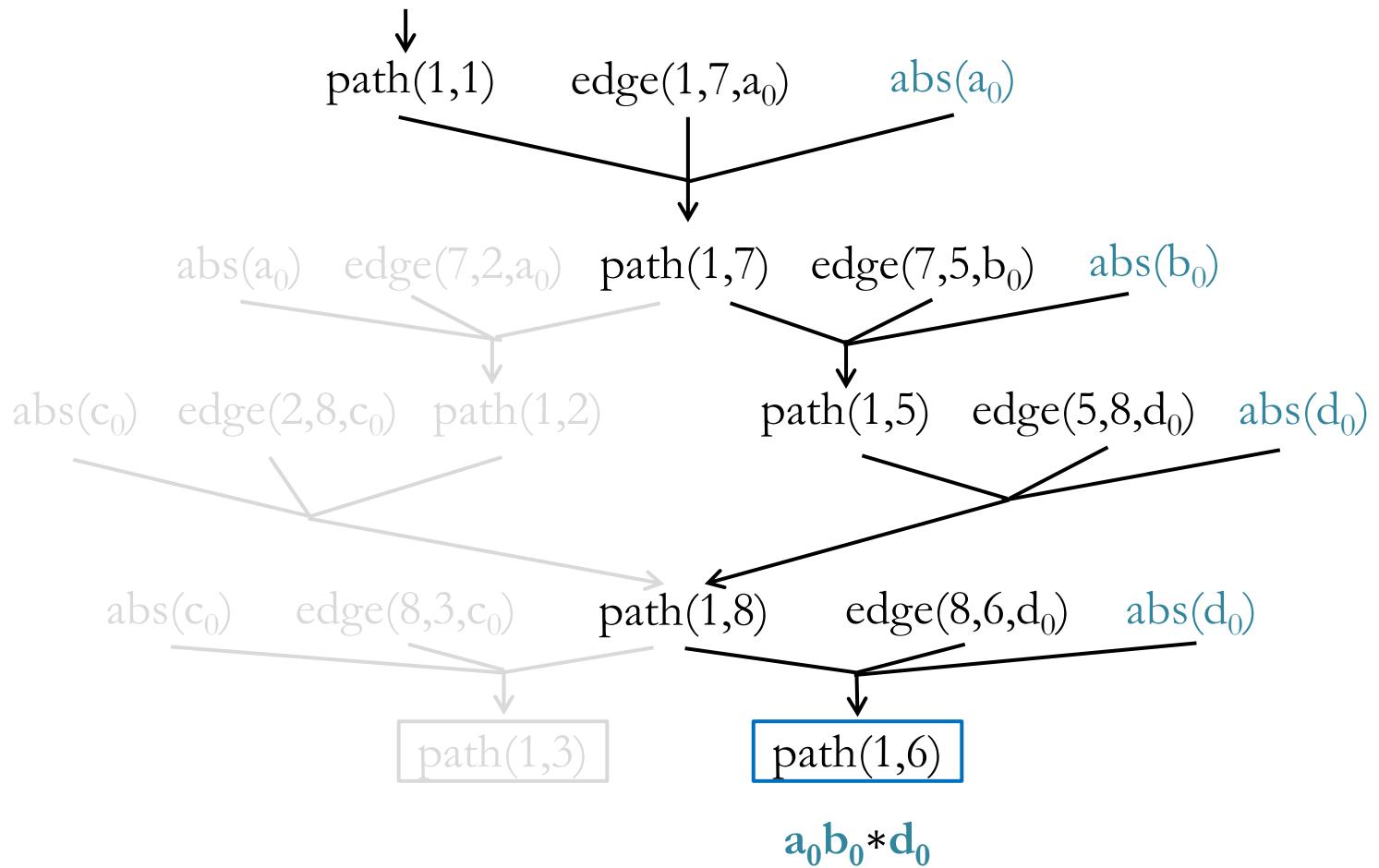
# Iteration 1 - Derivation Graph



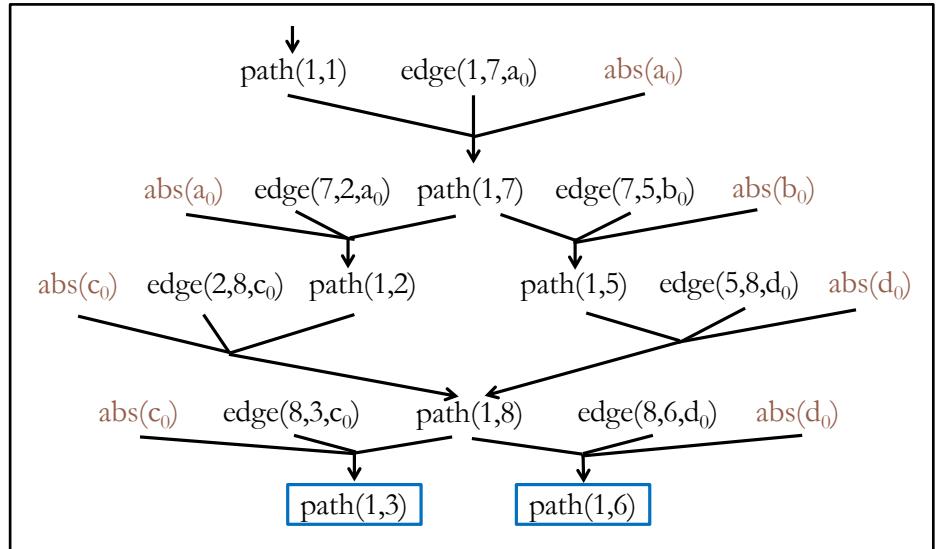
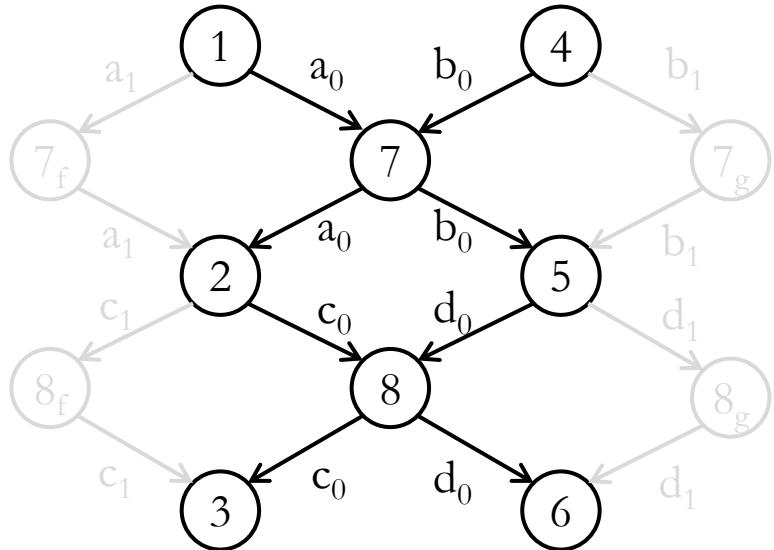
# Iteration 1 - Derivation Graph



# Iteration 1 - Derivation Graph



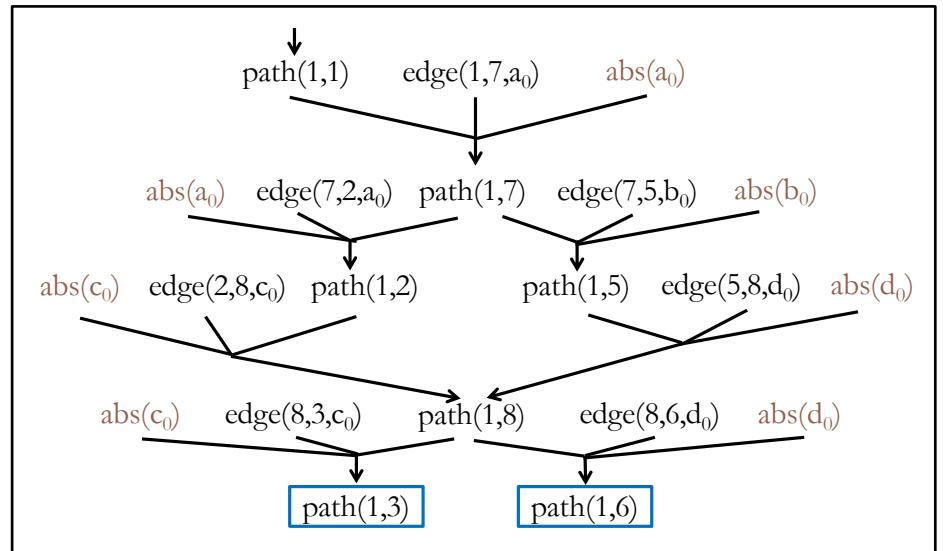
# Iteration 1 - Derivation Graph



Query	Eliminated Abstractions
q <sub>1</sub> : path(1, 3)	a <sub>0</sub> *c <sub>0</sub> * (4/16)
q <sub>2</sub> : path(1, 6)	a <sub>0</sub> *c <sub>0</sub> d <sub>0</sub> , a <sub>0</sub> b <sub>0</sub> *d <sub>0</sub> (3/16)

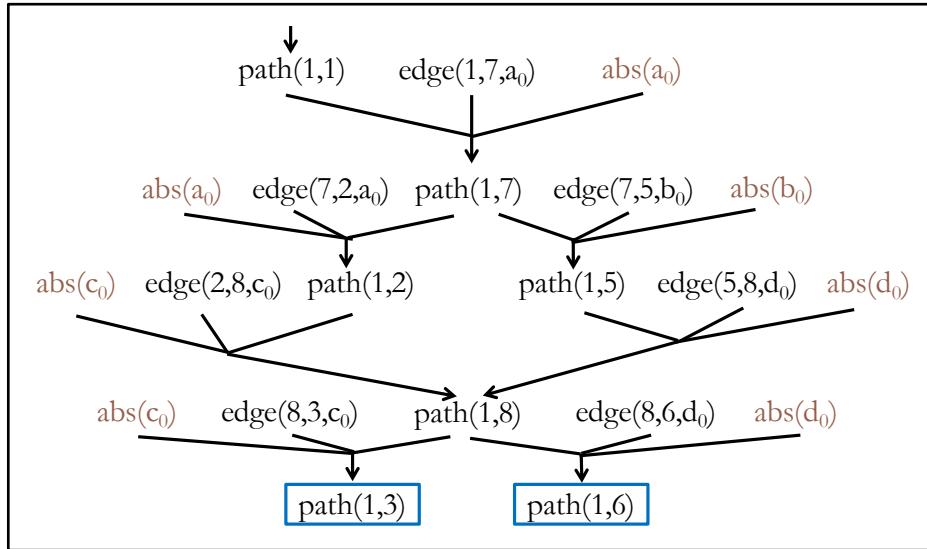
abs(a<sub>0</sub>)⊕abs(a<sub>1</sub>), abs(b<sub>0</sub>)⊕abs(b<sub>1</sub>),  
 abs(c<sub>0</sub>)⊕abs(c<sub>1</sub>), abs(d<sub>0</sub>)⊕abs(d<sub>1</sub>).

# Encoding in MaxSAT



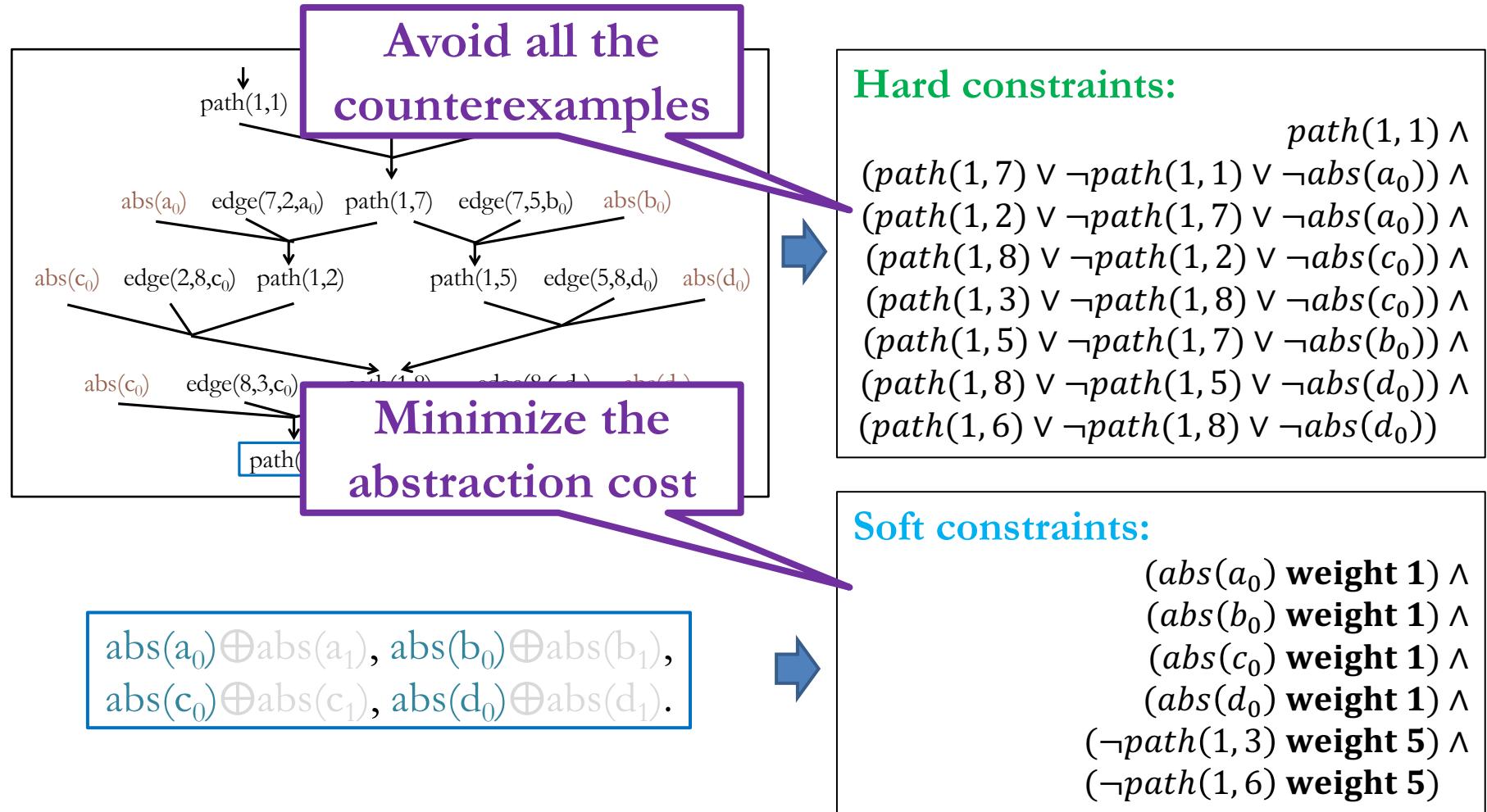
$\text{abs}(a_0) \oplus \text{abs}(a_1), \text{abs}(b_0) \oplus \text{abs}(b_1),$   
 $\text{abs}(c_0) \oplus \text{abs}(c_1), \text{abs}(d_0) \oplus \text{abs}(d_1).$

# Encoding in MaxSAT



$\text{abs}(a_0) \oplus \text{abs}(a_1)$ ,  $\text{abs}(b_0) \oplus \text{abs}(b_1)$ ,  
 $\text{abs}(c_0) \oplus \text{abs}(c_1)$ ,  $\text{abs}(d_0) \oplus \text{abs}(d_1)$ .

# Encoding in MaxSAT



# Encoding in MaxSAT

## Solution:

$path(1, 1) = \text{true}$ ,  $path(1, 2) = \text{false}$ ,  
 $path(1, 3) = \text{false}$ ,  $path(1, 5) = \text{false}$ ,  
 $path(1, 6) = \text{false}$ ,  $path(1, 7) = \text{false}$ ,  
 $path(1, 8) = \text{false}$ ,

$abs(a_0) = \text{false}$ ,  $abs(b_0) = \text{true}$ ,  
 $abs(c_0) = \text{true}$ ,  $abs(d_0) = \text{true}$ .



$a_1 b_0 c_0 d_0$

## Hard constraints:

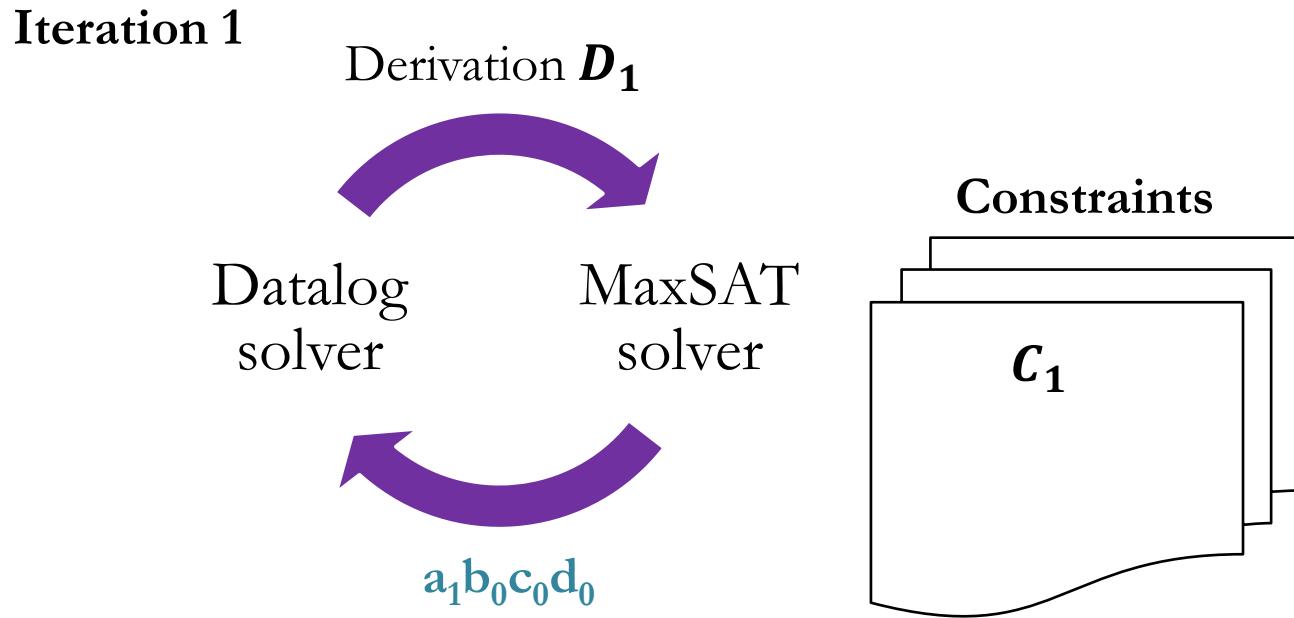
$path(1, 1) \wedge$   
 $(path(1, 7) \vee \neg path(1, 1) \vee \neg abs(a_0)) \wedge$   
 $(path(1, 2) \vee \neg path(1, 7) \vee \neg abs(a_0)) \wedge$   
 $(path(1, 8) \vee \neg path(1, 2) \vee \neg abs(c_0)) \wedge$   
 $(path(1, 3) \vee \neg path(1, 8) \vee \neg abs(c_0)) \wedge$   
 $(path(1, 5) \vee \neg path(1, 7) \vee \neg abs(b_0)) \wedge$   
 $(path(1, 8) \vee \neg path(1, 5) \vee \neg abs(d_0)) \wedge$   
 $(path(1, 6) \vee \neg path(1, 8) \vee \neg abs(d_0))$

## Soft constraints:

Query	Eliminated Abstractions
$q_1: path(1, 3)$	$a_0^* c_0^*$ (4/16)
$q_2: path(1, 6)$	$a_0^* c_0 d_0$ , $a_0 b_0^* d_0$ (3/16)

$(abs(a_0) \text{ weight } 1) \wedge$   
 $(abs(b_0) \text{ weight } 1) \wedge$   
 $(abs(c_0) \text{ weight } 1) \wedge$   
 $(abs(d_0) \text{ weight } 1) \wedge$   
 $(\neg path(1, 3) \text{ weight } 5) \wedge$   
 $(\neg path(1, 6) \text{ weight } 5)$

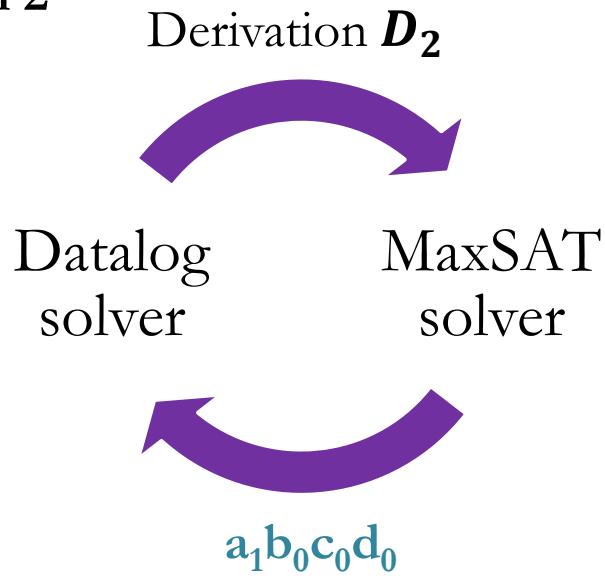
# Iteration 2 and Beyond



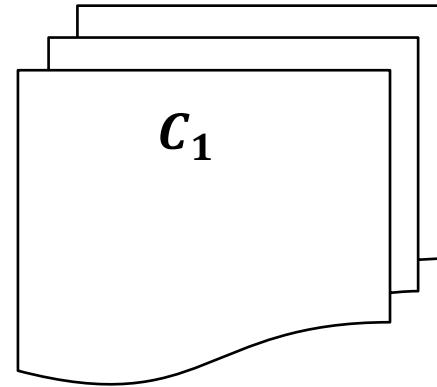
Query	Answer	Eliminated Abstractions
$q_1$ : path(1, 3)		$a_0^* c_0^*$ (4/16)
$q_2$ : path(1, 6)		$a_0^* c_0 d_0, a_0 b_0^* d_0$ (3/16)

# Iteration 2 and Beyond

Iteration 2



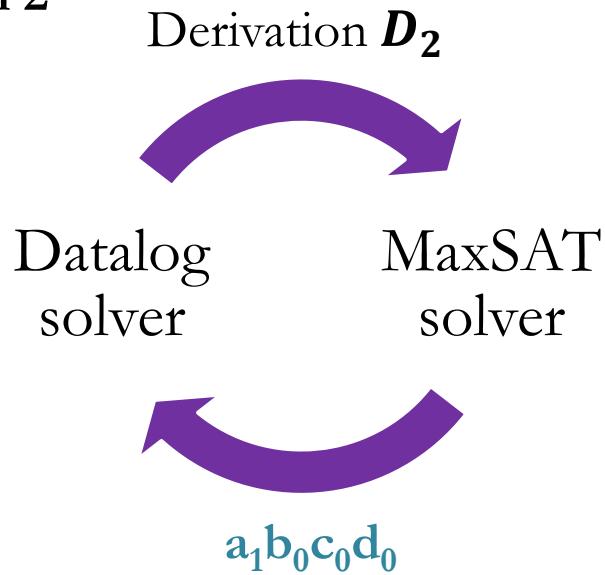
Constraints



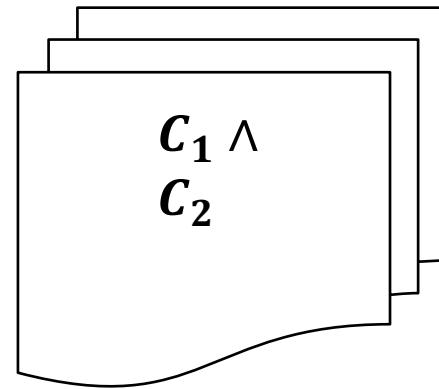
Query	Answer	Eliminated Abstractions
$q_1$ : path(1, 3)		$a_0^* c_0^*$ <span style="float: right;">(4/16)</span>
$q_2$ : path(1, 6)		$a_0^* c_0 d_0, a_0 b_0^* d_0$ <span style="float: right;">(3/16)</span>

# Iteration 2 and Beyond

## Iteration 2



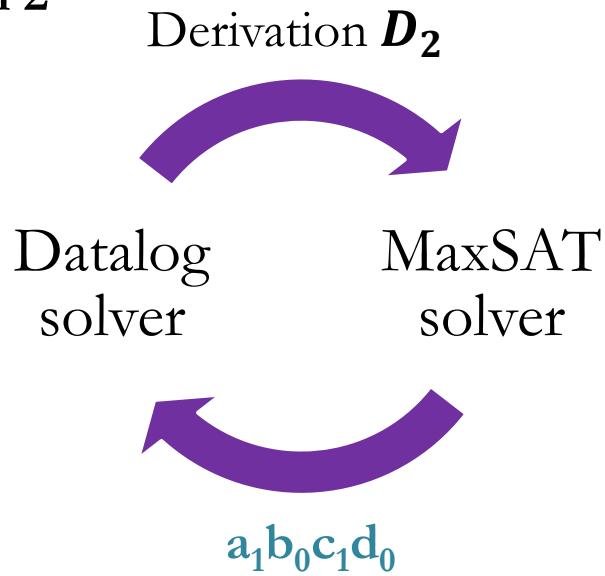
## Constraints



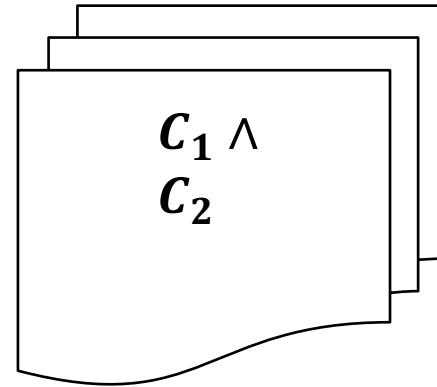
Query	Answer	Eliminated Abstractions
$q_1$ : path(1, 3)		$a_0^* c_0^*$ (4/16)
$q_2$ : path(1, 6)		$a_0^* c_0 d_0, a_0 b_0^* d_0$ (3/16)

# Iteration 2 and Beyond

## Iteration 2

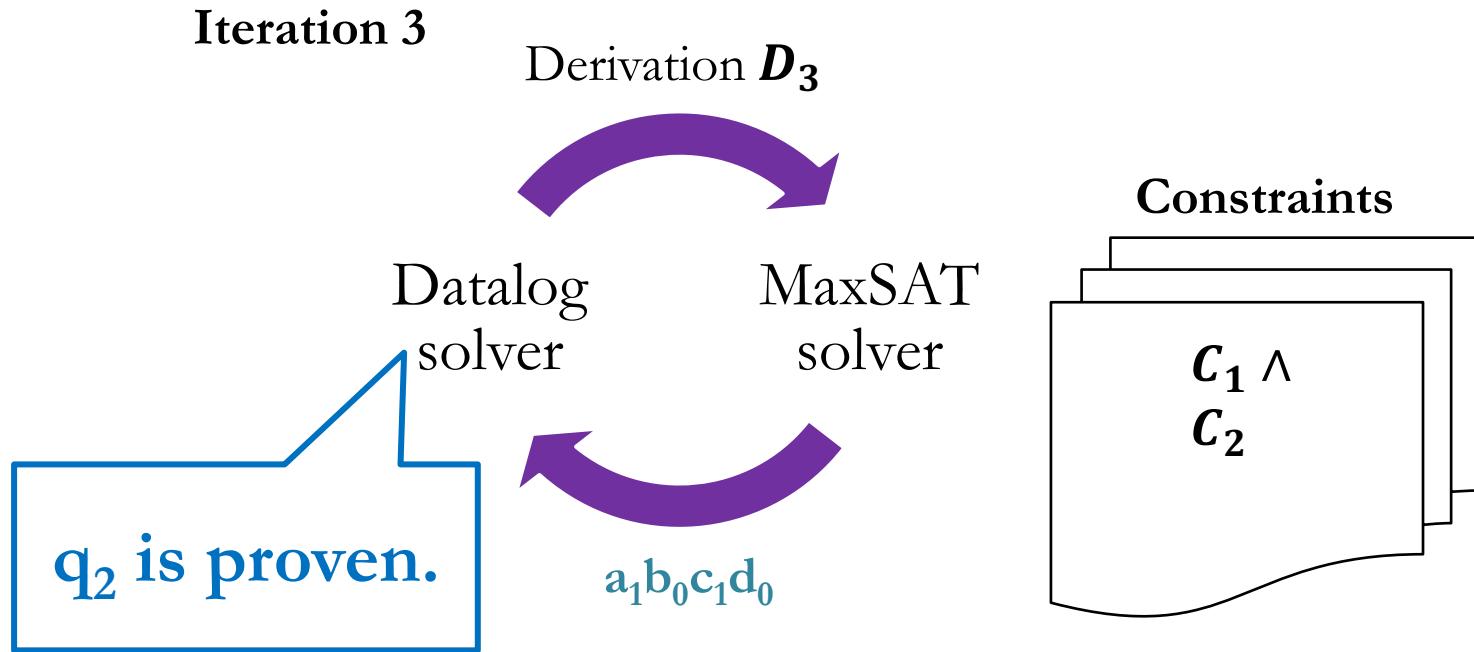


## Constraints



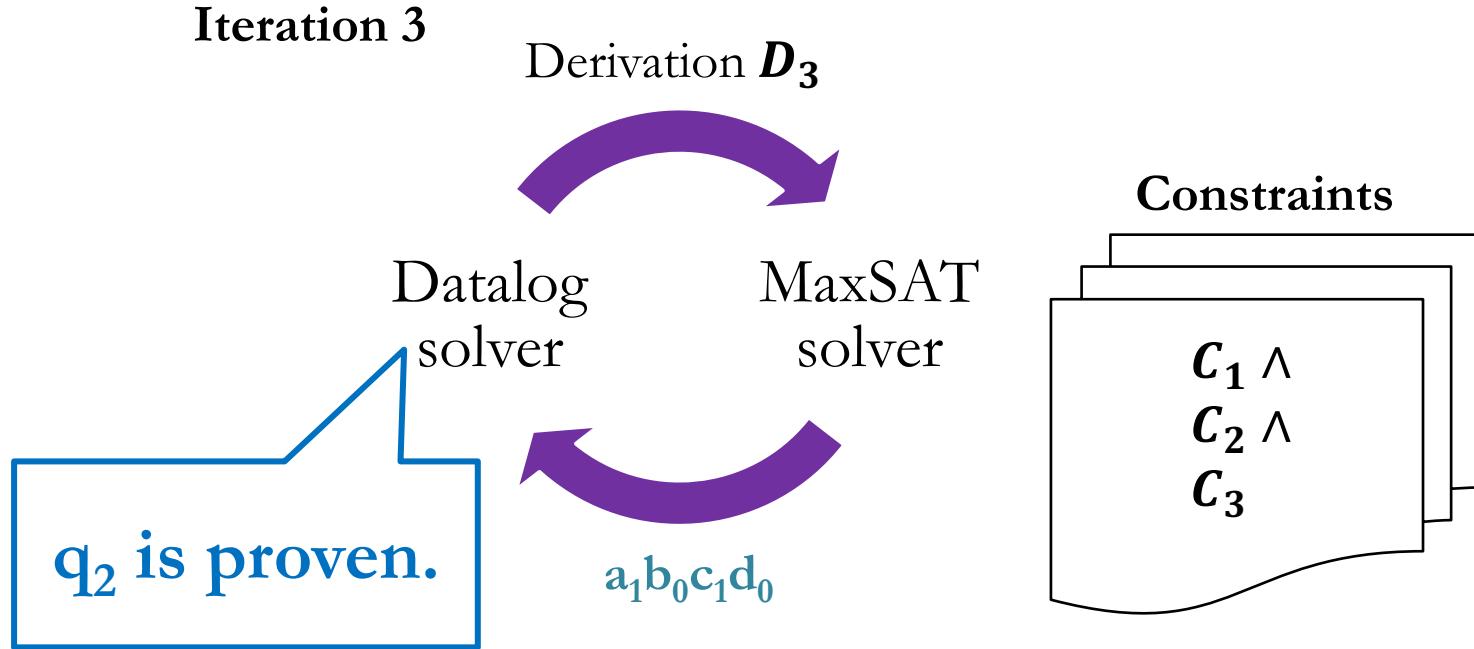
Query	Answer	Eliminated Abstractions
$q_1$ : path(1, 3)		$a_0^* c_0^*, a_1^* c_0^*$ (8/16)
$q_2$ : path(1, 6)		$a_0^* c_0 d_0, a_0 b_0^* d_0, a_1^* c_0 d_0$ (5/16)

# Iteration 2 and Beyond



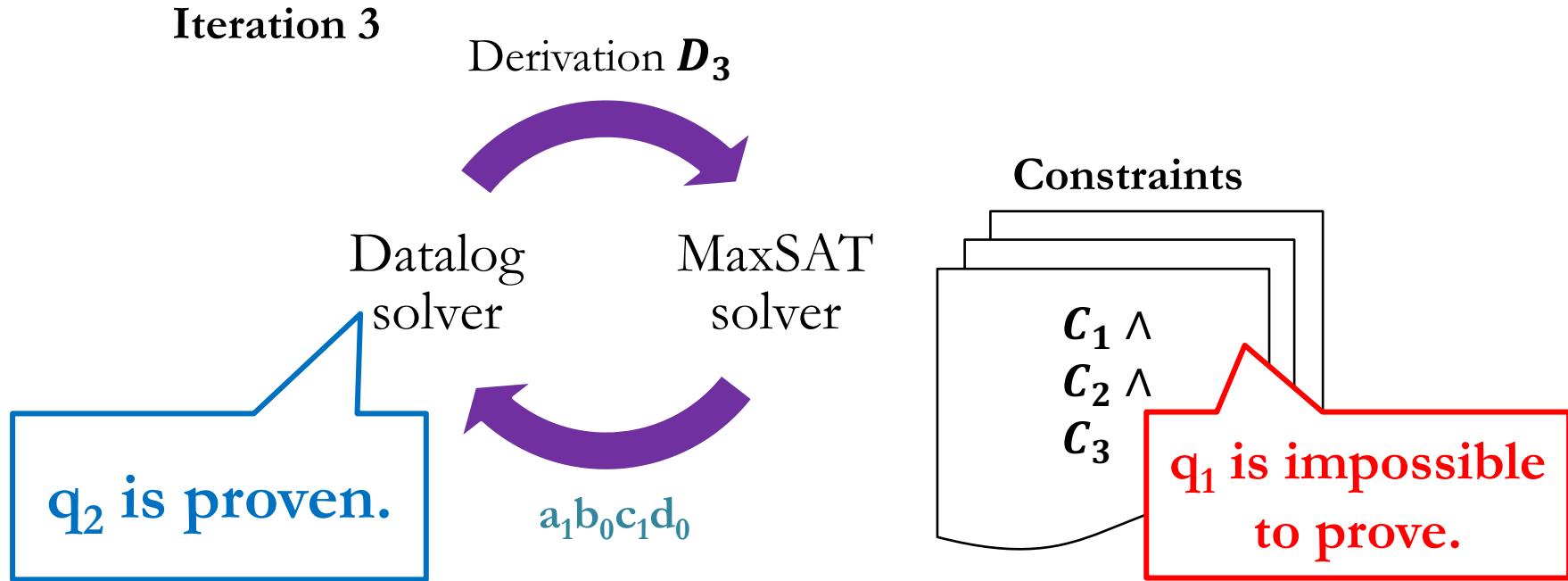
Query	Answer	Eliminated Abstractions	
q <sub>1</sub> : path(1, 3)		$a_0^* c_0^*, a_1^* c_0^*$	(8/16)
q <sub>2</sub> : path(1, 6)	✓ $a_1 b_0 c_1 d_0$	$a_0^* c_0 d_0, a_0 b_0^* d_0, a_1^* c_0 d_0$	(5/16)

# Iteration 2 and Beyond



Query	Answer	Eliminated Abstractions	
$q_1: \text{path}(1, 3)$		$a_0^* c_0^*, a_1^* c_0^*$	(8/16)
$q_2: \text{path}(1, 6)$	✓ $a_1 b_0 c_1 d_0$	$a_0^* c_0 d_0, a_0 b_0^* d_0, a_1^* c_0 d_0$	(5/16)

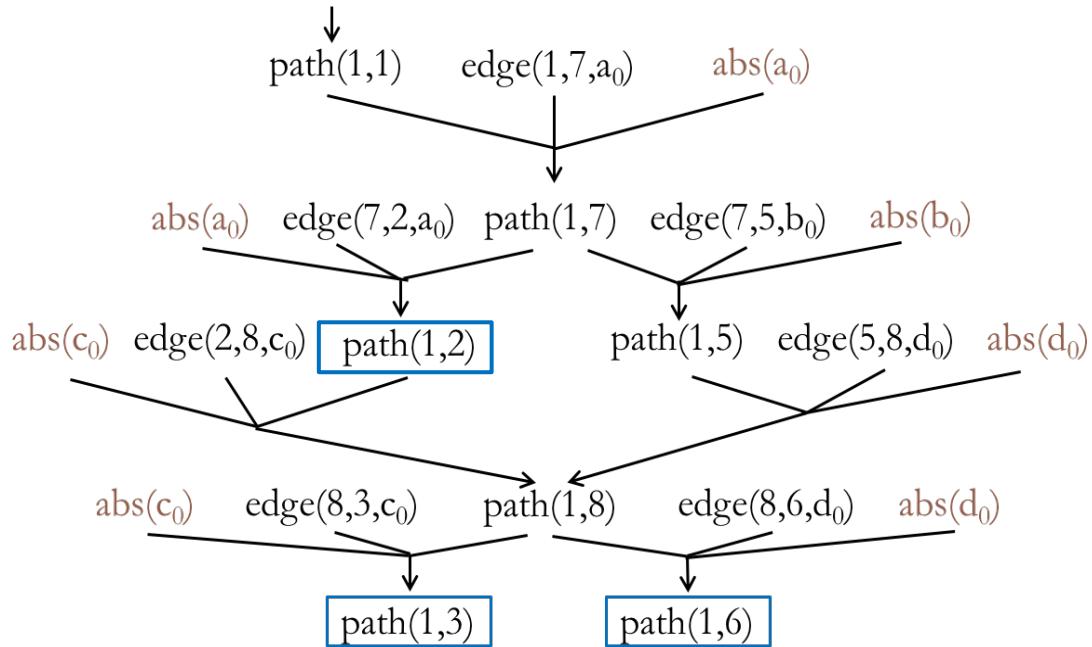
# Iteration 2 and Beyond



Query	Answer	Eliminated Abstractions
$q_1: \text{path}(1, 3)$	✗ Impossibility	$a_0^* c_0^*, a_1^* c_0^*, a_0^* c_1^*, a_1^* c_1^* (16/16)$
$q_2: \text{path}(1, 6)$	✓ $a_1 b_0 c_1 d_0$	$a_0^* c_0 d_0, a_0 b_0^* d_0, a_1^* c_0 d_0 (5/16)$

# Mixing Counterexamples

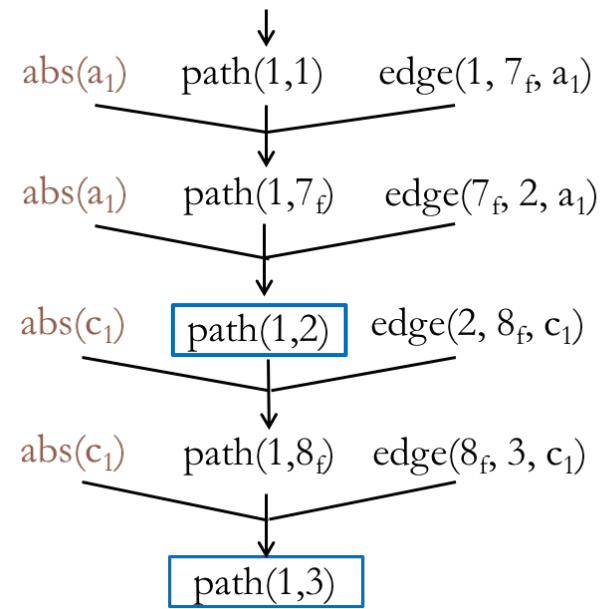
## Iteration 1



Eliminated  
Abstractions:

a<sub>0</sub>\*c<sub>0</sub>\*

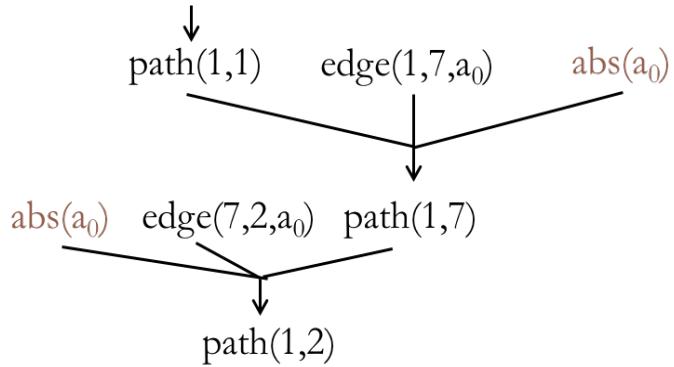
## Iteration 3



a<sub>1</sub>\*c<sub>1</sub>\*

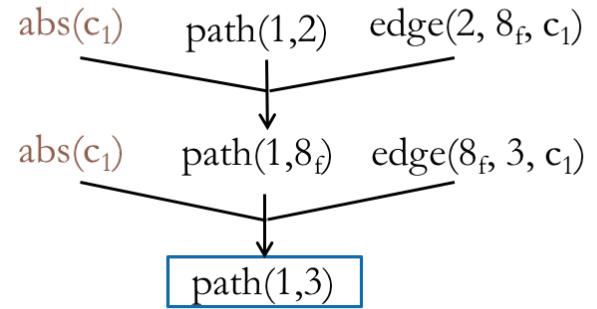
# Mixing Counterexamples

Iteration 1



Mixed!

Iteration 3



Eliminated  
Abstractions:

$a_0^*c_0^*$

$a_0^*c_1^*$

$a_1^*c_1^*$

# Experimental Setup

---

- ▶ Implemented using off-the-shelf solvers
  - ▶ Datalog: bddbddb
  - ▶ MaxSAT: MiFuMaX
- ▶ Applied to two analyses that are challenging to scale
  - ▶ **k-object-sensitive pointer analysis**
    - ▶ flow-insensitive, weak updates, cloning-based
  - ▶ **type-state analysis**
    - ▶ flow-sensitive, strong updates, summary-based
- ▶ Evaluated on 8 Java programs (250-450 KLOC each)

# Pointer Analysis Results

	queries		abstraction size		iterations	
	total	resolved	final	max	< 3% of max	
		current				
toba-s	7	7	0	170	18K	10
javasrc-p	46	46	0	470	18K	13
weblech	5	5	2	140	31K	10
hedc	47	47	6	730	29K	18
antlr	143	143	5	970	29K	15
luindex	138	138	67	1K	40K	26
lusearch	322	322	29	1K	39K	17
schroeder-m	51	51	25	450	58K	15

# Performance of Datalog Solver

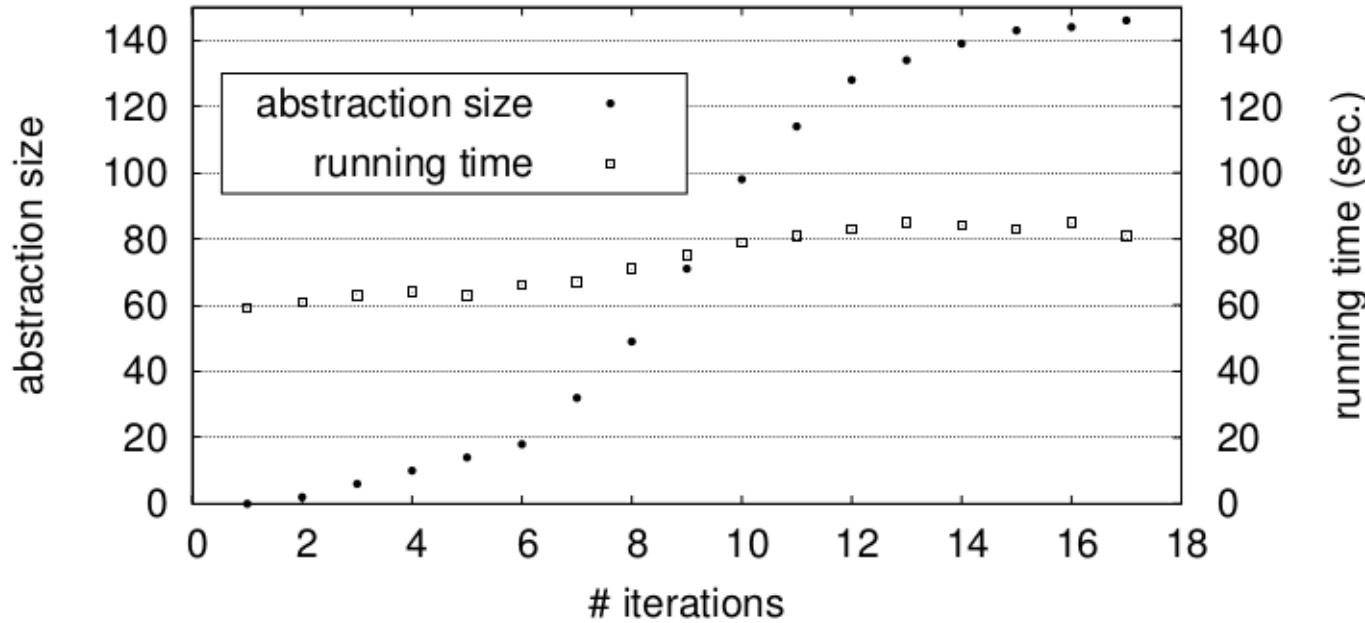
$k = 4$ , 3h28m

Baseline     $k = 3$ , 590s

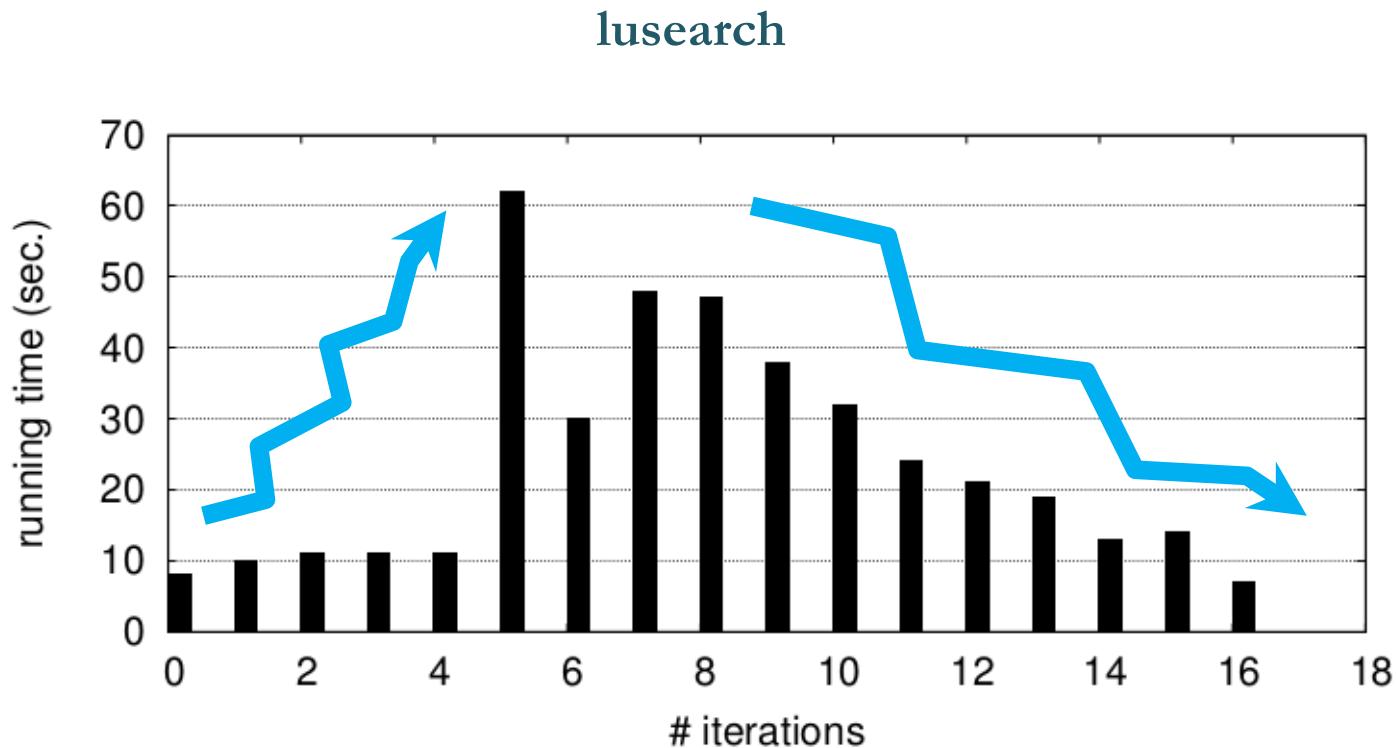
$k = 2$ , 214s

lusearch

$k = 1$ , 153s



# Performance of MaxSAT Solver



# Statistics of MaxSAT Formulae

---

	pointer analysis	
	variables	clauses
toba-s	0.7M	1.5M
javasrc-p	0.5M	0.9M
weblech	1.6M	3.3M
hedc	1.2M	2.7M
antlr	3.6M	6.9M
luindex	2.4M	5.6M
lusearch	2.1M	5.0M
schroeder-m	6.7M	23.7M

# Overview of Applications

---

- ▶ Abstraction Selection [PLDI 2014]
- ▶ Alarm Classification [FSE 2015]
- ▶ Alarm Resolution [OOSPLA 2017]

# How Do We Go From This ...

## Detected Races

**R1:** Race on field `org.apache.ftpserver.RequestHandler.m_request`

<code>org.apache.ftpserver.RequestHandler: 9</code>	<code>org.apache.ftpserver.RequestHandler: 18</code>
---	--

**R2:** Race on field `org.apache.ftpserver.RequestHandler.m_request`

<code>org.apache.ftpserver.RequestHandler: 17</code>	<code>org.apache.ftpserver.RequestHandler: 18</code>
--	--

**R3:** Race on field `org.apache.ftpserver.RequestHandler.m_writer`

<code>org.apache.ftpserver.RequestHandler: 19</code>	<code>org.apache.ftpserver.RequestHandler: 20</code>
--	--

**R4:** Race on field `org.apache.ftpserver.RequestHandler.m_reader`

<code>org.apache.ftpserver.RequestHandler: 21</code>	<code>org.apache.ftpserver.RequestHandler: 22</code>
--	--

**R5:** Race on field `org.apache.ftpserver.RequestHandler.m_controlSocket`

<code>org.apache.ftpserver.RequestHandler: 23</code>	<code>org.apache.ftpserver.RequestHandler: 24</code>
--	--

## Eliminated Races

**E1:** Race on field `org.apache.ftpserver.RequestHandler. m_isConnectionClosed`

<code>org.apache.ftpserver.RequestHandler: 13</code>	<code>org.apache.ftpserver.RequestHandler: 15</code>
--	--

# ... To This?

Detected Races	
<b>R1:</b> Race on field <code>org.apache.ftpserver.RequestHandler.m_request</code>	
<code>org.apache.ftpserver.RequestHandler: 9</code>	<code>org.apache.ftpserver.RequestHandler: 18</code>
<b>R2:</b> Race on field <code>org.apache.ftpserver.RequestHandler.m_request</code>	
<code>org.apache.ftpserver.RequestHandler: 17</code>	<code>org.apache.ftpserver.RequestHandler: 18</code>
<b>R3:</b> Race on field <code>org.apache.ftpserver.RequestHandler.m_writer</code>	
<code>org.apache.ftpserver.RequestHandler: 19</code>	<code>org.apache.ftpserver.RequestHandler: 20</code>
<b>R4:</b> Race on field <code>org.apache.ftpserver.RequestHandler.m_reader</code>	
<code>org.apache.ftpserver.RequestHandler: 21</code>	<code>org.apache.ftpserver.RequestHandler: 22</code>
<b>R5:</b> Race on field <code>org.apache.ftpserver.RequestHandler.m_controlSocket</code>	
<code>org.apache.ftpserver.RequestHandler: 23</code>	<code>org.apache.ftpserver.RequestHandler: 24</code>



## Eliminated Races

**E1:** Race on field `org.apache.ftpserver.RequestHandler. m_isConnectionClosed`

`org.apache.ftpserver.RequestHandler: 13`    `org.apache.ftpserver.RequestHandler: 15`

# ... To This?

Detected Races	
R1: Race on field org.apache.ftpserver.RequestHandler.m_request	 
org.apache.ftpserver.RequestHandler: 9	org.apache.ftpserver.RequestHandler: 18
Eliminated Races	
E1: Race on field org.apache.ftpserver.RequestHandler. m_isConnectionClosed	
org.apache.ftpserver.RequestHandler: 13	org.apache.ftpserver.RequestHandler: 15
E2: Race on field org.apache.ftpserver.RequestHandler.m_request	
org.apache.ftpserver.RequestHandler: 17	org.apache.ftpserver.RequestHandler: 18
E3: Race on field org.apache.ftpserver.RequestHandler.m_writer	
org.apache.ftpserver.RequestHandler: 19	org.apache.ftpserver.RequestHandler: 20
E4: Race on field org.apache.ftpserver.RequestHandler.m_reader	
org.apache.ftpserver.RequestHandler: 21	org.apache.ftpserver.RequestHandler: 22
E5: Race on field org.apache.ftpserver.RequestHandler.m_controlSocket	
org.apache.ftpserver.RequestHandler: 23	org.apache.ftpserver.RequestHandler: 24

# An Example: Static Datarace Analysis

## **Input relations:**

$\text{next}(p1, p2)$ ,  $\text{mayAlias}(p1, p2)$ ,  $\text{guarded}(p1, p2)$

## **Output relations:**

$\text{parallel}(p1, p2)$ ,  $\text{race}(p1, p2)$

## **Constraints:**

$\text{parallel}(p3, p2) :- \text{parallel}(p1, p2), \text{next}(p3, p1).$

$\text{parallel}(p1, p2) :- \text{parallel}(p2, p1).$

$\text{race}(p1, p2) :- \text{parallel}(p1, p2), \text{mayAlias}(p1, p2), \neg \text{guarded}(p1, p2).$

...

# An Example: Static Datarace Analysis

## **Input relations:**

$\text{next}(p_1, p_2)$ ,  $\text{mayAlias}(p_1, p_2)$ ,  $\text{guarded}(p_1, p_2)$

program point  $p_1$  is immediate successor of  $p_2$ .

$p_1$  &  $p_2$  may access the same memory location.

## **Output relations:**

$\text{parallel}(p_1, p_2)$ ,  $\text{race}(p_1, p_2)$

$p_1$  &  $p_2$  may happen in parallel.

$p_1$  &  $p_2$  are guarded by the same lock.

## **Constraints:**

$p_1$  &  $p_2$  may have a datarace.

$\text{parallel}(p_3, p_2) \vdash \text{parallel}(p_1, p_2), \text{next}(p_3, p_1)$ .

$\text{parallel}(p_1, p_2) \vdash \text{parallel}(p_2, p_1)$ .

If  $p_1$  &  $p_2$  may happen in parallel, and  $p_3$  is successor of  $p_1$ , then  $p_3$  &  $p_2$  may happen in parallel.

$\text{race}(p_1, p_2) \vdash \text{parallel}(p_1, p_2), \text{mayAlias}(p_1, p_2), \neg \text{guarded}(p_1, p_2)$ .

...

If  $p_2$  &  $p_1$  may happen in parallel, then  $p_1$  &  $p_2$  may happen in parallel.

If  $p_1$  &  $p_2$  may happen in parallel, and they may access the same memory location, and they are not guarded by the same lock, then  $p_1$  &  $p_2$  may have a datarace.

# An Example: Static Datarace Analysis

## Input relations:

`next(p1, p2)`, `mayAlias(p1, p2)`, `guarded(p1, p2)`

## Output relations:

`parallel(p1, p2)`, `race(p1, p2)`

```
a = 1;  
if (a > 2) { // p1  
    ... // p3  
}
```

“Soft” Rule

## Constraints:

`parallel(p3, p2) :- parallel(p1, p2), next(p3, p1).`

weight 5

`parallel(p1, p2) :- parallel(p2, p1).`

`race(p1, p2) :- parallel(p1, p2), mayAlias(p1, p2),  $\neg$ guarded(p1, p2).`

...

`$\neg$ race(x2, x1).`

weight 25

“Hard” Rule

# An Example: Static Datarace Analysis

---

```
1 public class RequestHandler {  
2     Request request;  
3     FtpWriter writer;  
4     BufferedReader reader;  
5     Socket controlSocket;  
6     boolean isConnectionClosed;  
7     ...  
8     public Request getRequest() {  
9         return request;  
10    }  
11    public void close() {  
12        synchronized (this) {  
13            if (isClosed)  
14                return;  
15            isClosed = true;  
16        }  
17        request.clear();  
18        request = null;  
19        writer.close();  
20        writer = null;  
21        reader.close();  
22        reader = null;  
23        controlSocket.close();  
24        controlSocket = null;  
25    }  
}
```

Source code snippet from Apache FTP Server

# An Example: Static Datarace Analysis

```
1 public class RequestHandler {  
2     Request request;  
3     FtpWriter writer;  
4     BufferedReader reader;  
5     Socket controlSocket;  
6     boolean isConnectionClosed;  
7     ...  
8     public Request getRequest() {  
9         return request;  
10    }  
11    public void close() {  
12        synchronized (this) {  
13            if (isClosed)  
14                return;  
15            isClosed = true;  
16        }  
17        request.clear();  
18        request = null;  
19        writer.close();  
20        writer = null;  
21        reader.close();  
22        reader = null;  
23        controlSocket.close();  
24        controlSocket = null;  
25    }  
  
R1
```

Source code snippet from Apache FTP Server

# An Example: Static Datarace Analysis

```
1 public class RequestHandler {  
2     Request request;  
3     FtpWriter writer;  
4     BufferedReader reader;  
5     Socket controlSocket;  
6     boolean isConnectionClosed;  
7     ...  
8     public Request getRequest() {  
9         return request;  
10    }
```

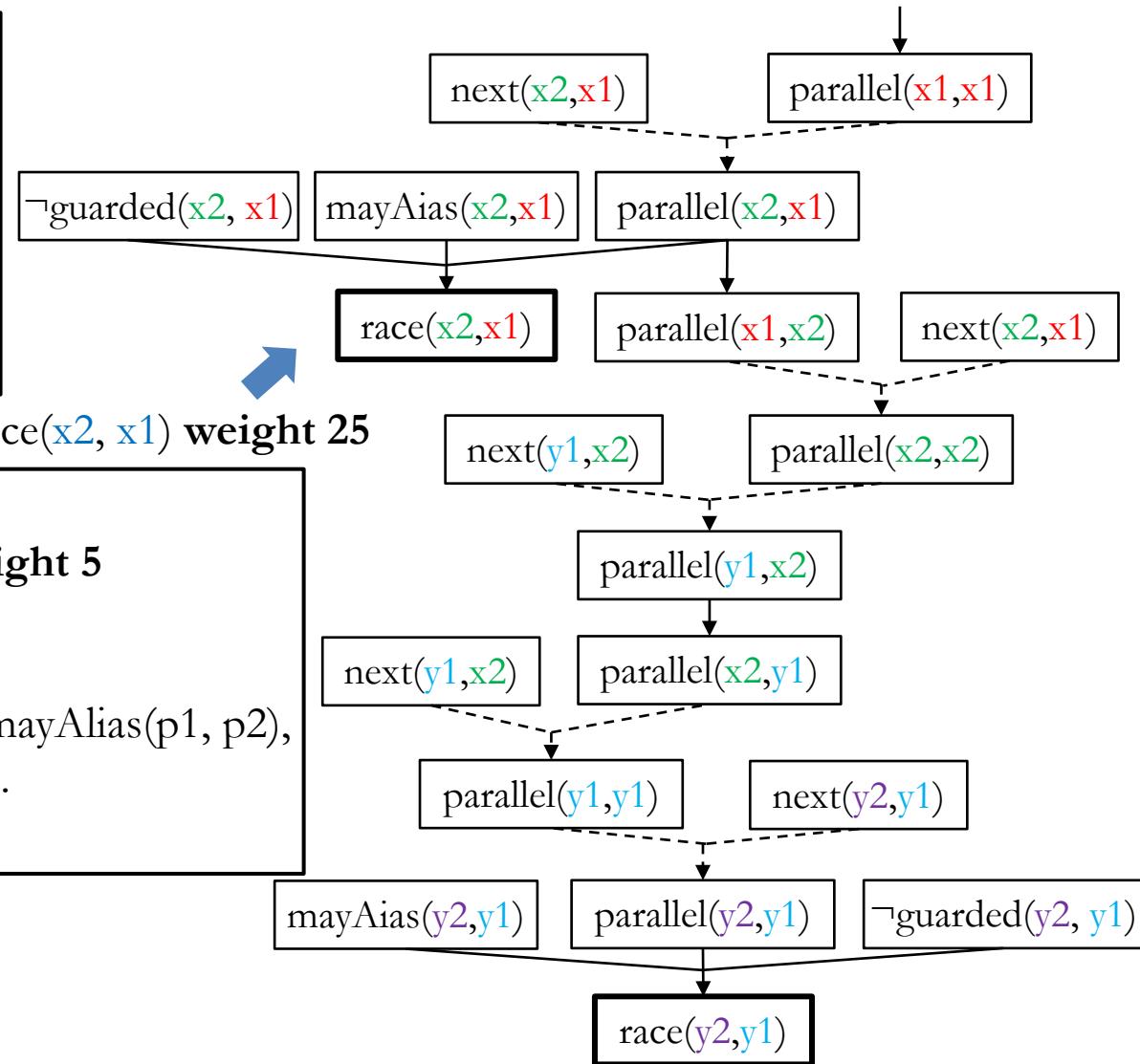
```
11    public void close() {  
12        synchronized (this) {  
13            if (isClosed)  
14                return;  
15            isClosed = true;  
16        }  
17        request.clear();  
18        request = null;  
19        writer.close();  
20        writer = null;  
21        reader.close();  
22        reader = null;  
23        controlSocket.close();  
24        controlSocket = null;  
25    }
```

R2  
R3  
R4  
R5

Source code snippet from Apache FTP Server

# How Does Generalization Work?

```
17     ...  
18     request.clear(); // x1  
19     request = null; // x2  
20     writer.close(); // y1  
21     writer = null; // y2  
    ...
```



```
parallel(p3, p2) :- parallel(p1, p2),  
    next (p3, p1). weight 5
```

```
parallel(p1, p2) :- parallel(p2, p1).
```

```
race(p1, p2) :- parallel(p1, p2), mayAlias(p1, p2),  
    \neg guarded(p1, p2).
```

...

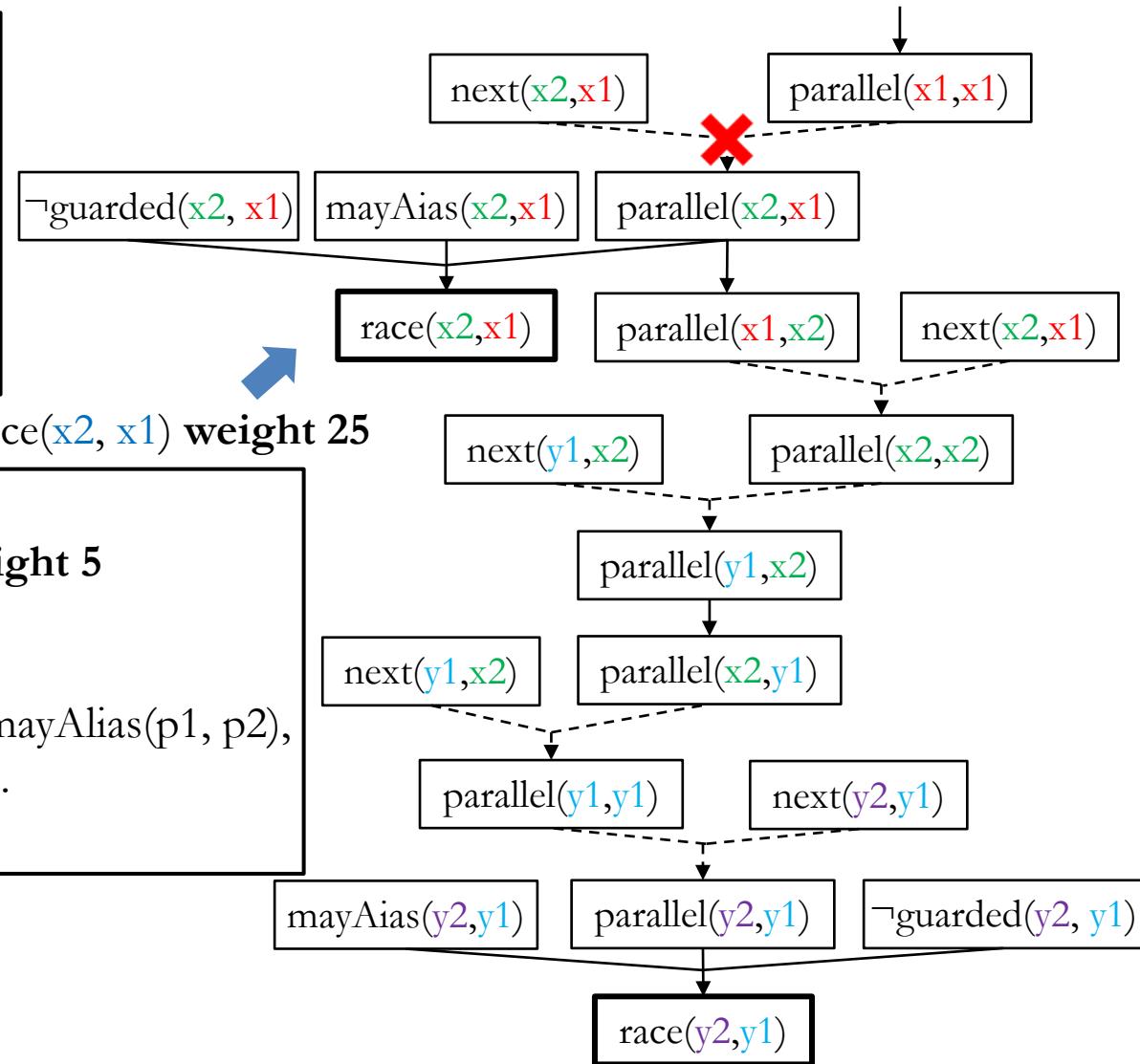
# How Does Generalization Work?

```
17     ...  
18     request.clear(); // x1  
19     request = null; // x2  
20     writer.close(); // y1  
21     writer = null; // y2  
    ...
```

```
parallel(p3, p2) :- parallel(p1, p2),  
    next(p3, p1). weight 5
```

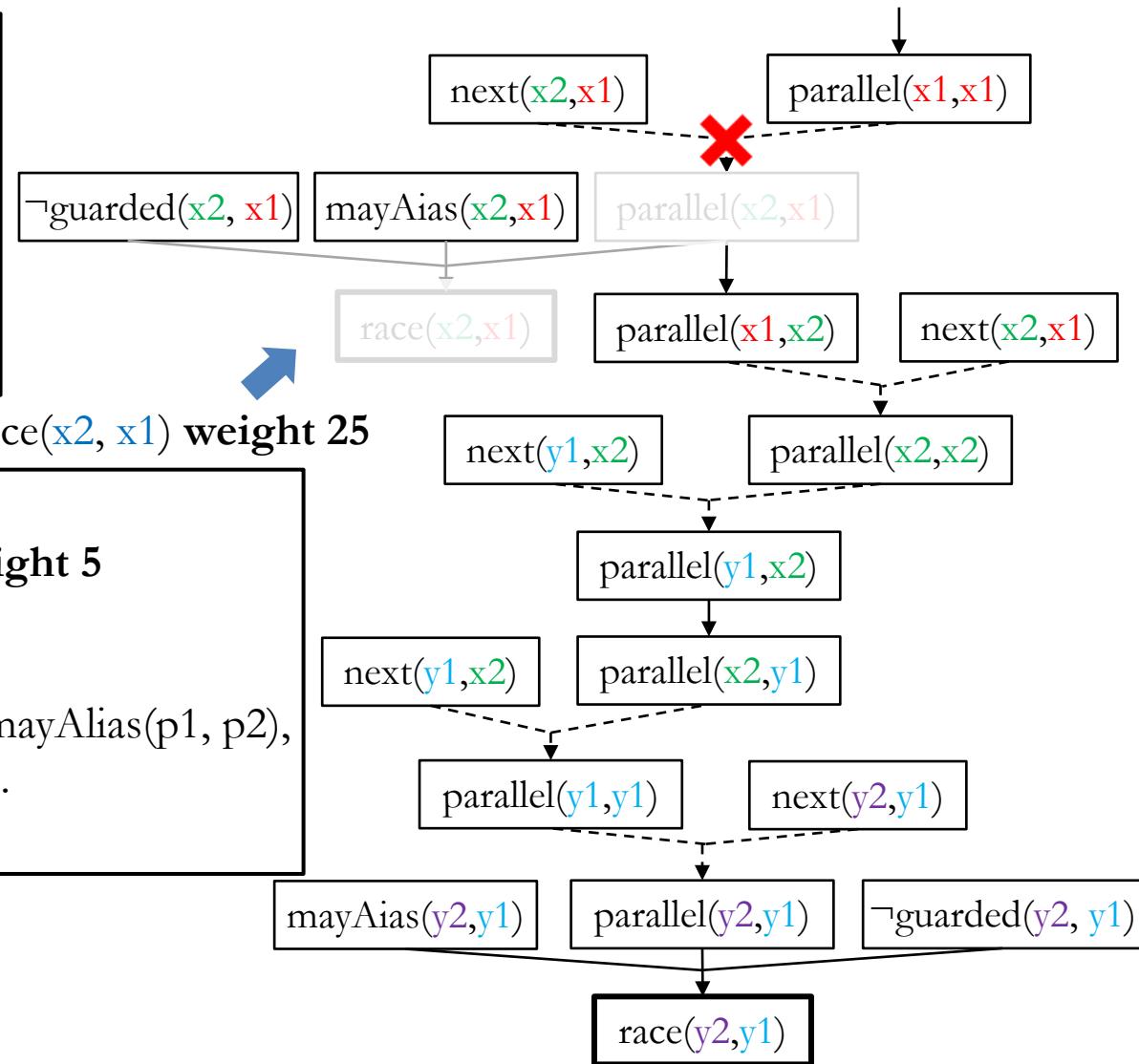
```
parallel(p1, p2) :- parallel(p2, p1).
```

```
race(p1, p2) :- parallel(p1, p2), mayAlias(p1, p2),  
     $\neg$ guarded(p1, p2).  
    ...
```



# How Does Generalization Work?

```
17     ...  
18     request.clear(); // x1  
19     request = null; // x2  
20     writer.close(); // y1  
21     writer = null; // y2  
    ...
```



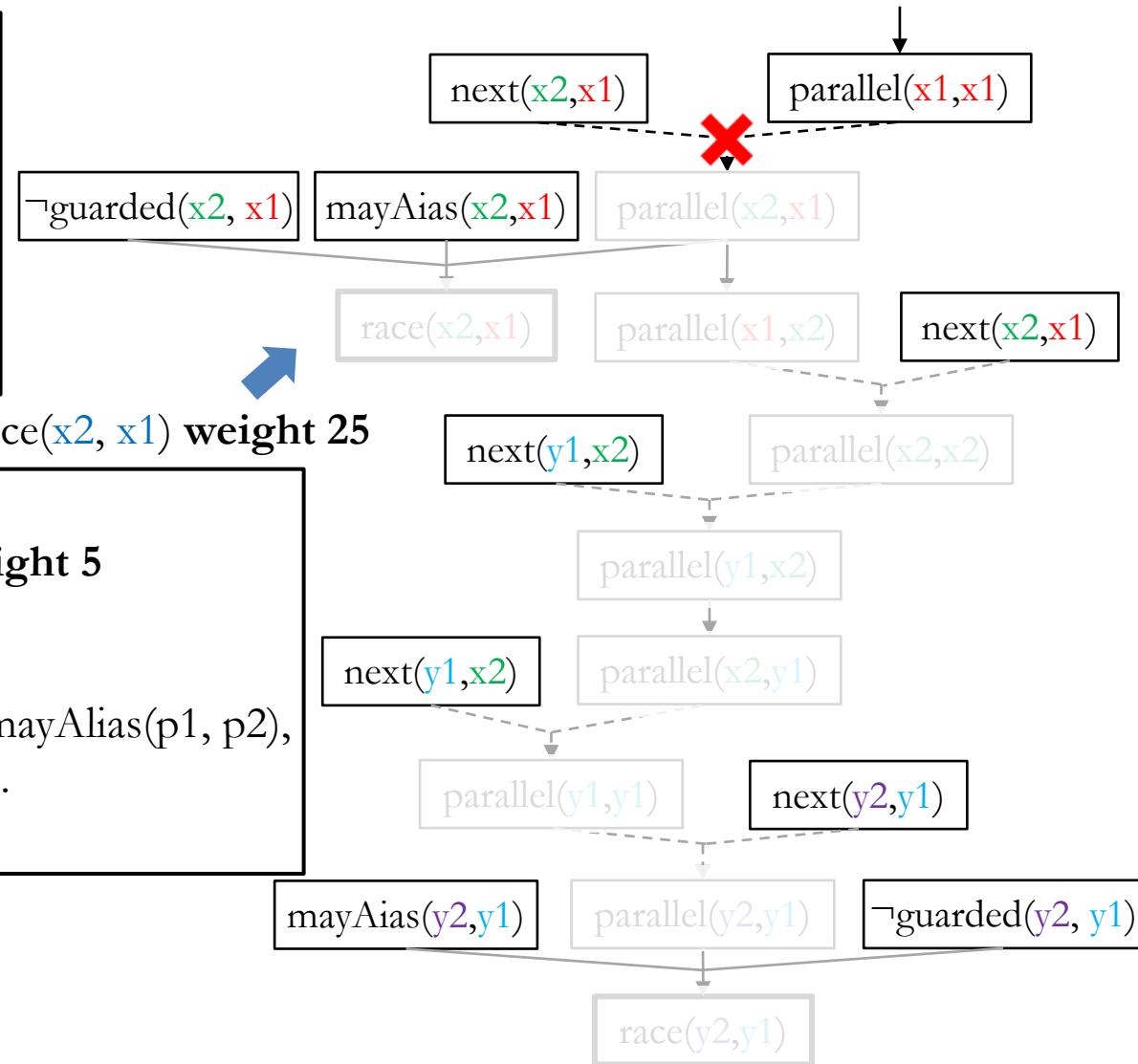
# How Does Generalization Work?

```
17     ...  
18     request.clear(); // x1  
19     request = null; // x2  
20     writer.close(); // y1  
21     writer = null; // y2  
22     ...
```

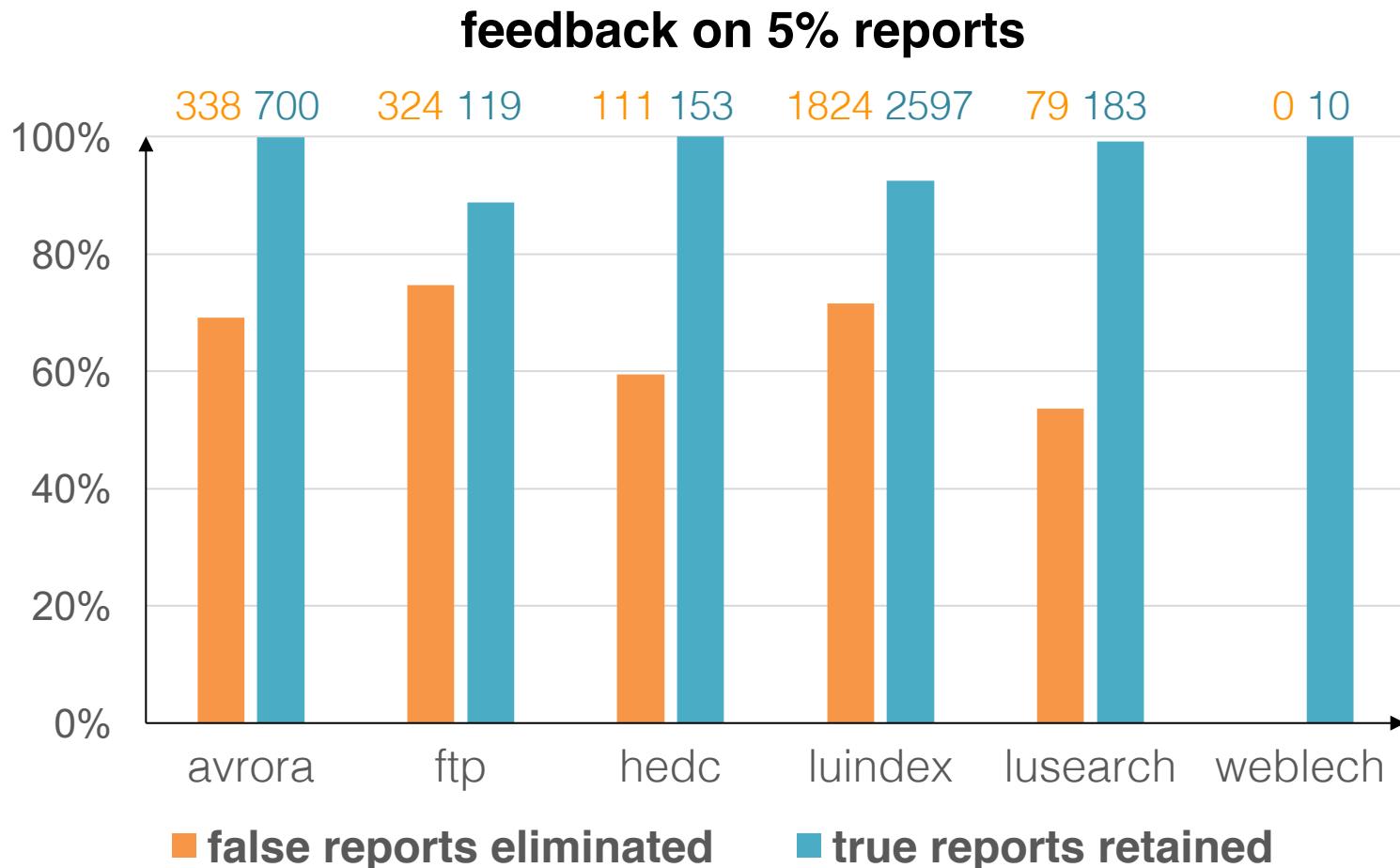
```
parallel(p3, p2) :- parallel(p1, p2),  
    next(p3, p1). weight 5
```

```
parallel(p1, p2) :- parallel(p2, p1).
```

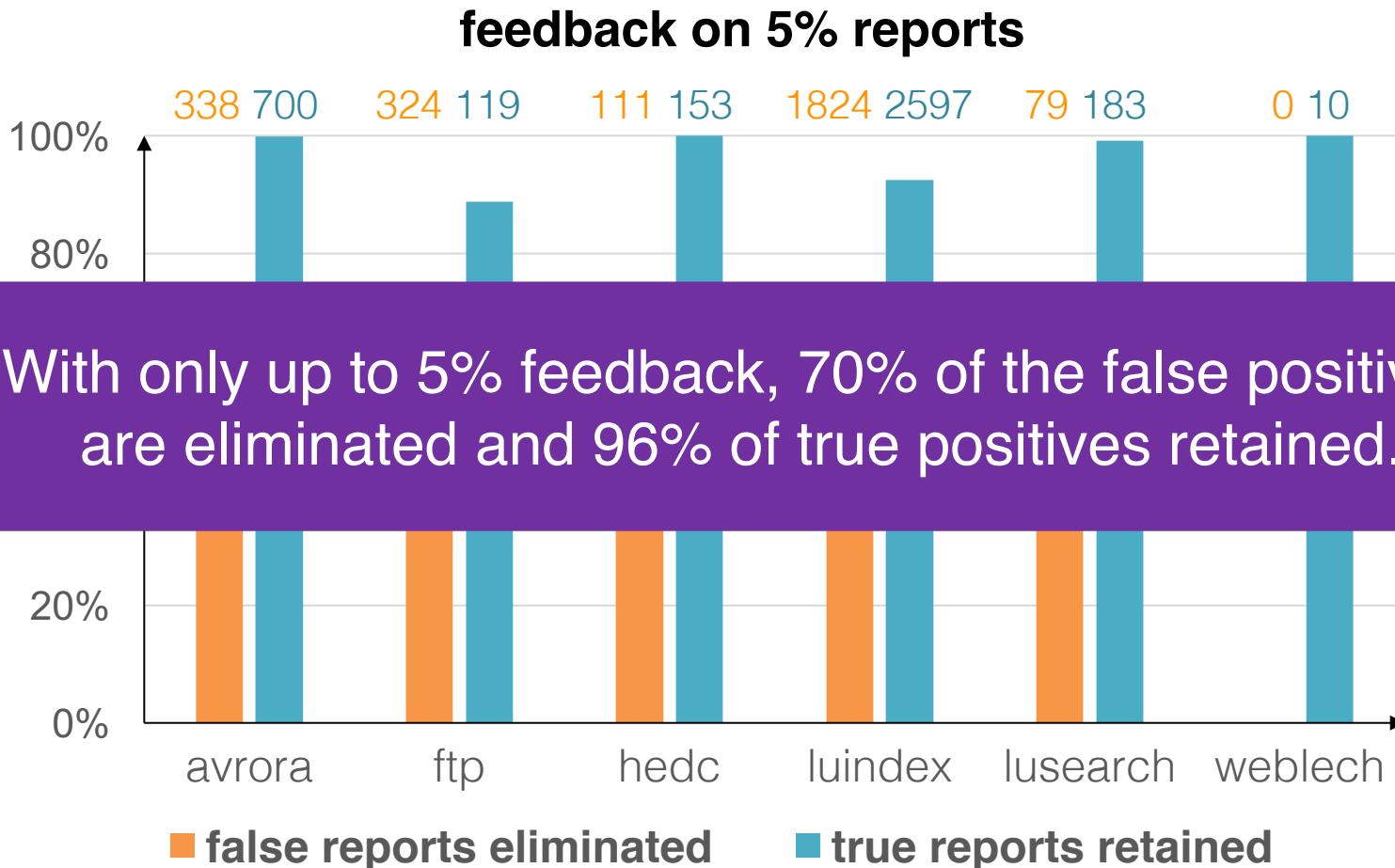
```
race(p1, p2) :- parallel(p1, p2), mayAlias(p1, p2),  
     $\neg$ guarded(p1, p2).  
...
```



# Accuracy of Alarm Classification

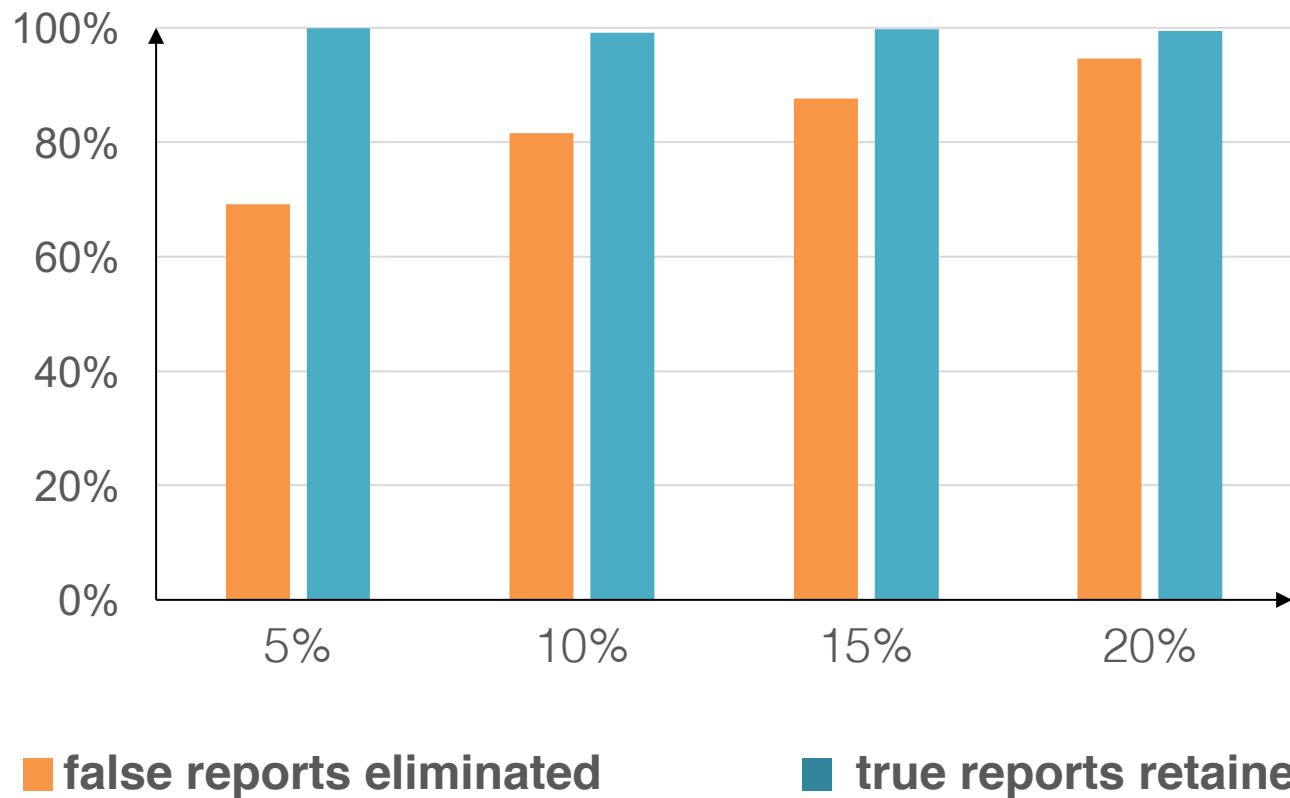


# Accuracy of Alarm Classification



# Impact of Varying Amount of Feedback

**338 false and 700 true reports**



# Overview of Applications

---

- ▶ Abstraction Selection [PLDI 2014]
- ▶ Alarm Classification [FSE 2015]
- ▶ Alarm Resolution [OOSPLA 2017]

# Example Revisited: Static Datarace Analysis

Can this statement be  
executed by different threads  
in parallel?

```
8 public Request getRequest() {  
9     return request;  
10}
```

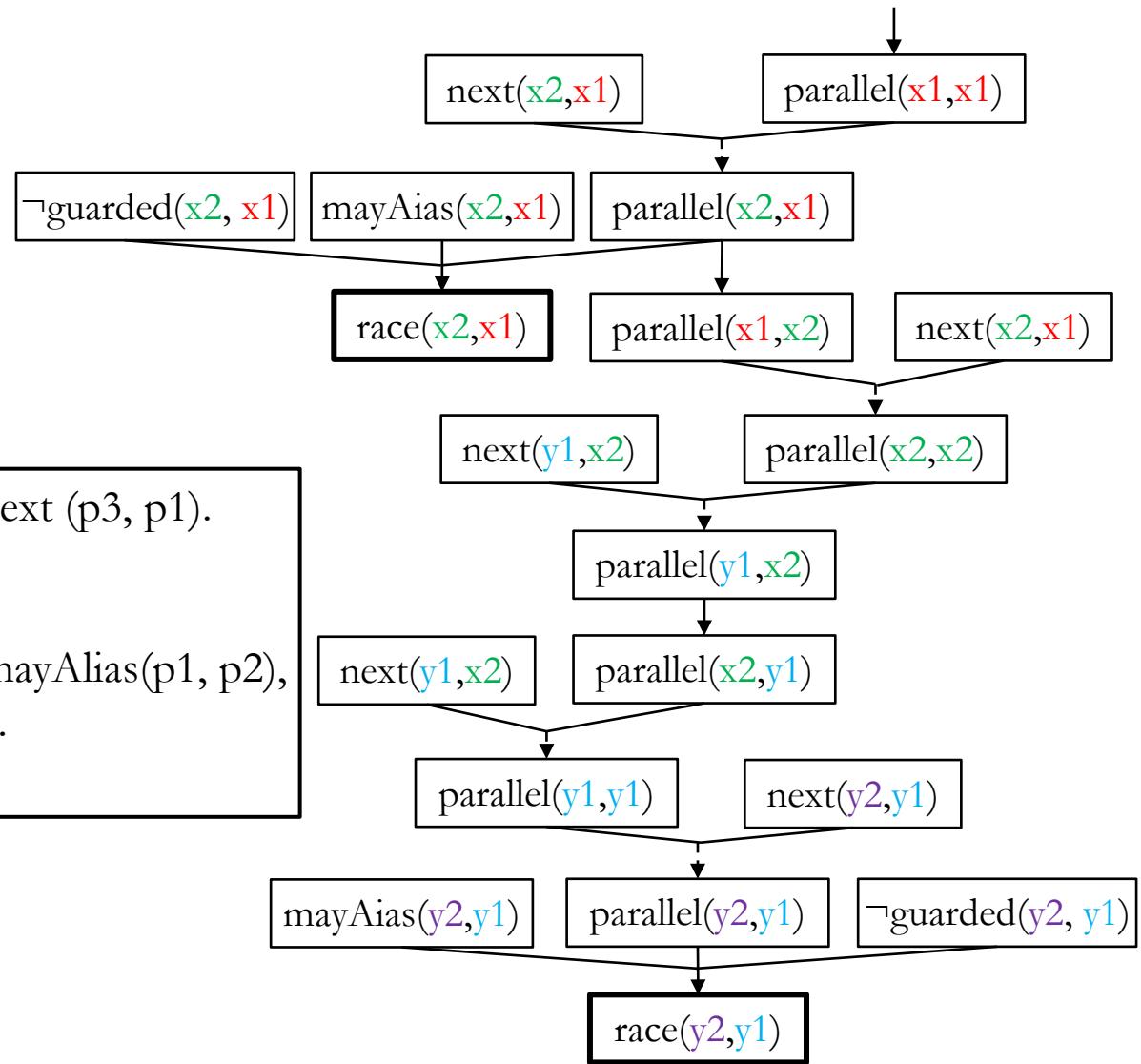
```
11 public void close() {  
12     synchronized (this) {  
13         if (isClosed)  
14             return;  
15         isClosed = true;  
16     }  
17     request.clear();  
18     request = null;  
19     writer.close();  
20     writer = null;  
21     reader.close();  
22     reader = null;  
23     controlSocket.close();  
24     controlSocket = null;  
25 }
```

Source code snippet from Apache FTP Server

# Illustration: Space of Questions

```
17     ...  
18     request.clear(); // x1  
19     request = null; // x2  
20     writer.close(); // y1  
21     writer = null; // y2  
    ...
```

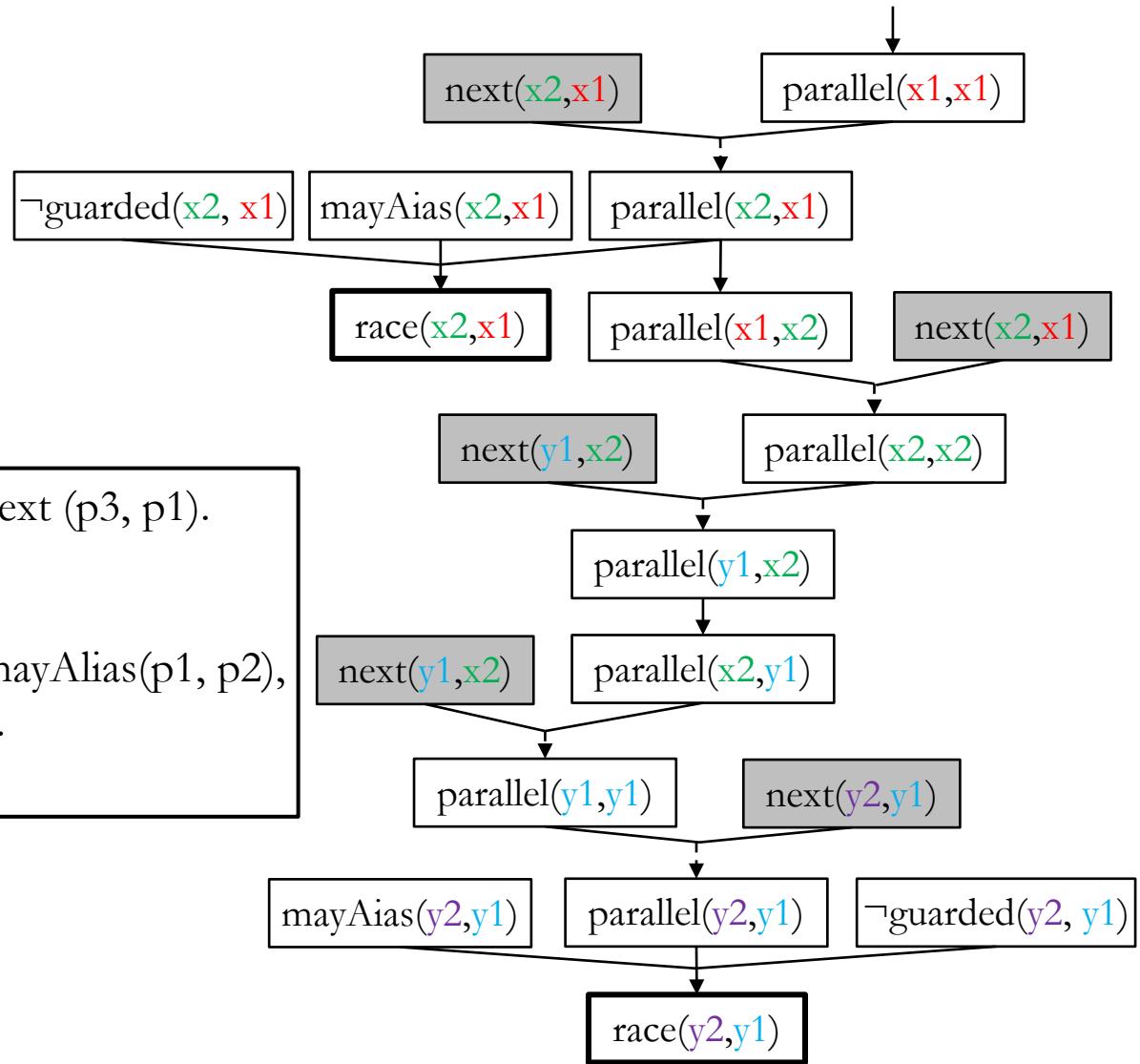
```
parallel(p3, p2) :- parallel(p1, p2), next(p3, p1).  
parallel(p1, p2) :- parallel(p2, p1).  
  
race(p1, p2) :- parallel(p1, p2), mayAlias(p1, p2),  
    \neg guarded(p1, p2).  
    ...
```



# Illustration: Space of Questions

```
17     ...  
18     request.clear(); // x1  
19     request = null; // x2  
20     writer.close(); // y1  
21     writer = null; // y2  
    ...
```

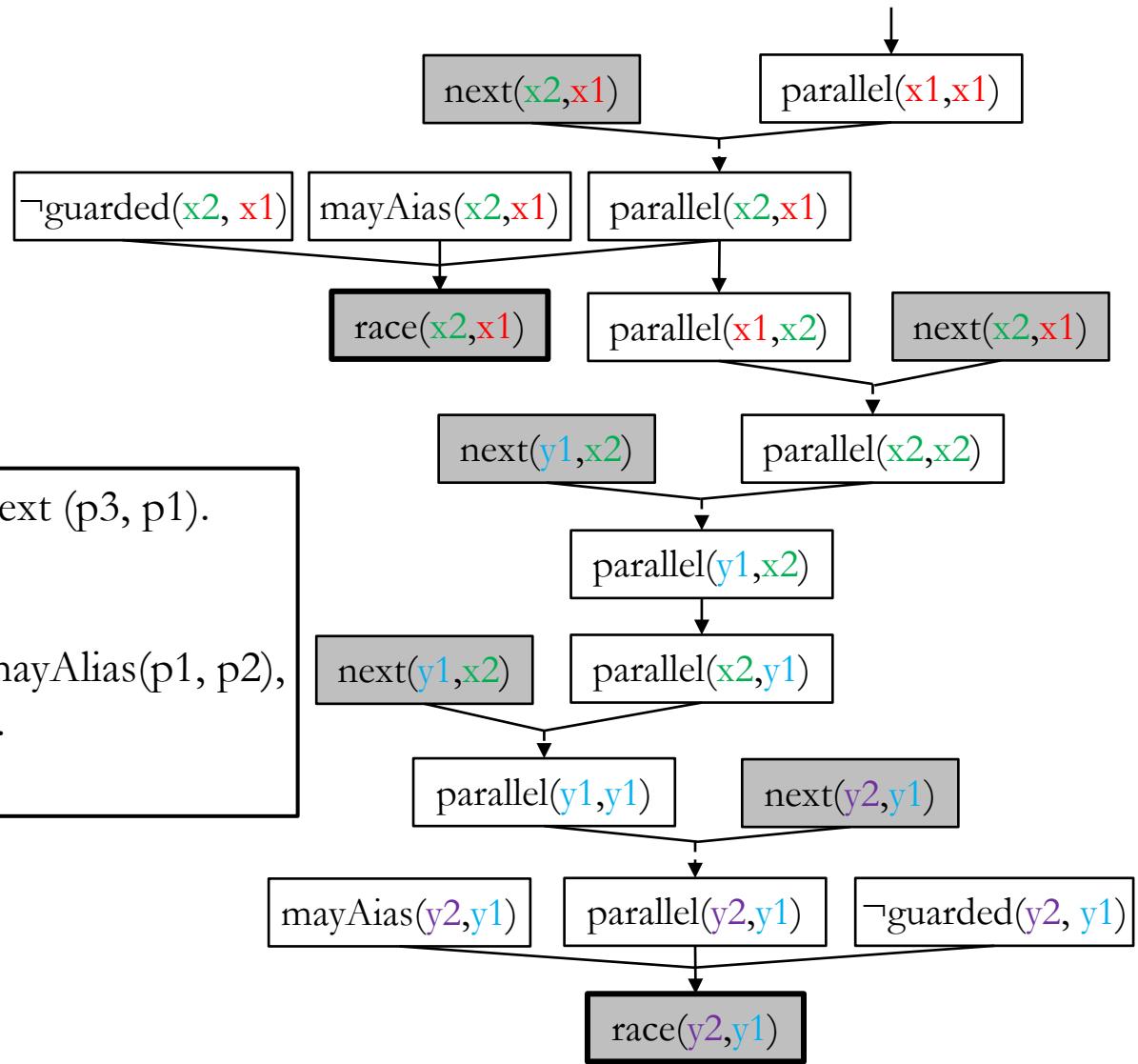
```
parallel(p3, p2) :- parallel(p1, p2), next(p3, p1).  
parallel(p1, p2) :- parallel(p2, p1).  
  
race(p1, p2) :- parallel(p1, p2), mayAlias(p1, p2),  
               $\neg$ guarded(p1, p2).  
...  
...
```



# Illustration: Space of Questions

```
17     ...  
18     request.clear(); // x1  
19     request = null; // x2  
20     writer.close(); // y1  
21     writer = null; // y2  
    ...
```

```
parallel(p3, p2) :- parallel(p1, p2), next(p3, p1).  
parallel(p1, p2) :- parallel(p2, p1).  
  
race(p1, p2) :- parallel(p1, p2), mayAlias(p1, p2),  
     $\neg$ guarded(p1, p2).  
...
```



# Two Key Objectives

---

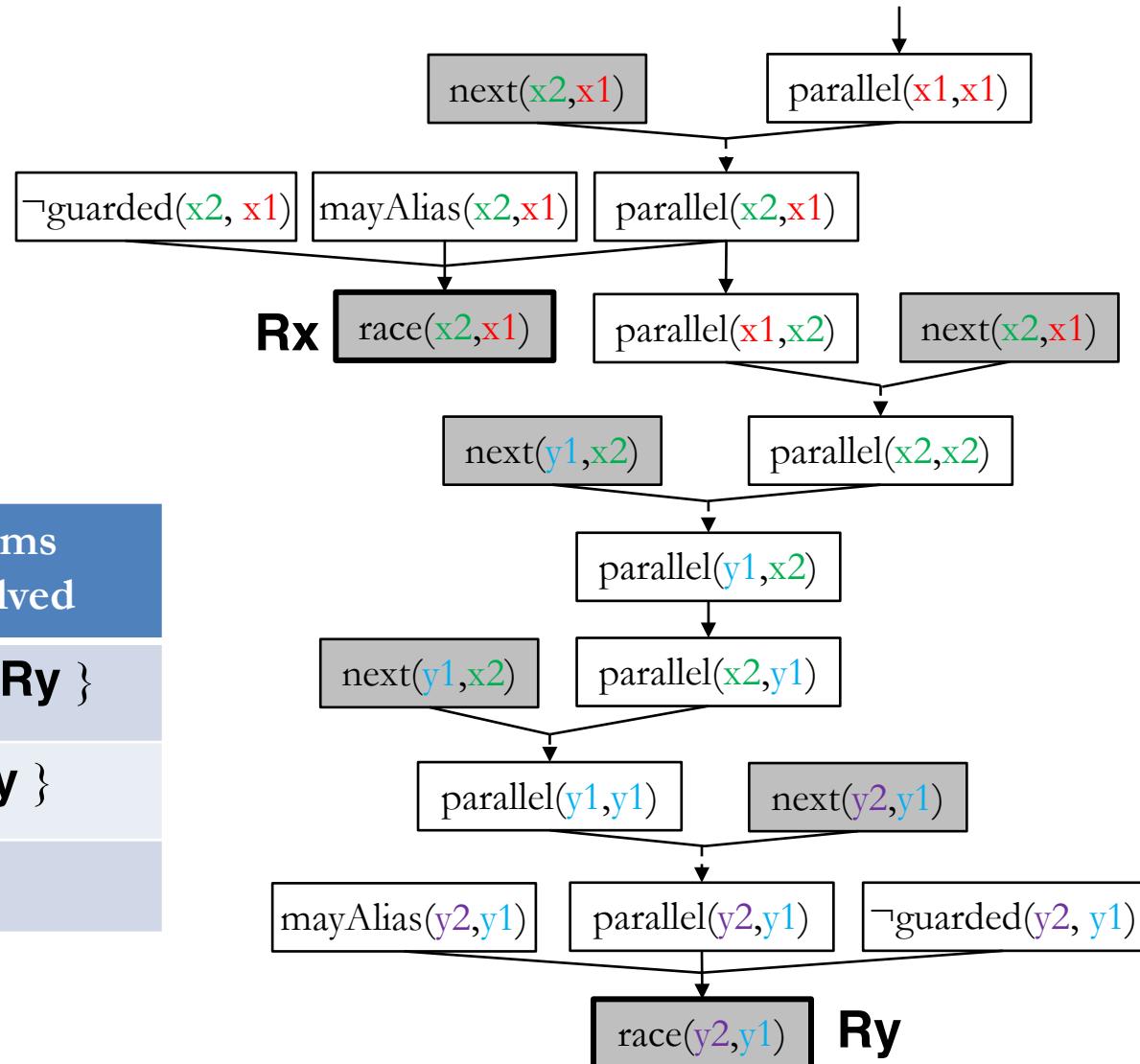
- ▶ **Generalization**
  - ▶ Number of Questions << Number of Alarms Resolved
- ▶ **Prioritization**
  - ▶ Prioritize Questions Likely to Resolve the Most Alarms

# Illustration: Payoff Comparison

```

17   ...
18   request.clear(); // x1
19   request = null; // x2
20   writer.close(); // y1
21   writer = null; // y2
...

```



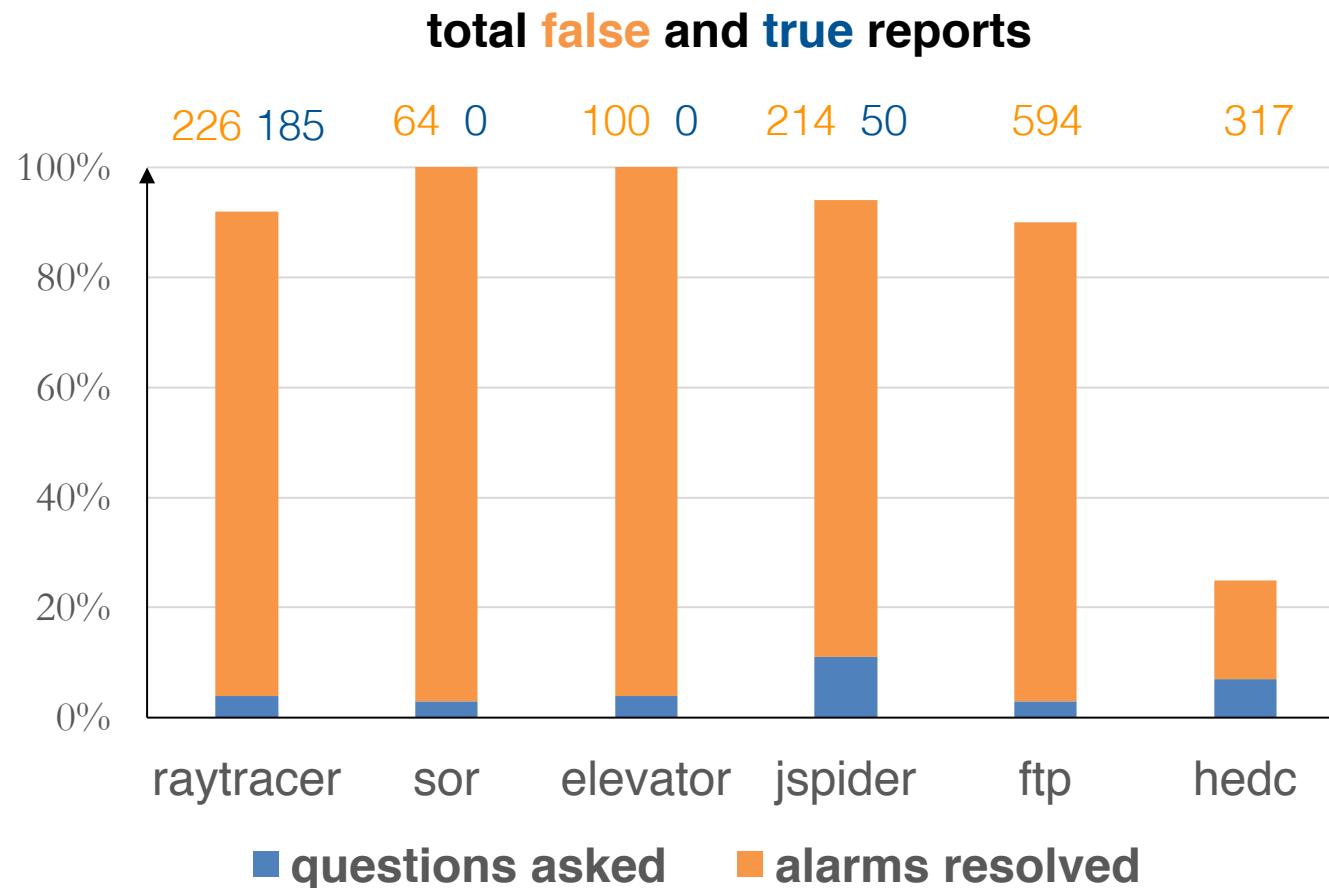
Questions Asked	Alarms Resolved
{ parallel(x1, x1) }	{ Rx, Ry }
{ parallel(y1, y1) }	{ Ry }
{ mayAlias(y2, y1) }	∅

# Highlights of Overall Approach

---

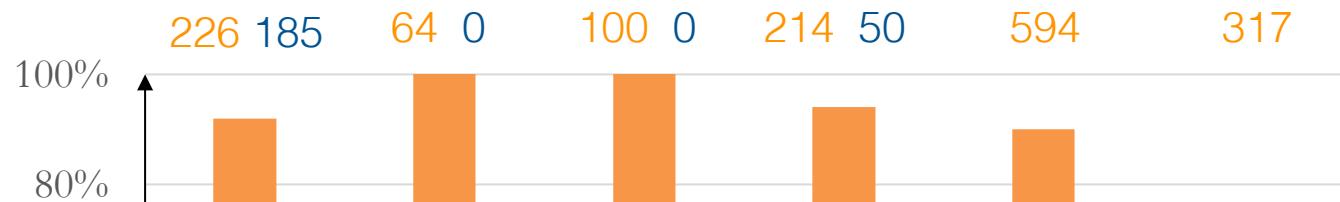
- ▶ Iterative: leverages labels of past questions in choosing future questions
- ▶ Maximizes the expected payoff in each iteration
  - ▶ Payoff = # Alarms Resolved / # Questions Asked
- ▶ Non-linear optimization objective
  - ▶ Binary search on payoff by solving sequence of MaxSAT instances
- ▶ Data-driven: leverages heuristics to guess likely labels
  - ▶ Static, Dynamic, Aggregated

# Empirical Results: Generalization



# Empirical Results: Generalization

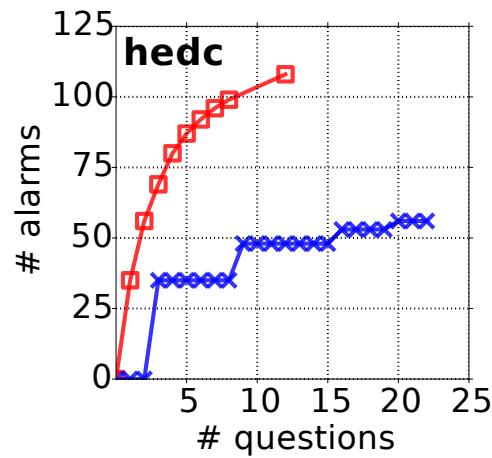
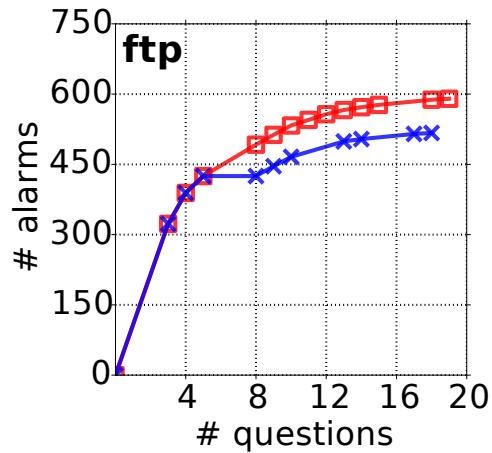
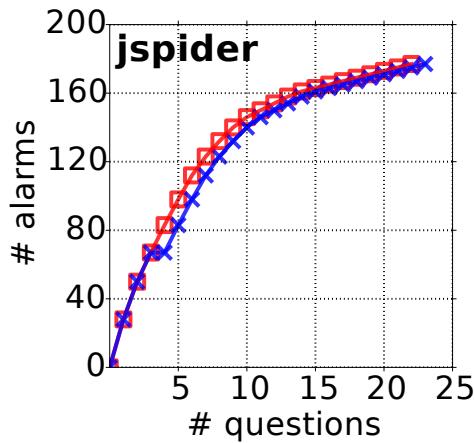
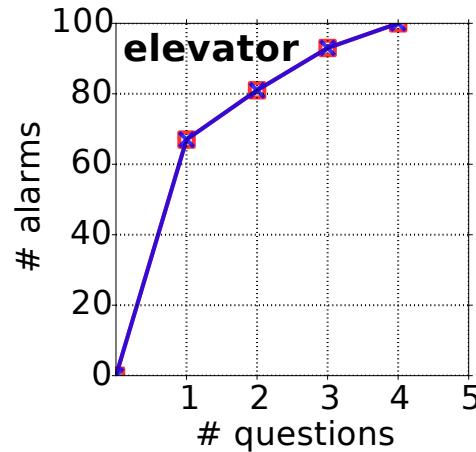
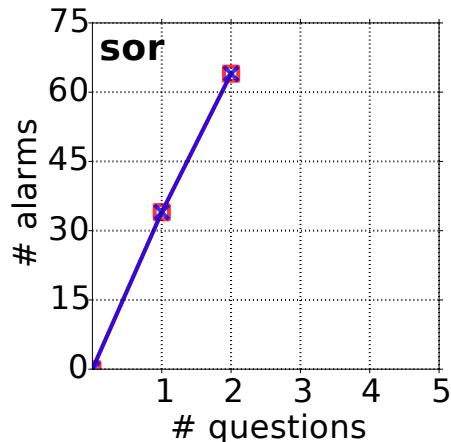
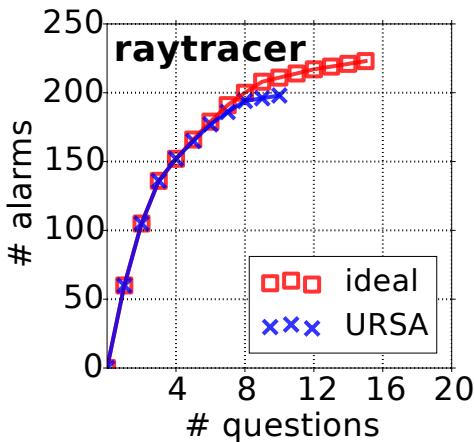
total **false and true reports**



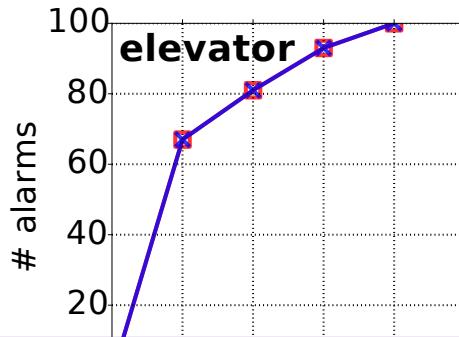
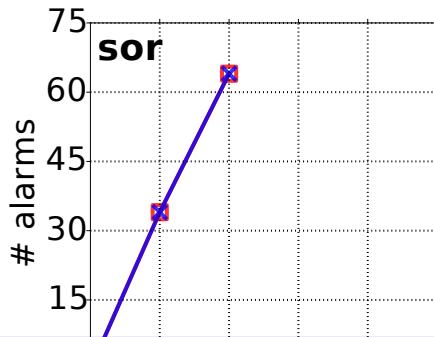
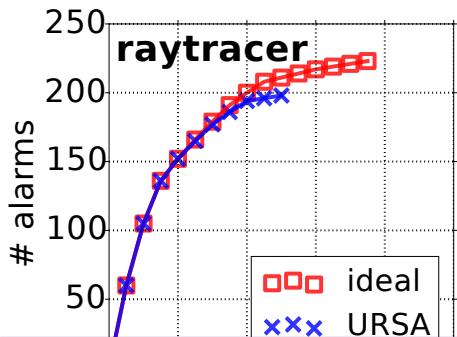
With only up to 5% questions, 74% of the false alarms are resolved with average payoff of 12X per question.



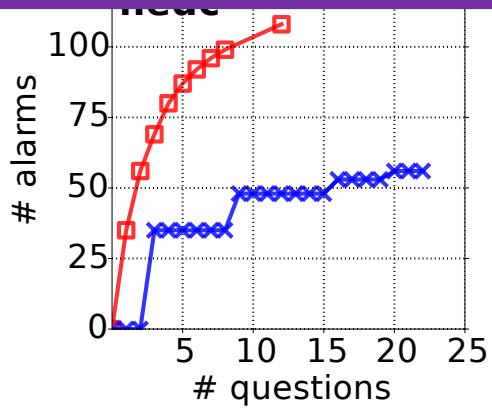
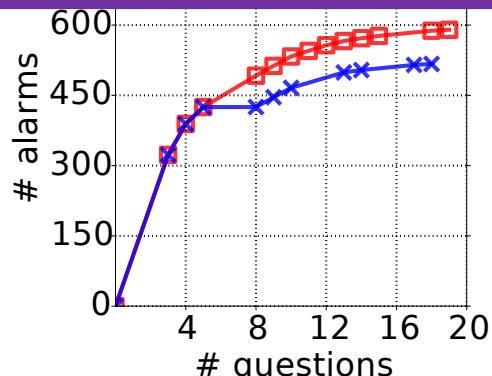
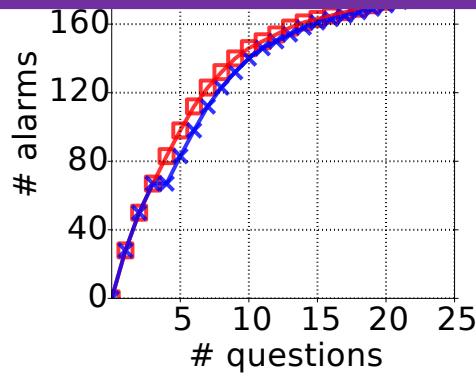
# Empirical Results: Prioritization



# Empirical Results: Prioritization



Earlier iterations yield higher payoffs and match the performance of the ideal setting.



# Summary: Applications in Software Analysis

	Hard Constraints	Soft Constraints
Automated Verification [PLDI'14]	analysis rules $\text{abstraction}_1 \oplus \dots \oplus \text{abstraction}_n$	$\neg \text{result}_i$ weight $w_i$ <b>query resolution award</b> $\text{abstraction}_j$ weight $w_j$ <b>abstraction cost</b>
Static Bug Detection [FSE'15]	analysis rules	analysis rule $_i$ weight $w_i$ <b>confidence of writer</b> $\neg \text{result}_j$ weight $w_j$ <b>confidence of user</b>
Interactive Verification [OOPSLA'17]	analysis rules	$\neg \text{cause}_i$ weight $w_i$ <b>cost of inspection</b> $\text{result}_j$ weight $w_j$ <b>reward of resolution</b>

# Other Applications

## ▶ Statistical Relational Learning

```
wrote(p, t) :- advisedBy(s, p), wrote(s, t). weight 3
p == q :- advisedBy(s, p), advisedBy(s, q). weight 5
professor(p) :- advisedBy(_, p). weight 20
wrote("Tom", "paper1").
wrote("Tom", "paper2").
wrote("Jerry", "paper1").
wrote("Chuck", "paper2").
professor("Jerry").
```

Given constraints and facts,  
find most likely answer to:

advisedBy("Tom", ?)

## ▶ Mathematical Programming

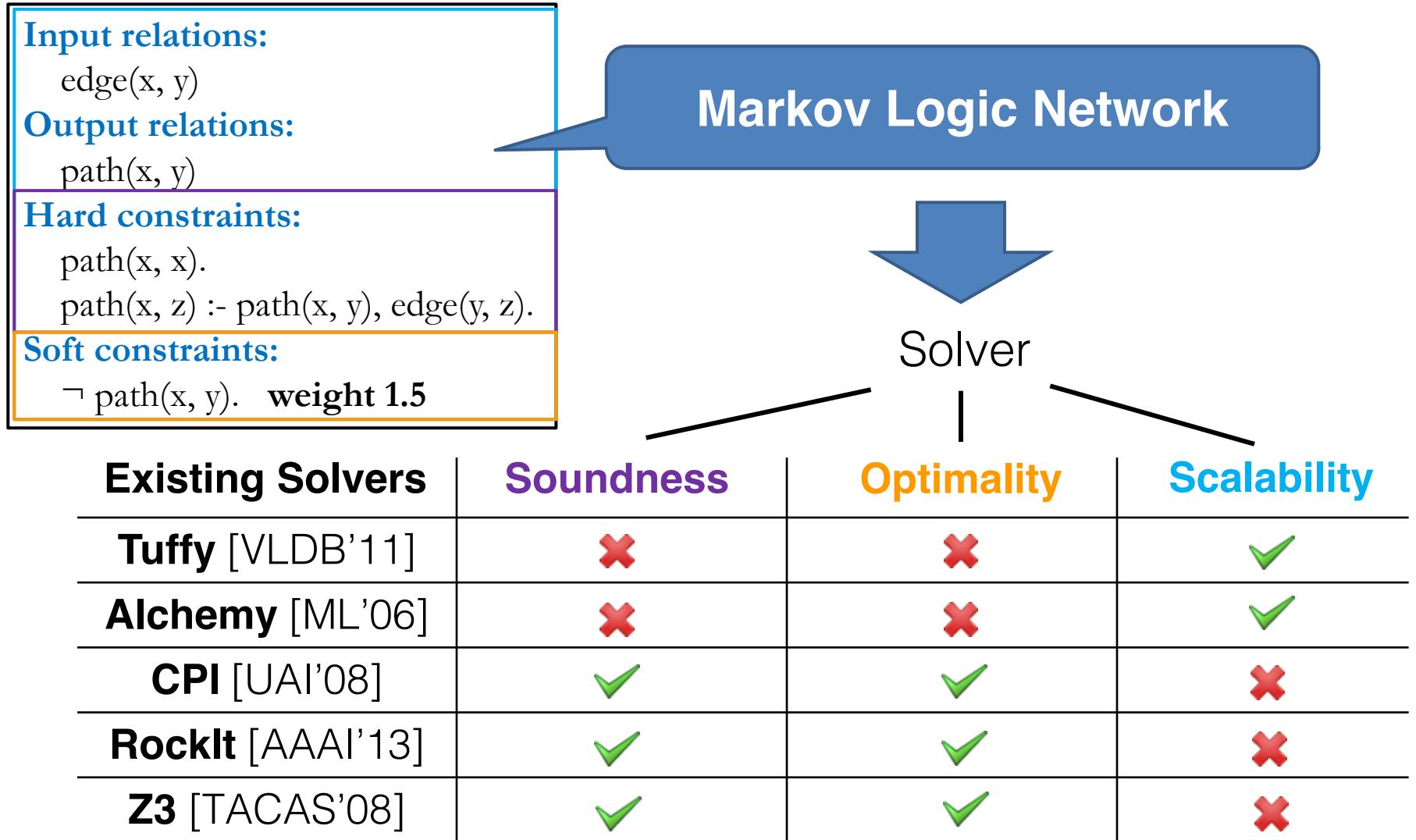
```
totalShelf [] += stock[p] * space [p]
totalProfit[] += stock[p] * profit[p]
product(p) -> stock[p]      >= minStock[p]
product(p) -> stock[p]      <= maxStock[p]
true        -> totalShelf[] <= maxShelf[]
lang:solve:variable(`stock)
lang:solve:max(`totalProfit)
```

# Talk Outline

---

- ▶ Background
- ▶ Part I: Applications in Software Analysis
- ▶ Part II: Techniques for MaxSAT Solving
- ▶ Conclusion

# The Inference Problem

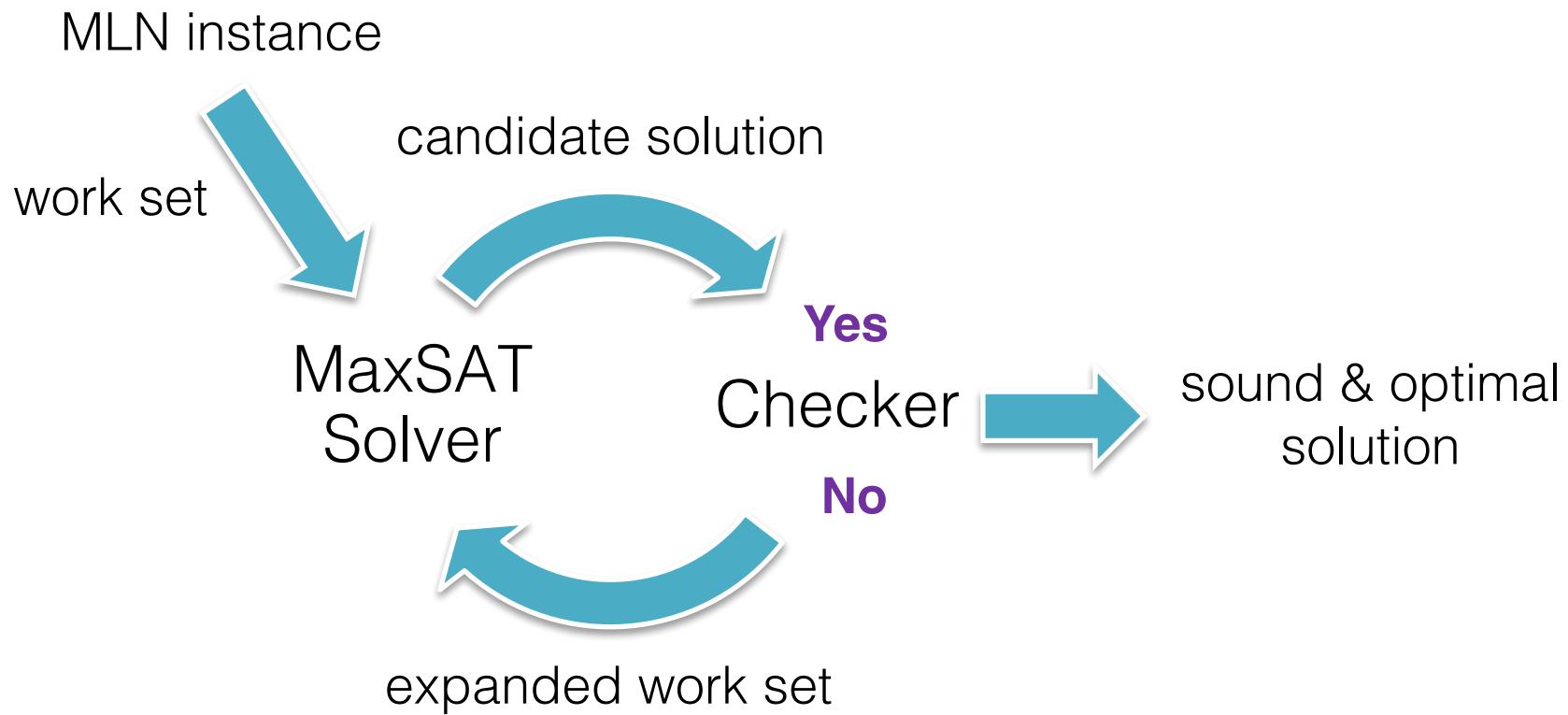


# Overview of Techniques

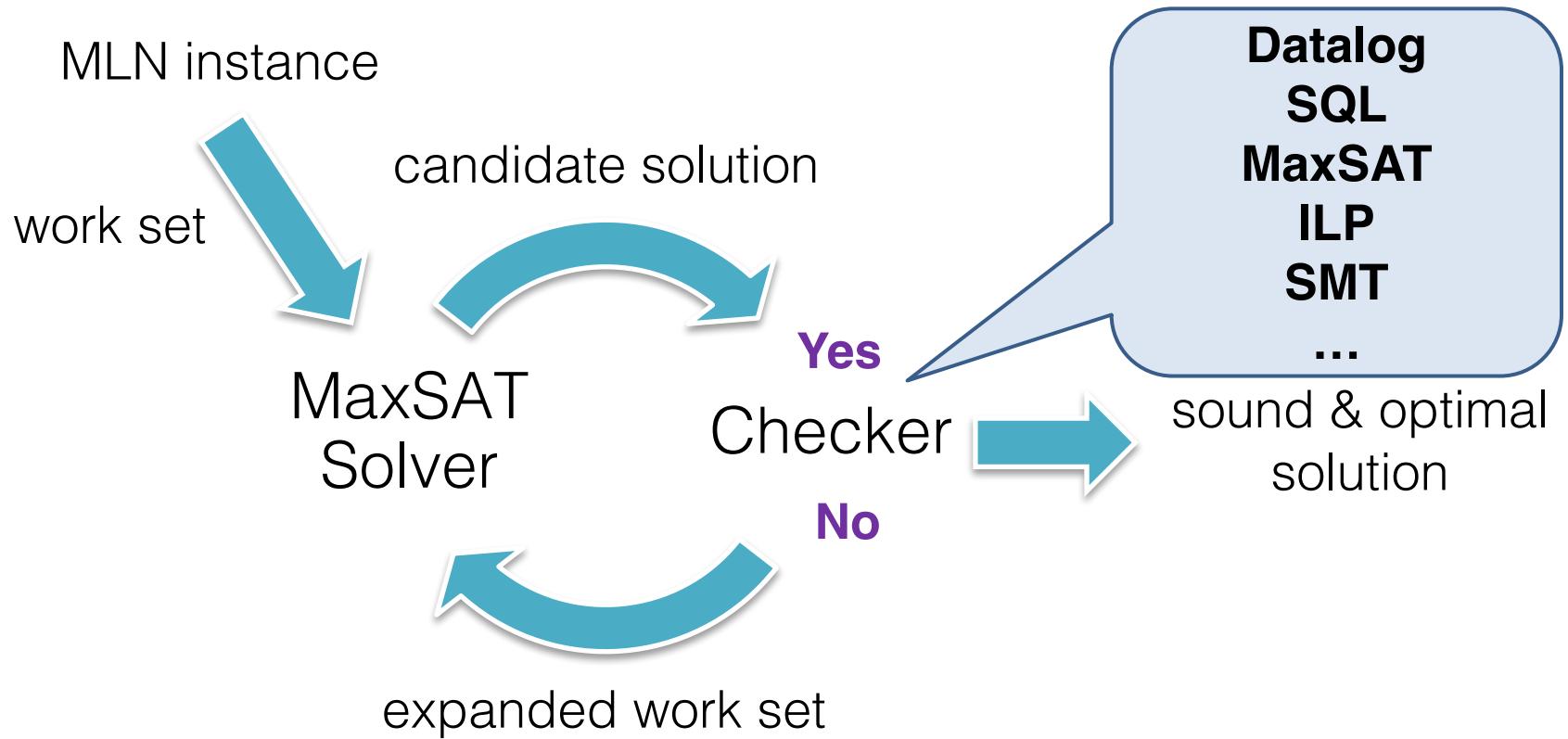
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- ▶ General Framework [SAT 2015]
- ▶ Bottom-Up Solving [AAAI 2016]
- ▶ Top-Down Solving [POPL 2016]

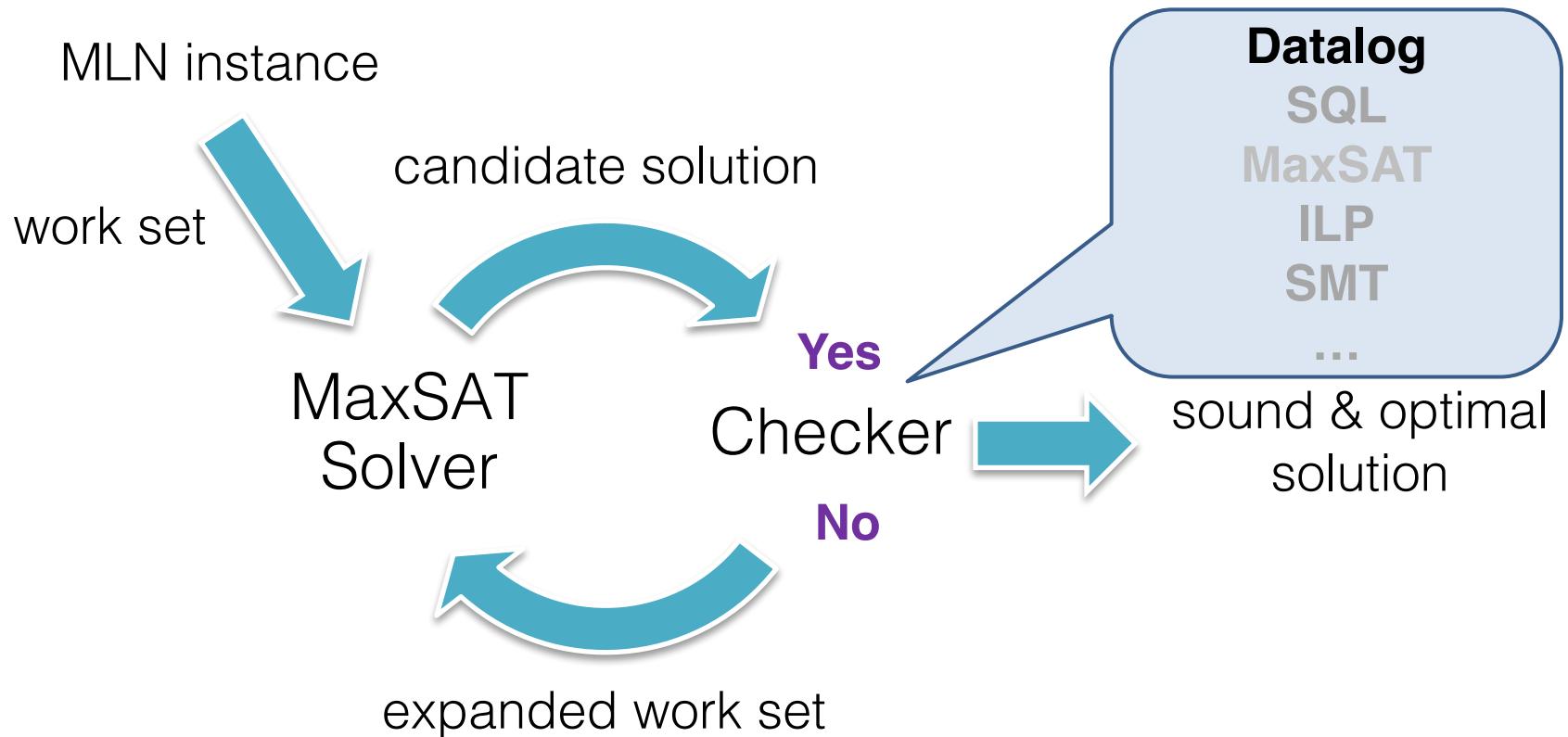
# Framework Architecture



# Framework Instances

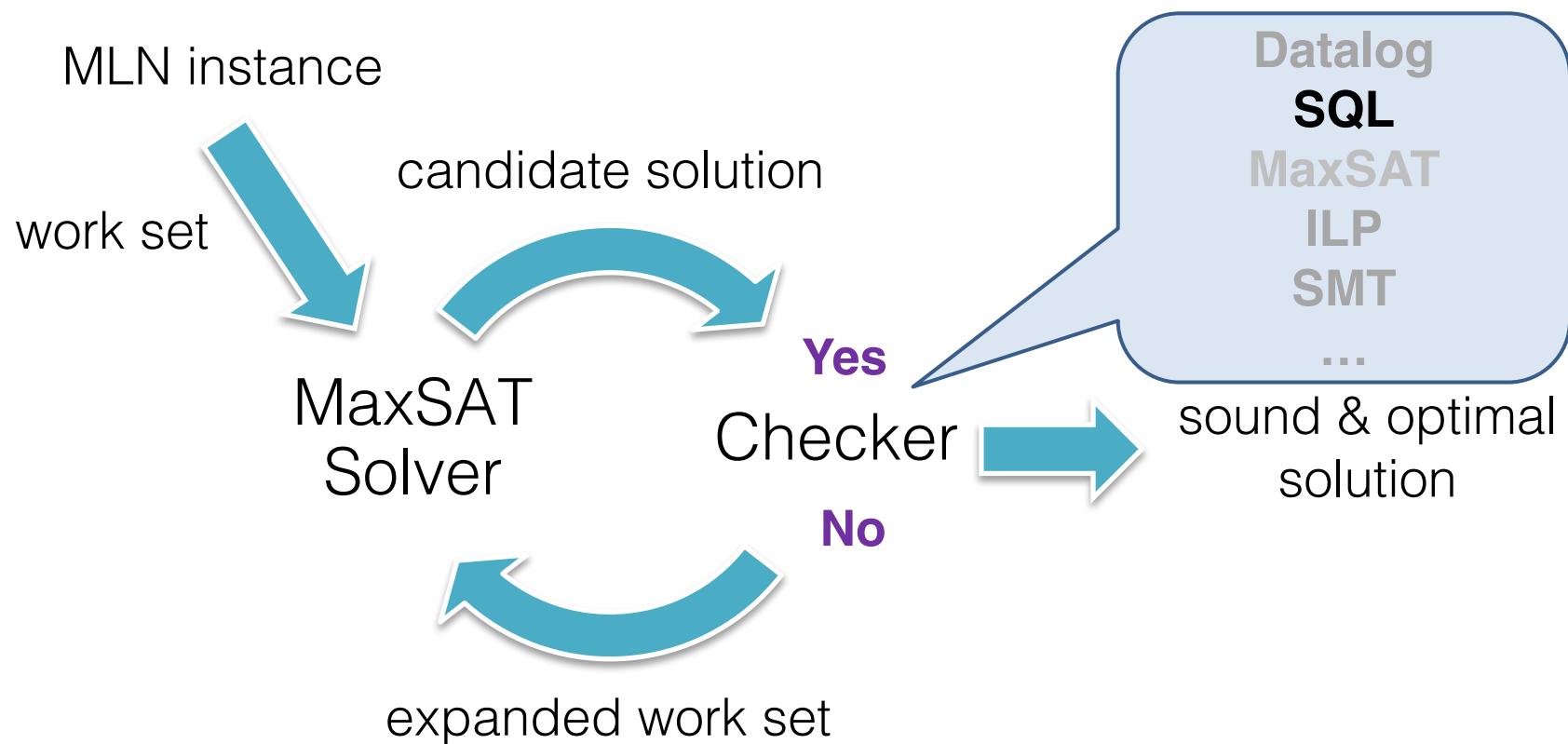


# Framework Instance: Abstraction Refinement



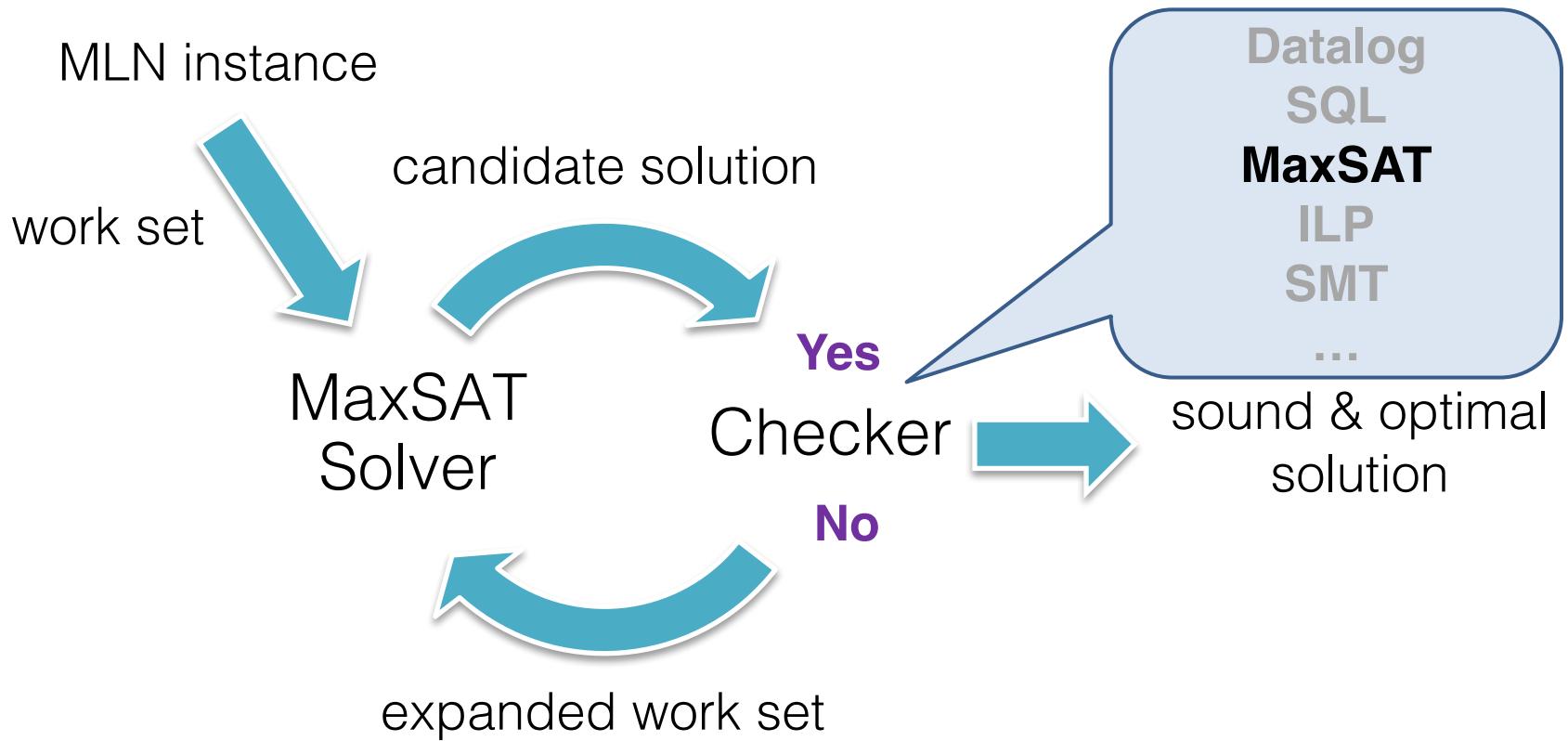
On Abstraction Refinement for Program Analyses in Datalog  
[PLDI 2014]

# Framework Instance: Bottom-Up Solving



Scaling Relational Inference Using Proofs and Refutations  
[AAAI 2016]

# Framework Instance: Top-Down Solving



Query-Guided Maximum Satisfiability  
[POPL 2016]

# Overview of Techniques

---

- ▶ General Framework [SAT 2015]
- ▶ Bottom-Up Solving [AAAI 2016]
- ▶ Top-Down Solving [POPL 2016]

# Bottom-Up Solving

---

Follows **cutting-plane method [Riedl'09]** with three new insights for better scalability on our applications:

1. Exploits **high-level structure of MLN** to efficiently find new ground constraints violated current solution.
2. Accelerates convergence by eagerly grounding **Horn constraints** using Datalog solver.
3. **Terminates earlier** by checking objective value (rather than set of violated soft constraints) for saturation.

# Example

**Input relations:**

$\text{edge}(x, y)$

**Output relations:**

$\text{path}(x, y)$

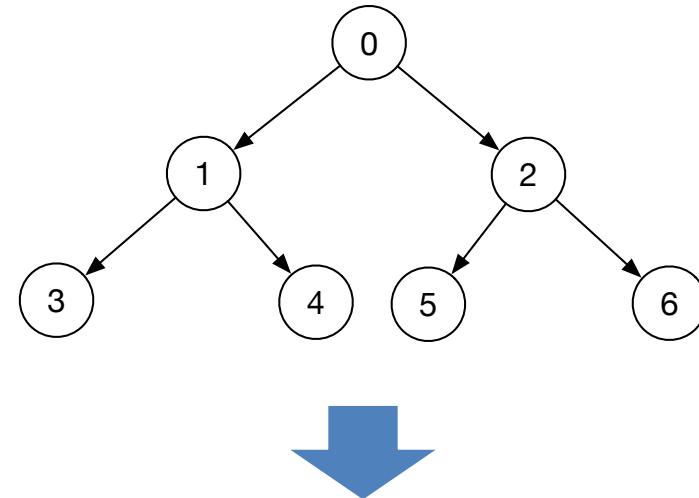
**Hard constraints:**

$\text{path}(x, x).$

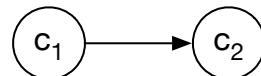
$\text{path}(x, z) :- \text{path}(x, y), \text{edge}(y, z).$

**Soft constraints:**

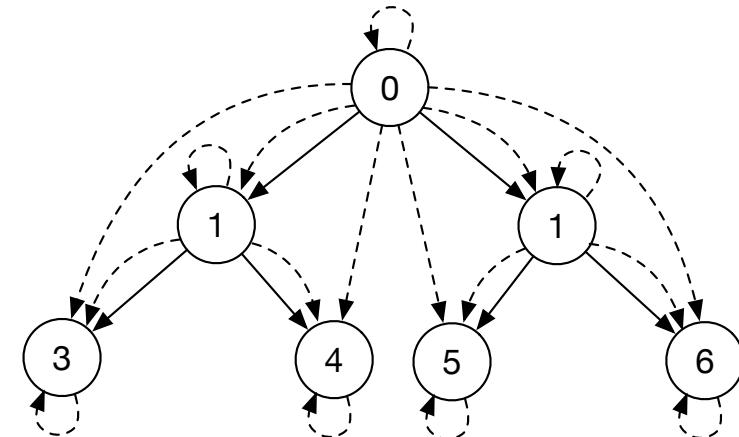
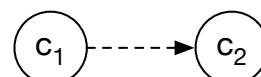
$\neg \text{path}(x, y).$  weight 1.5



$\text{edge}(c_1, c_2)$



$\text{path}(c_1, c_2)$



# Example: Iteration 1 - Solve

**Input relations:**

$\text{edge}(x, y)$

**Output relations:**

$\text{path}(x, y)$

**Hard constraints:**

$\text{path}(x, x).$

$\text{path}(x, z) :- \text{path}(x, y), \text{edge}(y, z).$

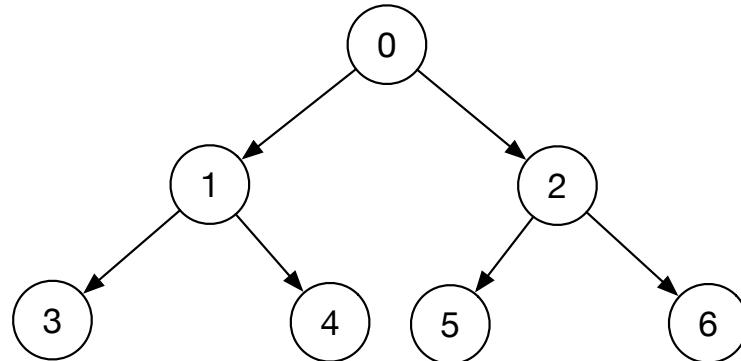
**Soft constraints:**

$\neg \text{path}(x, y).$  weight 1.5

**Hard clauses:**

$\text{edge}(0,1) \wedge \dots \wedge \text{edge}(2,6)$

**Soft clauses:**



# Example: Iteration 1 - Check

**Input relations:**

$\text{edge}(x, y)$

**Output relations:**

$\text{path}(x, y)$

**Hard constraints:**

$\text{path}(x, x).$

$\text{path}(x, z) \leftarrow \text{path}(x, y), \text{edge}(y, z).$

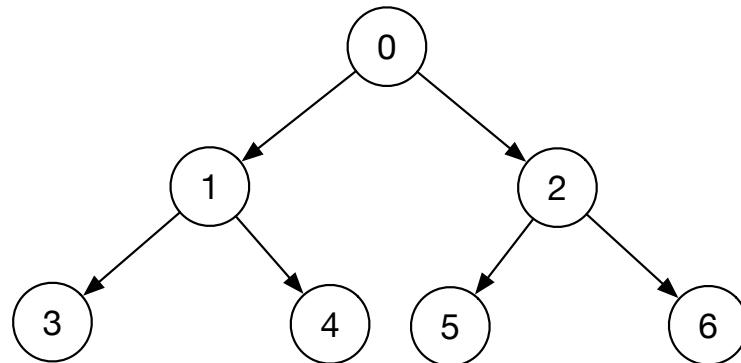
**Soft constraints:**

$\neg \text{path}(x, y).$  weight 1.5

**Hard clauses:**

$\text{edge}(0,1) \wedge \dots \wedge \text{edge}(2,6)$

**Soft clauses:**



# Example: Iteration 2 - Solve

**Input relations:**

$\text{edge}(x, y)$

**Output relations:**

$\text{path}(x, y)$

**Hard constraints:**

$\text{path}(x, x).$

$\text{path}(x, z) \leftarrow \text{path}(x, y), \text{edge}(y, z).$

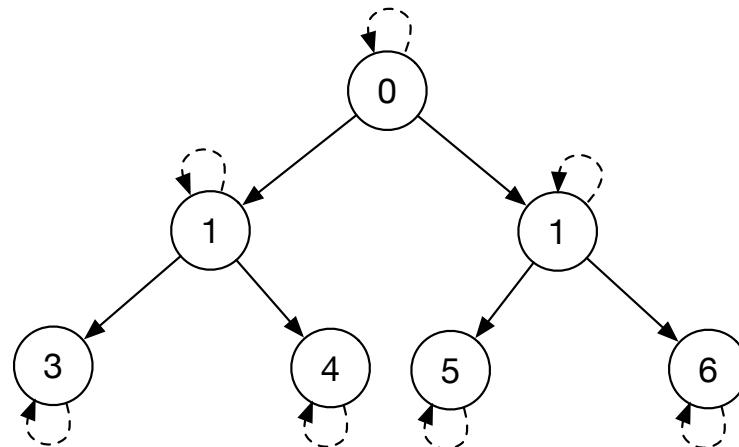
**Soft constraints:**

$\neg \text{path}(x, y).$  weight 1.5

**Hard clauses:**

$\text{edge}(0,1) \wedge \dots \wedge \text{edge}(2,6) \wedge$   
 $\text{path}(0,0) \wedge \dots \wedge \text{path}(6,6)$

**Soft clauses:**



# Example: Iteration 2 - Check

**Input relations:**

$\text{edge}(x, y)$

**Output relations:**

$\text{path}(x, y)$

**Hard constraints:**

$\text{path}(x, x).$

$\text{path}(x, z) \leftarrow \text{path}(x, y), \text{edge}(y, z).$

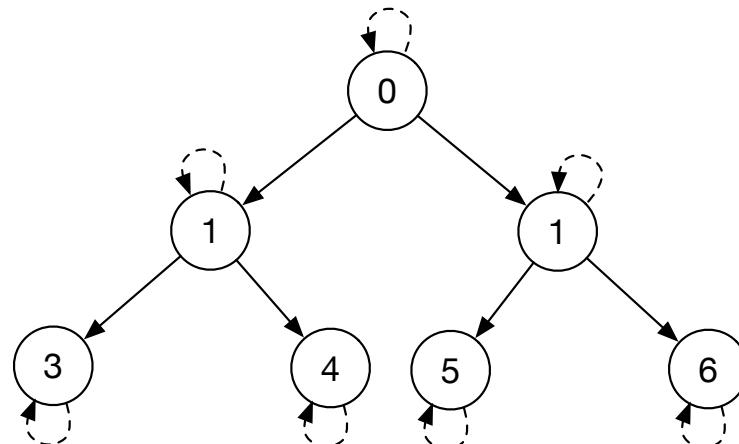
**Soft constraints:**

$\neg \text{path}(x, y).$  weight 1.5

**Hard clauses:**

$\text{edge}(0,1) \wedge \dots \wedge \text{edge}(2,6) \wedge$   
 $\text{path}(0,0) \wedge \dots \wedge \text{path}(6,6)$

**Soft clauses:**



# Example: Iteration 3 - Solve

**Input relations:**

$\text{edge}(x, y)$

**Output relations:**

$\text{path}(x, y)$

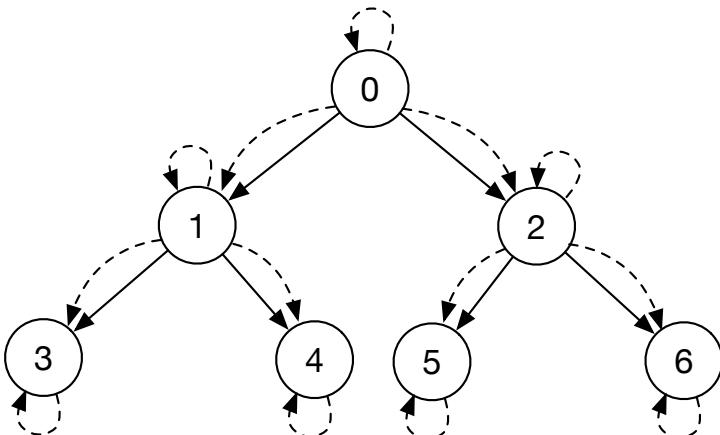
**Hard constraints:**

$\text{path}(x, x).$

$\text{path}(x, z) \leftarrow \text{path}(x, y), \text{edge}(y, z).$

**Soft constraints:**

$\neg \text{path}(x, y).$  weight 1.5



**Hard clauses:**

$\text{edge}(0,1) \wedge \dots \wedge \text{edge}(2,6) \wedge$

$\text{path}(0,0) \wedge \dots \wedge \text{path}(6,6) \wedge$

$\text{path}(0,1) \vee \neg \text{path}(0,0) \vee \neg \text{edge}(0,1) \wedge$

$\text{path}(0,2) \vee \neg \text{path}(0,0) \vee \neg \text{edge}(0,2) \wedge$

$\dots \wedge$

$\text{path}(2,6) \vee \neg \text{path}(2,2) \vee \neg \text{edge}(2,6)$

**Soft clauses:**

$(\neg \text{path}(0,0) \text{ weight } 1.5) \wedge$

$\dots \wedge$

$(\neg \text{path}(6,6) \text{ weight } 1.5)$

# Example: Iteration 3 - Check

**Input relations:**

$\text{edge}(x, y)$

**Output relations:**

$\text{path}(x, y)$

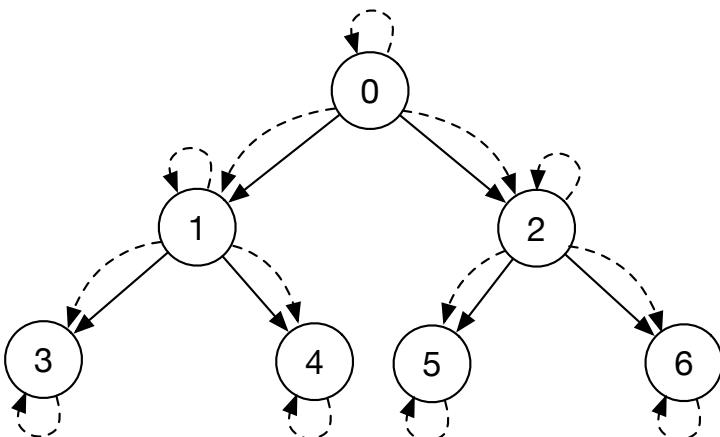
**Hard constraints:**

$\text{path}(x, x).$

$\text{path}(x, z) \leftarrow \text{path}(x, y), \text{edge}(y, z).$

**Soft constraints:**

$\neg \text{path}(x, y).$  weight 1.5



**Hard clauses:**

$$\begin{aligned} &\text{edge}(0,1) \wedge \dots \wedge \text{edge}(2,6) \wedge \\ &\text{path}(0,0) \wedge \dots \wedge \text{path}(6,6) \wedge \\ &\text{path}(0,1) \vee \neg \text{path}(0,0) \vee \neg \text{edge}(0,1) \wedge \\ &\text{path}(0,2) \vee \neg \text{path}(0,0) \vee \neg \text{edge}(0,2) \wedge \\ &\dots \wedge \\ &\text{path}(2,6) \vee \neg \text{path}(2,2) \vee \neg \text{edge}(2,6) \end{aligned}$$

**Soft clauses:**

$$\begin{aligned} &(\neg \text{path}(0,0) \text{ weight } 1.5) \wedge \\ &\dots \wedge \\ &(\neg \text{path}(6,6) \text{ weight } 1.5) \end{aligned}$$

# Example: Iteration 4 - Solve

**Input relations:**

$\text{edge}(x, y)$

**Output relations:**

$\text{path}(x, y)$

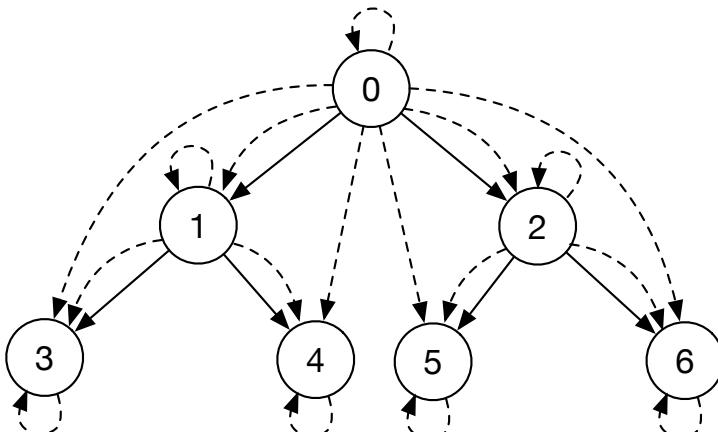
**Hard constraints:**

$\text{path}(x, x).$

$\text{path}(x, z) \leftarrow \text{path}(x, y), \text{edge}(y, z).$

**Soft constraints:**

$\neg \text{path}(x, y).$  weight 1.5



**Hard clauses:**

$\text{edge}(0,1) \wedge \dots \wedge \text{edge}(2,6) \wedge$

$\text{path}(0,0) \wedge \dots \wedge \text{path}(6,6) \wedge$

$\text{path}(0,1) \vee \neg \text{path}(0,0) \vee \neg \text{edge}(0,1) \wedge$

$\text{path}(0,2) \vee \neg \text{path}(0,0) \vee \neg \text{edge}(0,2) \wedge$

$\dots \wedge$

$\text{path}(2,6) \vee \neg \text{path}(2,2) \vee \neg \text{edge}(2,6) \wedge$

$\text{path}(0,3) \vee \neg \text{path}(0,1) \vee \neg \text{edge}(1,3) \wedge$

$\dots \wedge$

$\text{path}(0,6) \vee \neg \text{path}(0,2) \vee \neg \text{edge}(2,6)$

**Soft clauses:**

$(\neg \text{path}(0,0) \text{ weight } 1.5) \wedge$

$\dots \wedge$

$(\neg \text{path}(6,6) \text{ weight } 1.5) \wedge$

$(\neg \text{path}(0,1) \text{ weight } 1.5) \wedge$

$\dots \wedge$

$(\neg \text{path}(2,6) \text{ weight } 1.5)$

# Example: Iteration 4 - Check

**Input relations:**

$\text{edge}(x, y)$

**Output relations:**

$\text{path}(x, y)$

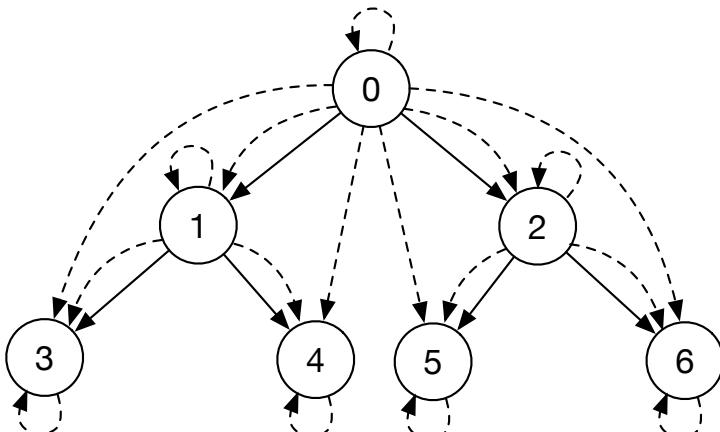
**Hard constraints:**

$\text{path}(x, x).$

$\text{path}(x, z) \leftarrow \text{path}(x, y), \text{edge}(y, z).$

**Soft constraints:**

$\neg \text{path}(x, y).$  weight 1.5



**Hard clauses:**

$$\begin{aligned} & \text{edge}(0,1) \wedge \dots \wedge \text{edge}(2,6) \wedge \\ & \text{path}(0,0) \wedge \dots \wedge \text{path}(6,6) \wedge \\ & \text{path}(0,1) \vee \neg \text{path}(0,0) \vee \neg \text{edge}(0,1) \wedge \\ & \text{path}(0,2) \vee \neg \text{path}(0,0) \vee \neg \text{edge}(0,2) \wedge \\ & \dots \wedge \\ & \text{path}(2,6) \vee \neg \text{path}(2,2) \vee \neg \text{edge}(2,6) \wedge \\ & \text{path}(0,3) \vee \neg \text{path}(0,1) \vee \neg \text{edge}(1,3) \wedge \\ & \dots \wedge \\ & \text{path}(0,6) \vee \neg \text{path}(0,2) \vee \neg \text{edge}(2,6) \end{aligned}$$

**Soft clauses:**

$$\begin{aligned} & (\neg \text{path}(0,0) \text{ weight } 1.5) \wedge \\ & \dots \wedge \\ & (\neg \text{path}(6,6) \text{ weight } 1.5) \wedge \\ & (\neg \text{path}(0,1) \text{ weight } 1.5) \wedge \\ & \dots \wedge \\ & (\neg \text{path}(2,6) \text{ weight } 1.5) \end{aligned}$$

# Example: Iteration 5 - Solve

**Input relations:**

$\text{edge}(x, y)$

**Output relations:**

$\text{path}(x, y)$

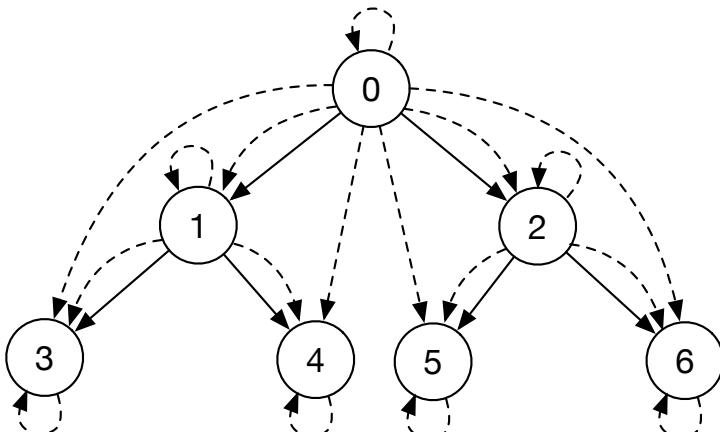
**Hard constraints:**

$\text{path}(x, x).$

$\text{path}(x, z) \leftarrow \text{path}(x, y), \text{edge}(y, z).$

**Soft constraints:**

$\neg \text{path}(x, y).$  weight 1.5



**Hard clauses:**

$$\begin{aligned} & \text{edge}(0,1) \wedge \dots \wedge \text{edge}(2,6) \wedge \\ & \text{path}(0,0) \wedge \dots \wedge \text{path}(6,6) \wedge \\ & \text{path}(0,1) \vee \neg \text{path}(0,0) \vee \neg \text{edge}(0,1) \wedge \\ & \text{path}(0,2) \vee \neg \text{path}(0,0) \vee \neg \text{edge}(0,2) \wedge \\ & \dots \wedge \\ & \text{path}(2,6) \vee \neg \text{path}(2,2) \vee \neg \text{edge}(2,6) \wedge \\ & \text{path}(0,3) \vee \neg \text{path}(0,1) \vee \neg \text{edge}(1,3) \wedge \\ & \dots \wedge \\ & \text{path}(0,6) \vee \neg \text{path}(0,2) \vee \neg \text{edge}(2,6) \end{aligned}$$

**Soft clauses:**

$$\begin{aligned} & (\neg \text{path}(0,0) \text{ weight } 1.5) \wedge \\ & \dots \wedge \\ & (\neg \text{path}(6,6) \text{ weight } 1.5) \wedge \\ & (\neg \text{path}(0,1) \text{ weight } 1.5) \wedge \\ & \dots \wedge \\ & (\neg \text{path}(2,6) \text{ weight } 1.5) \wedge \\ & (\neg \text{path}(0,3) \text{ weight } 1.5) \wedge \\ & \dots \wedge \\ & (\neg \text{path}(0,6) \text{ weight } 1.5) \end{aligned}$$

# Example: Iteration 5 - Check

- 1) All hard constraints are satisfied
- 2) No new violated soft constraints  
=> sound to terminate

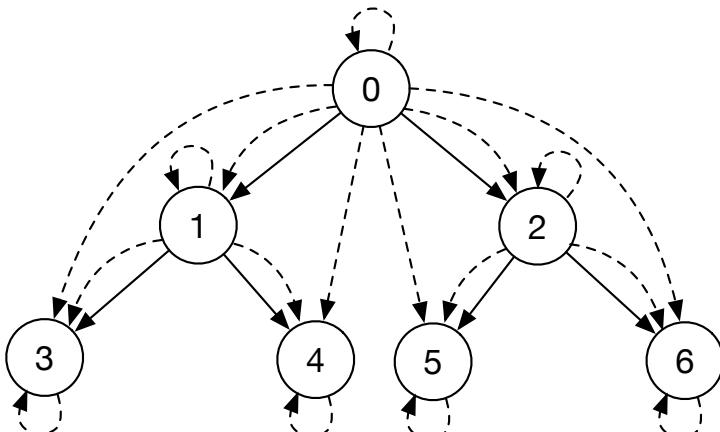
Hard constraints:

$\text{path}(x, x).$

$\text{path}(x, z) :- \text{path}(x, y), \text{edge}(y, z).$

Soft constraints:

$\neg \text{path}(x, y).$  weight 1.5



Hard clauses:

$$\begin{aligned} & \text{edge}(0,1) \wedge \dots \wedge \text{edge}(2,6) \wedge \\ & \text{path}(0,0) \wedge \dots \wedge \text{path}(6,6) \wedge \\ & \text{path}(0,1) \vee \neg \text{path}(0,0) \vee \neg \text{edge}(0,1) \wedge \\ & \text{path}(0,2) \vee \neg \text{path}(0,0) \vee \neg \text{edge}(0,2) \wedge \\ & \dots \wedge \\ & \text{path}(2,6) \vee \neg \text{path}(2,2) \vee \neg \text{edge}(2,6) \wedge \\ & \text{path}(0,3) \vee \neg \text{path}(0,1) \vee \neg \text{edge}(1,3) \wedge \\ & \dots \wedge \\ & \text{path}(0,6) \vee \neg \text{path}(0,2) \vee \neg \text{edge}(2,6) \end{aligned}$$

Soft clauses:

$$\begin{aligned} & (\neg \text{path}(0,0) \text{ weight } 1.5) \wedge \\ & \dots \wedge \\ & (\neg \text{path}(6,6) \text{ weight } 1.5) \wedge \\ & (\neg \text{path}(0,1) \text{ weight } 1.5) \wedge \\ & \dots \wedge \\ & (\neg \text{path}(2,6) \text{ weight } 1.5) \wedge \\ & (\neg \text{path}(0,3) \text{ weight } 1.5) \wedge \\ & \dots \wedge \\ & (\neg \text{path}(0,6) \text{ weight } 1.5) \end{aligned}$$

# Horn-Guided Optimization

**Input relations:**

$\text{edge}(x, y)$

**Output relations:**

$\text{path}(x, y)$

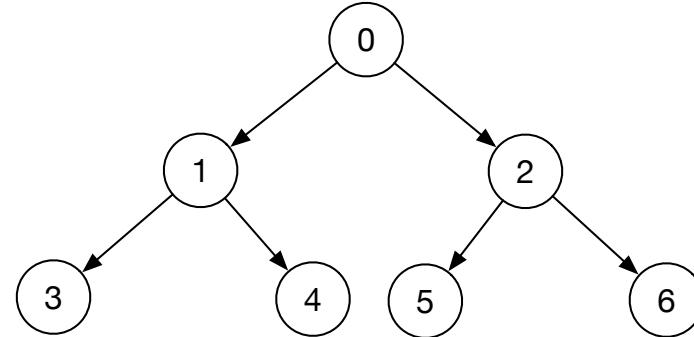
**Hard constraints:**

$\text{path}(x, x)$ . **Horn Rules!**

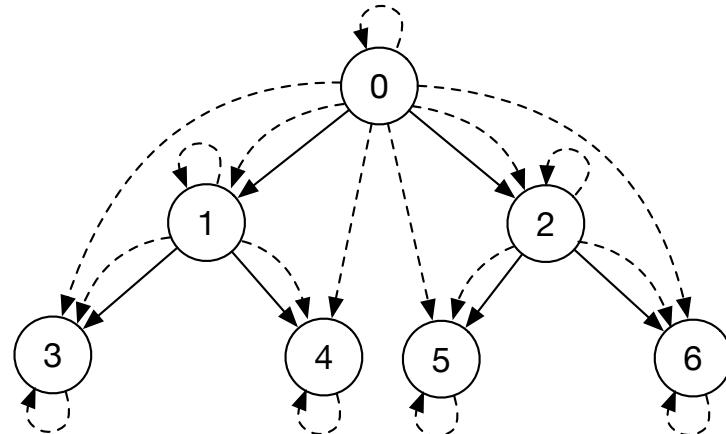
$\text{path}(x, z) \leftarrow \text{path}(x, y), \text{edge}(y, z)$ .

**Soft constraints:**

$\neg \text{path}(x, y)$ . **weight 1.5**



Preprocess  
hard Horn rules



# Horn-Guided Optimization

- 1) All hard constraints are satisfied
- 2) No new violated soft constraints  
=> sound to terminate

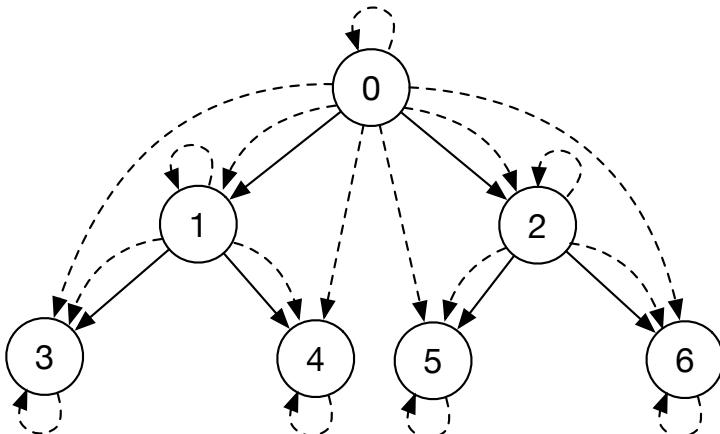
Hard constraints:

$\text{path}(x, x).$

$\text{path}(x, z) \leftarrow \text{path}(x, y), \text{edge}(y, z).$

Soft constraints:

$\neg \text{path}(x, y). \quad \text{weight } 1.5$



Hard clauses:

Soft clauses:

$(\neg \text{path}(0,0) \text{ weight } 1.5) \wedge$   
...  $\wedge$   
 $(\neg \text{path}(6,6) \text{ weight } 1.5) \wedge$   
 $(\neg \text{path}(0,1) \text{ weight } 1.5) \wedge$   
...  $\wedge$   
 $(\neg \text{path}(2,6) \text{ weight } 1.5) \wedge$   
 $(\neg \text{path}(0,3) \text{ weight } 1.5) \wedge$   
...  $\wedge$   
 $(\neg \text{path}(0,6) \text{ weight } 1.5)$

# Performance Evaluation

	total # ground clauses	# iterations		total time (hours : mins)		# ground clauses	
		Lazy	Guided	Lazy	Guided	Lazy	Guided
avrora	$1.8 \times 10^{26}$	492	12	6:31	0:25	0.8M	1.6M
ftp	$3.7 \times 10^{23}$	463	5	7:53	0:08	1.2M	1.4M
hedc	$1.9 \times 10^{24}$	354	6	1:55	0:06	0.8M	0.9M
luindex	$1.6 \times 10^{25}$	481	7	4:07	0:12	0.6M	1.1M
lusearch	$1.7 \times 10^{25}$	429	6	2:38	0:14	0.6M	1.0M
weblech	$4.4 \times 10^{24}$	416	6	1:59	0:07	0.6M	0.9M

# Overview of Techniques

---

- ▶ General Framework [SAT 2015]
- ▶ Bottom-Up Solving [AAAI 2016]
- ▶ Top-Down Solving [POPL 2016]

# Queries in Different Domains

## Program Reasoning:

Does variable **head** alias with variable **tail** on line 50 in Complex.java?



## Information Retrieval:

Is **Dijkstra** most likely an author of “**Structured Programming**”?



# Queries in MaxSAT

	$a$	$\wedge$	(C1)			
	$\neg a$	$\vee b$	$\wedge$	(C2)		
4		$\neg b$	$\vee c$	$\wedge$	(C3)	
2			$\neg c$	$\vee d$	$\wedge$	(C4)
7				$\neg d$		(C5)

QUERIES = {a, d}

# Query-Guided Maximum Satisfiability (Q-MaxSAT)

	$a$	$\wedge$	(C1)		
	$\neg a$	$\vee b$	$\wedge$	(C2)	
4		$\neg b$	$\vee c$	$\wedge$	(C3)
2		$\neg c$	$\vee d$	$\wedge$	(C4)
7		$\neg d$			(C5)

## Q-MaxSAT:

Given a MaxSAT formula  $\varphi$  and a set of queries  $Q \subseteq V$ , a solution to the Q-MaxSAT instance  $(\varphi, Q)$  is a partial solution  $\alpha_Q: Q \rightarrow \{0, 1\}$  such that

$$\exists \alpha \in \text{MaxSAT}(\varphi). (\forall v \in Q. \alpha_Q = \alpha(v))$$

QUERIES = {a, d}

# Query-Guided Maximum Satisfiability (Q-MaxSAT)

	$a$	$\wedge$	(C1)		
	$\neg a$	$\vee b$	$\wedge$	(C2)	
4		$\neg b$	$\vee c$	$\wedge$	(C3)
2		$\neg c$	$\vee d$	$\wedge$	(C4)
7		$\neg d$			(C5)

## Q-MaxSAT:

Given a MaxSAT formula  $\varphi$  and a set of queries  $Q \subseteq V$ , a solution to the Q-MaxSAT instance  $(\varphi, Q)$  is a partial solution  $\alpha_Q: Q \rightarrow \{0, 1\}$  such that

$$\exists \alpha \in \text{MaxSAT}(\varphi). (\forall v \in Q. \alpha_Q = \alpha(v))$$

QUERIES = {a, d}

Solution: a = true, d = false

# Query-Guided Maximum Satisfiability (Q-MaxSAT)

	$a$	$\wedge$	(C1)		
	$\neg a$	$\vee b$	$\wedge$	(C2)	
4		$\neg b$	$\vee c$	$\wedge$	(C3)
2		$\neg c$	$\vee d$	$\wedge$	(C4)
7		$\neg d$			(C5)

## Q-MaxSAT:

Given a MaxSAT formula  $\varphi$  and a set of queries  $Q \subseteq V$ , a solution to the Q-MaxSAT instance  $(\varphi, Q)$  is a partial solution  $\alpha_Q: Q \rightarrow \{0, 1\}$  such that

$$\exists \alpha \in \text{MaxSAT}(\varphi). (\forall v \in Q. \alpha_Q = \alpha(v))$$

QUERIES = {a, d}

MaxSAT Solution: a = true, b = true, c = true, d = false

**Our key idea:**

**Use a small set of clauses to succinctly  
summarize effect of unexplored clauses**

# Example

Queries = {v6}, formula =

v4	weight 100	Λ
v8	weight 100	Λ
¬ v7	weight 100	Λ
¬ v3 ∨ v1	weight 5	Λ
¬ v5 ∨ v2	weight 5	Λ
¬ v5 ∨ v3	weight 5	Λ
¬ v6 ∨ v5	weight 5	Λ
¬ v6 ∨ v7	weight 5	Λ
¬ v4 ∨ v6	weight 5	Λ
¬ v8 ∨ v6	weight 5	Λ
...		

# Example

Queries = {v6}, formula =

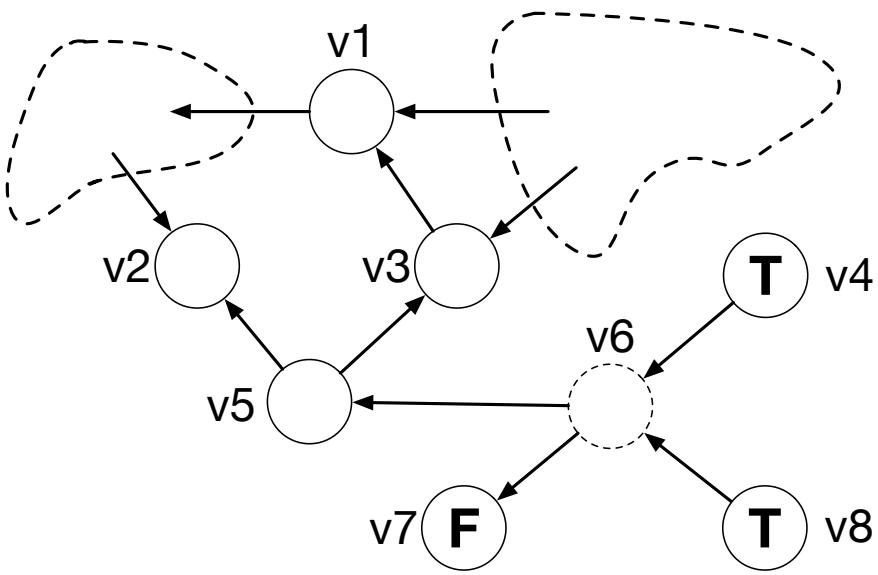
v4	weight 100	Λ
v8	weight 100	Λ
¬ v7	weight 100	Λ
¬ v3 ∨ v1	weight 5	Λ
¬ v5 ∨ v2	weight 5	Λ
¬ v5 ∨ v3	weight 5	Λ
¬ v6 ∨ v5	weight 5	Λ
¬ v6 ∨ v7	weight 5	Λ
¬ v4 ∨ v6	weight 5	Λ
¬ v8 ∨ v6	weight 5	Λ
...		

# Example

Queries = {v6}, formula =

v4	weight 100	Λ
v8	weight 100	Λ
¬ v7	weight 100	Λ
¬ v3 ∨ v1	weight 5	Λ
¬ v5 ∨ v2	weight 5	Λ
¬ v5 ∨ v3	weight 5	Λ
¬ v6 ∨ v5	weight 5	Λ
¬ v6 ∨ v7	weight 5	Λ
¬ v4 ∨ v6	weight 5	Λ
¬ v8 ∨ v6	weight 5	Λ
...		

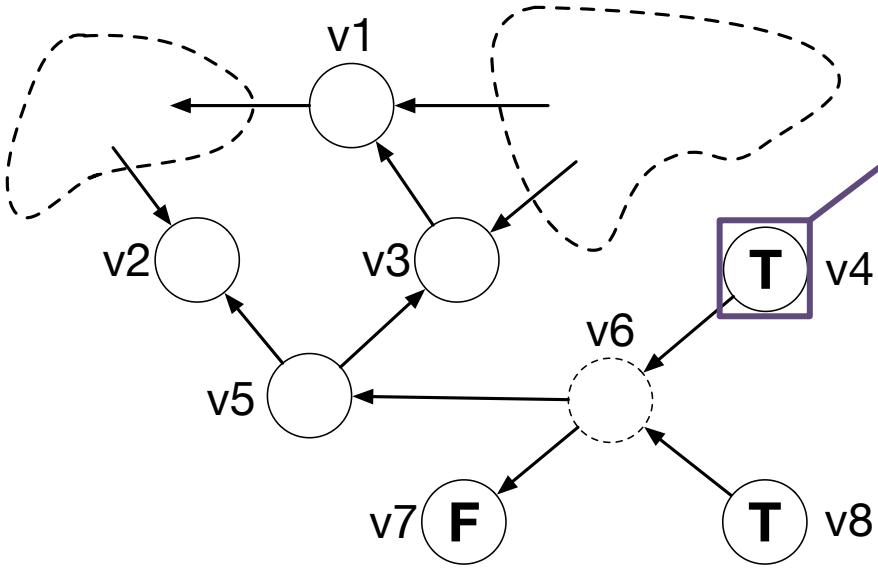
# Example



Queries = {v6}, formula =

v4	weight 100	$\wedge$
v8	weight 100	$\wedge$
¬ v7	weight 100	$\wedge$
¬ v3 $\vee$ v1	weight 5	$\wedge$
¬ v5 $\vee$ v2	weight 5	$\wedge$
¬ v5 $\vee$ v3	weight 5	$\wedge$
¬ v6 $\vee$ v5	weight 5	$\wedge$
¬ v6 $\vee$ v7	weight 5	$\wedge$
¬ v4 $\vee$ v6	weight 5	$\wedge$
¬ v8 $\vee$ v6	weight 5	$\wedge$
...		

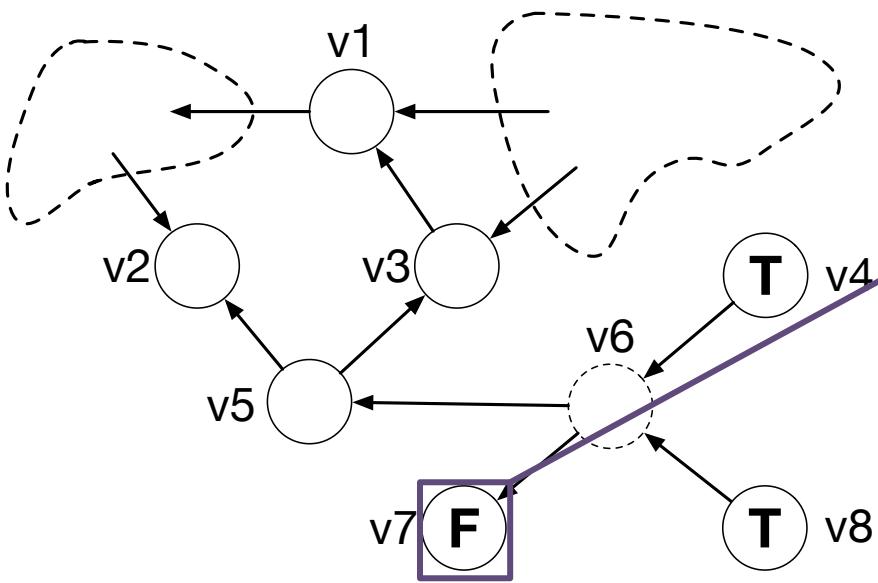
# Example



Queries = {v6}, formula =

v4	weight 100	^
v8	weight 100	^
¬ v7	weight 100	^
¬ v3 ∨ v1	weight 5	^
¬ v5 ∨ v2	weight 5	^
¬ v5 ∨ v3	weight 5	^
¬ v6 ∨ v5	weight 5	^
¬ v6 ∨ v7	weight 5	^
¬ v4 ∨ v6	weight 5	^
¬ v8 ∨ v6	weight 5	^
...		

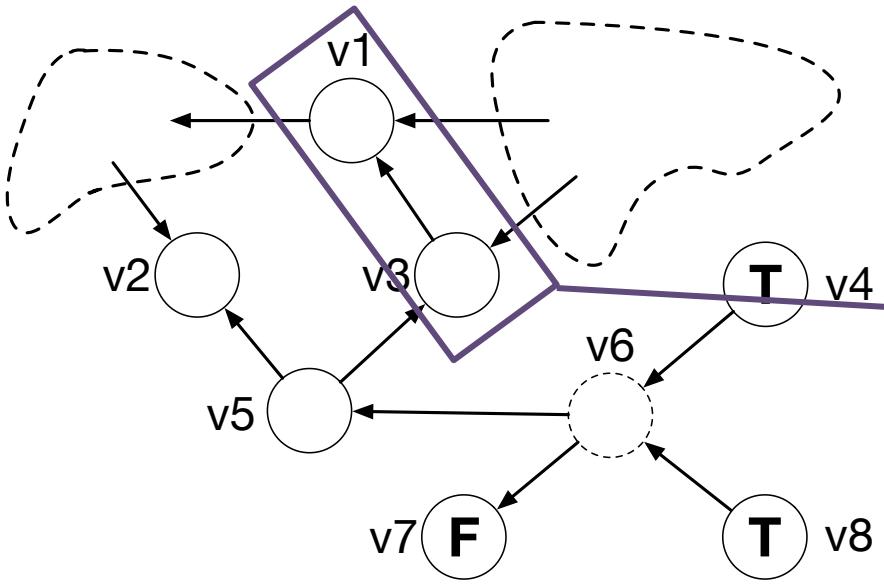
# Example



Queries = {v6}, formula =

v4	weight 100	^
v8	weight 100	^
¬ v7	weight 100	^
¬ v3 ∨ v1	weight 5	^
¬ v5 ∨ v2	weight 5	^
¬ v5 ∨ v3	weight 5	^
¬ v6 ∨ v5	weight 5	^
¬ v6 ∨ v7	weight 5	^
¬ v4 ∨ v6	weight 5	^
¬ v8 ∨ v6	weight 5	^
...		

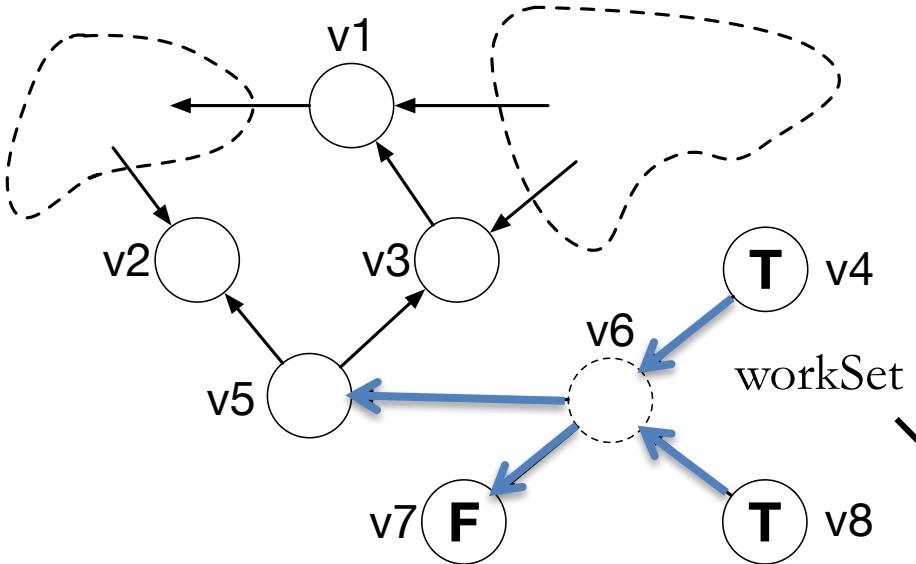
# Example



Queries = {v6}, formula =

v4	weight 100	$\wedge$
v8	weight 100	$\wedge$
¬ v7	weight 100	$\wedge$
¬ v3 $\vee$ v1	weight 5	$\wedge$
¬ v5 $\vee$ v2	weight 5	$\wedge$
¬ v5 $\vee$ v3	weight 5	$\wedge$
¬ v6 $\vee$ v5	weight 5	$\wedge$
¬ v6 $\vee$ v7	weight 5	$\wedge$
¬ v4 $\vee$ v6	weight 5	$\wedge$
¬ v8 $\vee$ v6	weight 5	$\wedge$
...		

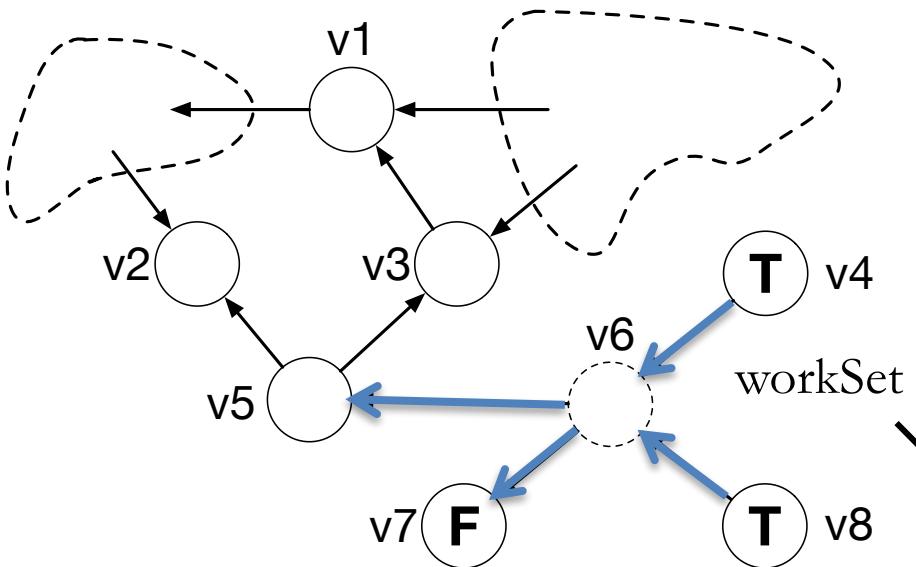
# Example: Iteration 1



Queries = {v6}, formula =

v4	weight 100	^
v8	weight 100	^
¬ v7	weight 100	^
¬ v3 ∨ v1	weight 5	^
¬ v5 ∨ v2	weight 5	^
¬ v5 ∨ v3	weight 5	^
¬ v6 ∨ v5	weight 5	^
¬ v6 ∨ v7	weight 5	^
¬ v4 ∨ v6	weight 5	^
¬ v8 ∨ v6	weight 5	^
...		

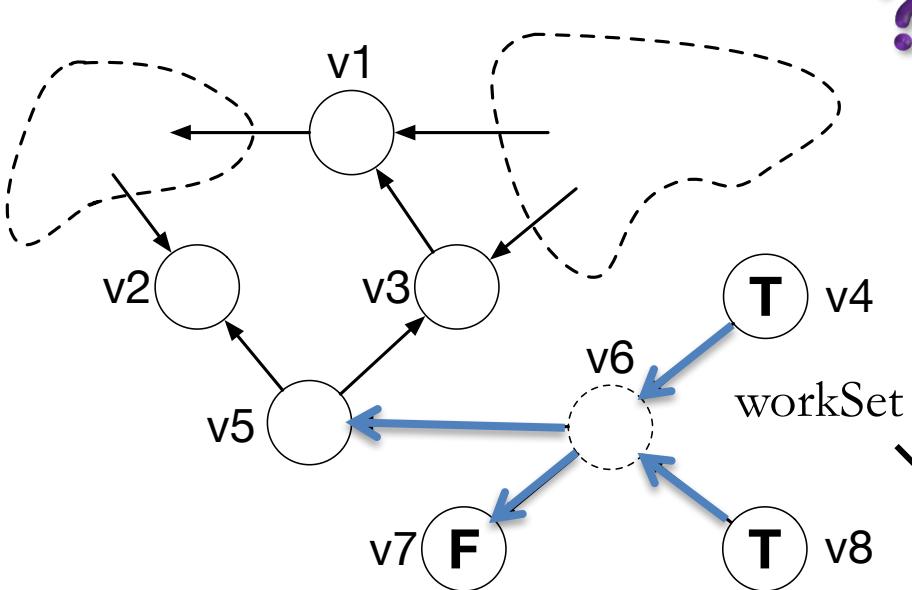
# Example: Iteration 1 (blue = true, red = false)



Queries = { $v_6$ }, formula =

$v_4$	weight 100	$\wedge$
$v_8$	weight 100	$\wedge$
$\neg v_7$	weight 100	$\wedge$
$\neg v_3 \vee v_1$	weight 5	$\wedge$
$\neg v_5 \vee v_2$	weight 5	$\wedge$
$\neg v_5 \vee v_3$	weight 5	$\wedge$
$\neg v_6 \vee v_5$	weight 5	$\wedge$
$\neg v_6 \vee v_7$	weight 5	$\wedge$
$\neg v_4 \vee v_6$	weight 5	$\wedge$
$\neg v_8 \vee v_6$	weight 5	$\wedge$
...		

# Example: Iteration 1 (blue = true, red = false)

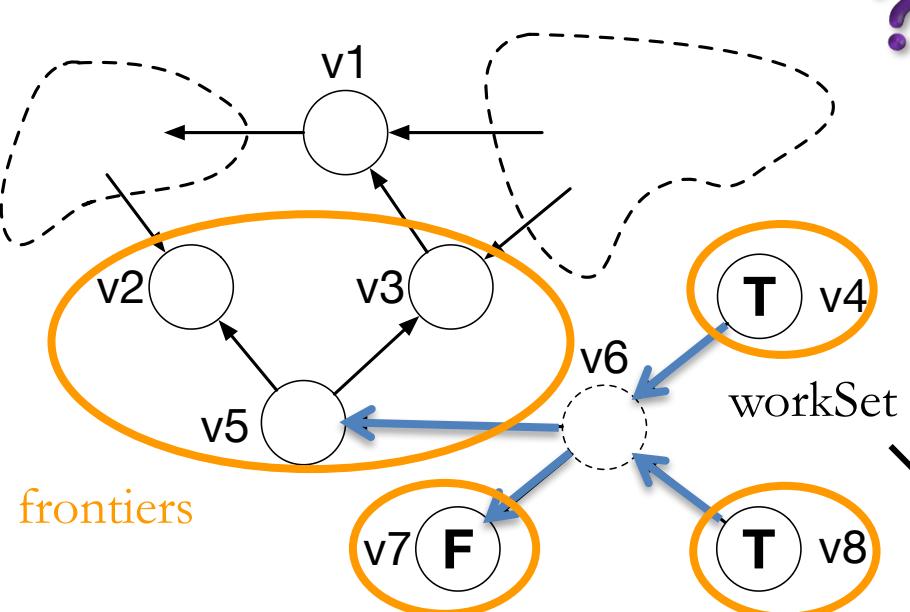


?

Queries = {v6}, formula =

v4	weight 100	^
v8	weight 100	^
¬ v7	weight 100	^
¬ v3 ∨ v1	weight 5	^
¬ v5 ∨ v2	weight 5	^
¬ v5 ∨ v3	weight 5	^
¬ v6 ∨ v5	weight 5	^
¬ v6 ∨ v7	weight 5	^
¬ v4 ∨ v6	weight 5	^
¬ v8 ∨ v6	weight 5	^
...		

# Example: Iteration 1 (blue = true, red = false)

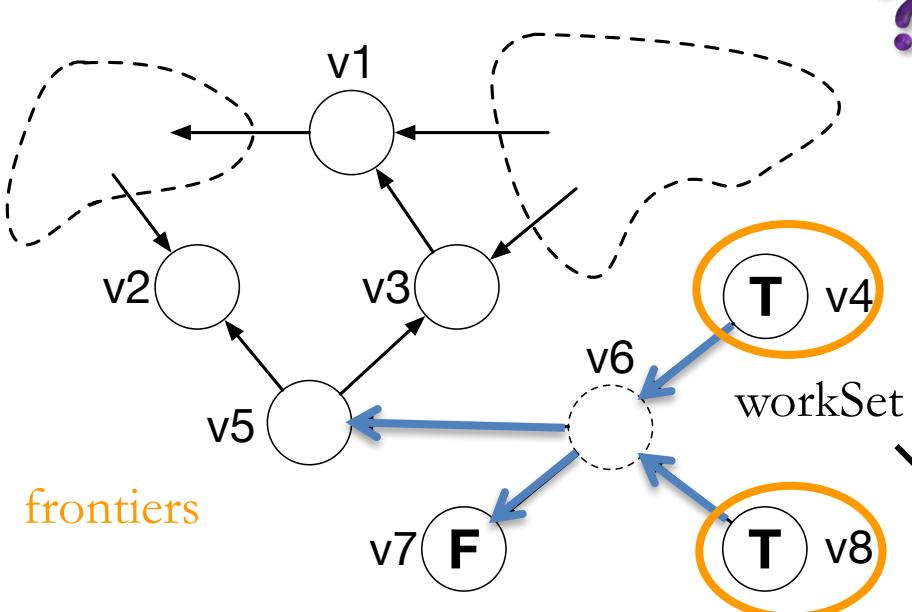


?

Queries = {v6}, formula =

v4	weight 100	Λ
v8	weight 100	Λ
¬ v7	weight 100	Λ
¬ v3 ∨ v1	weight 5	Λ
¬ v5 ∨ v2	weight 5	Λ
¬ v5 ∨ v3	weight 5	Λ
¬ v6 ∨ v5	weight 5	Λ
¬ v6 ∨ v7	weight 5	Λ
¬ v4 ∨ v6	weight 5	Λ
¬ v8 ∨ v6	weight 5	Λ
...		

# Example: Iteration 1 (blue = true, red = false)

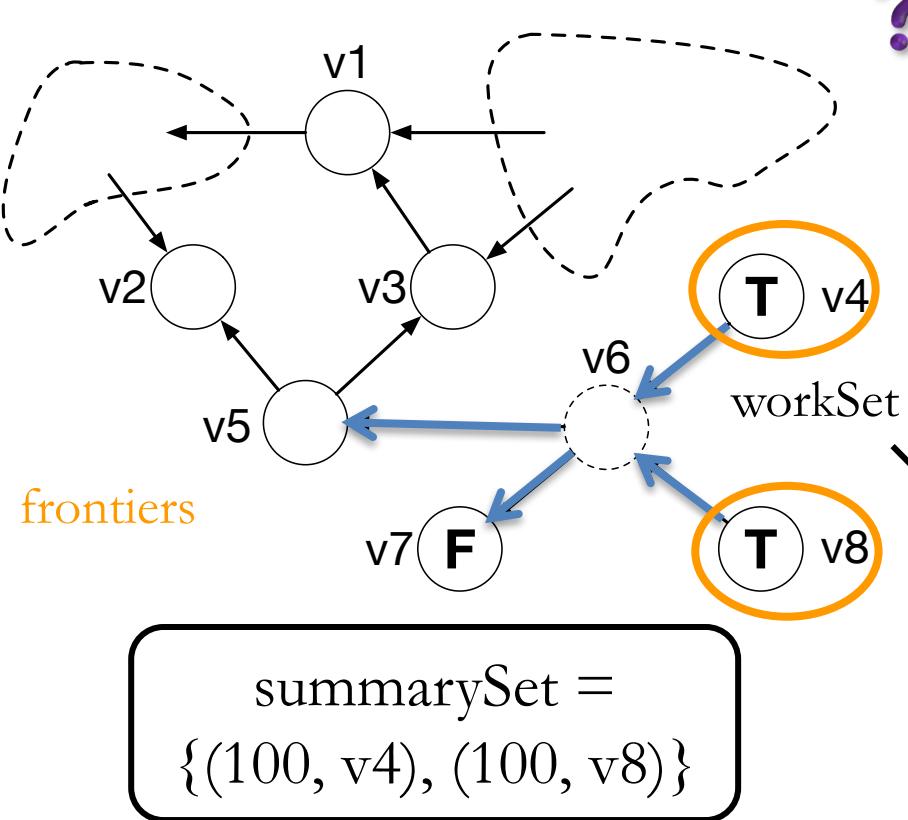


?

Queries = {v6}, formula =

v4	weight 100	Λ
v8	weight 100	Λ
¬ v7	weight 100	Λ
¬ v3 ∨ v1	weight 5	Λ
¬ v5 ∨ v2	weight 5	Λ
¬ v5 ∨ v3	weight 5	Λ
¬ v6 ∨ v5	weight 5	Λ
¬ v6 ∨ v7	weight 5	Λ
¬ v4 ∨ v6	weight 5	Λ
¬ v8 ∨ v6	weight 5	Λ
...		

# Example: Iteration 1 (blue = true, red = false)

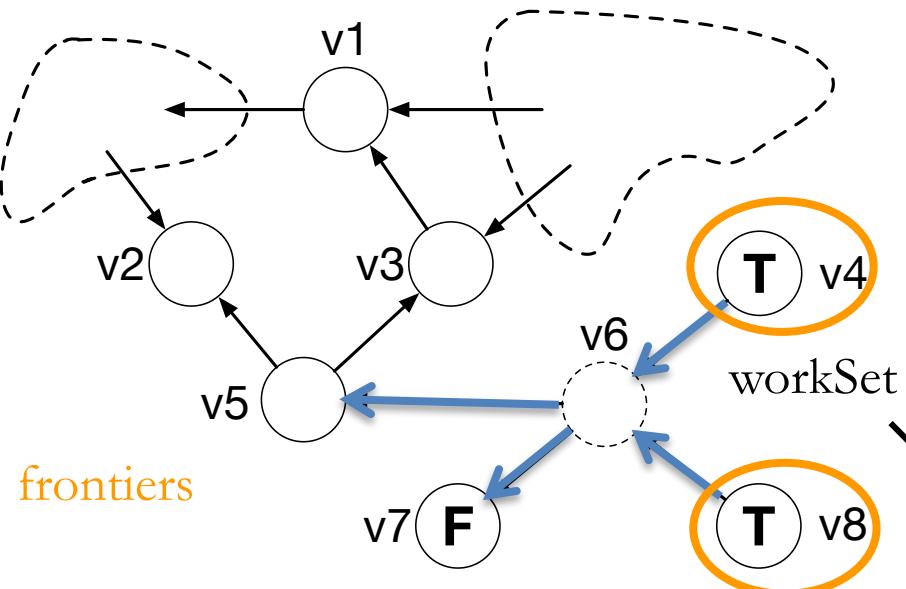


?

Queries = {v6}, formula =

v4	weight 100	$\wedge$
v8	weight 100	$\wedge$
¬ v7	weight 100	$\wedge$
¬ v3 $\vee$ v1	weight 5	$\wedge$
¬ v5 $\vee$ v2	weight 5	$\wedge$
¬ v5 $\vee$ v3	weight 5	$\wedge$
¬ v6 $\vee$ v5	weight 5	$\wedge$
¬ v6 $\vee$ v7	weight 5	$\wedge$
¬ v4 $\vee$ v6	weight 5	$\wedge$
¬ v8 $\vee$ v6	weight 5	$\wedge$
...		

# Example: Iteration 1 (blue = true, red = false)



frontiers

summarySet =  
 $\{(100, v4), (100, v8)\}$

220

$\max(\text{workSet} \cup \text{summarySet}) - \max(\text{workSet}) = 0$

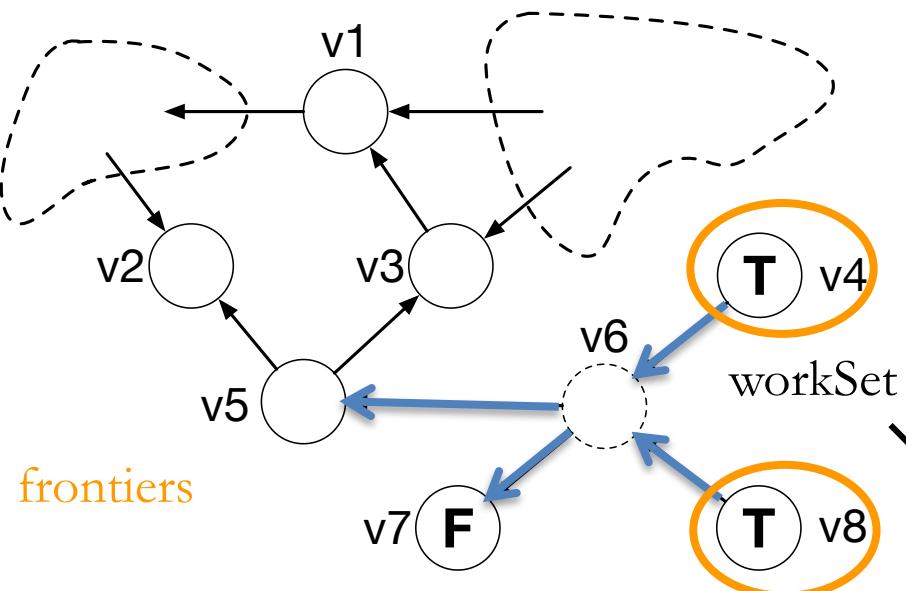
Queries =  $\{v6\}$ , formula =

v4	weight 100	$\wedge$
v8	weight 100	$\wedge$
└ v7	weight 100	$\wedge$
└ v3 $\vee$ v1	weight 5	$\wedge$
└ v5 $\vee$ v2	weight 5	$\wedge$
└ v5 $\vee$ v3	weight 5	$\wedge$
└ v6 $\vee$ v5	weight 5	$\wedge$
└ v6 $\vee$ v7	weight 5	$\wedge$
└ v4 $\vee$ v6	weight 5	$\wedge$
└ v8 $\vee$ v6	weight 5	$\wedge$
...		

20

?

# Example: Iteration 1 (blue = true, red = false)



frontiers

summarySet =  
 $\{(100, v4), (100, v8)\}$

220

$\max(\text{workSet} \cup \text{summarySet}) - \max(\text{workSet}) = 0$

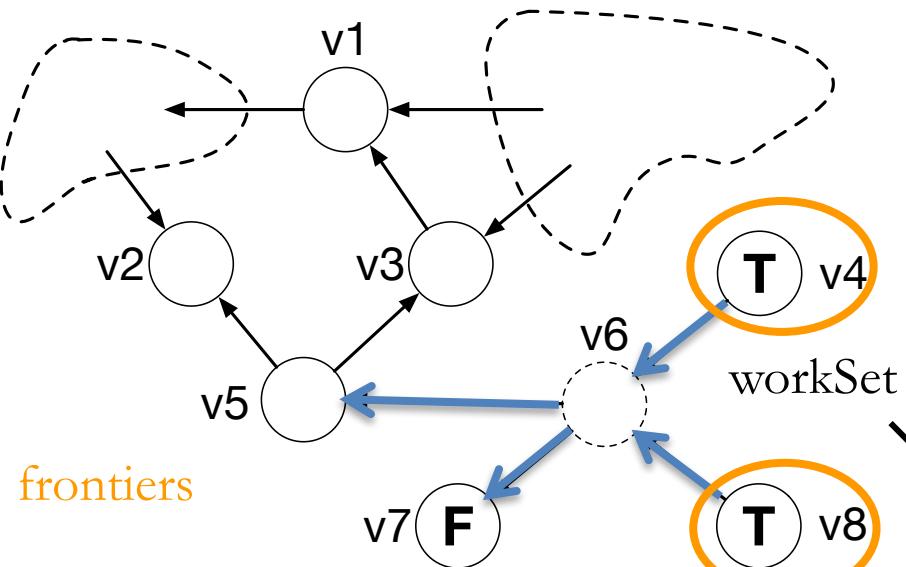


Queries =  $\{v6\}$ , formula =

v4	weight 100	$\wedge$
v8	weight 100	$\wedge$
└ v7	weight 100	$\wedge$
└ v3 $\vee$ v1	weight 5	$\wedge$
└ v5 $\vee$ v2	weight 5	$\wedge$
└ v5 $\vee$ v3	weight 5	$\wedge$
└ v6 $\vee$ v5	weight 5	$\wedge$
└ v6 $\vee$ v7	weight 5	$\wedge$
└ v4 $\vee$ v6	weight 5	$\wedge$
└ v8 $\vee$ v6	weight 5	$\wedge$
...		

20

# Example: Iteration 1 (blue = true, red = false)



frontiers

summarySet =  
 $\{(100, v4), (100, v8)\}$

220

$\max(\text{workSet} \cup \text{summarySet}) - \max(\text{workSet}) = 0$

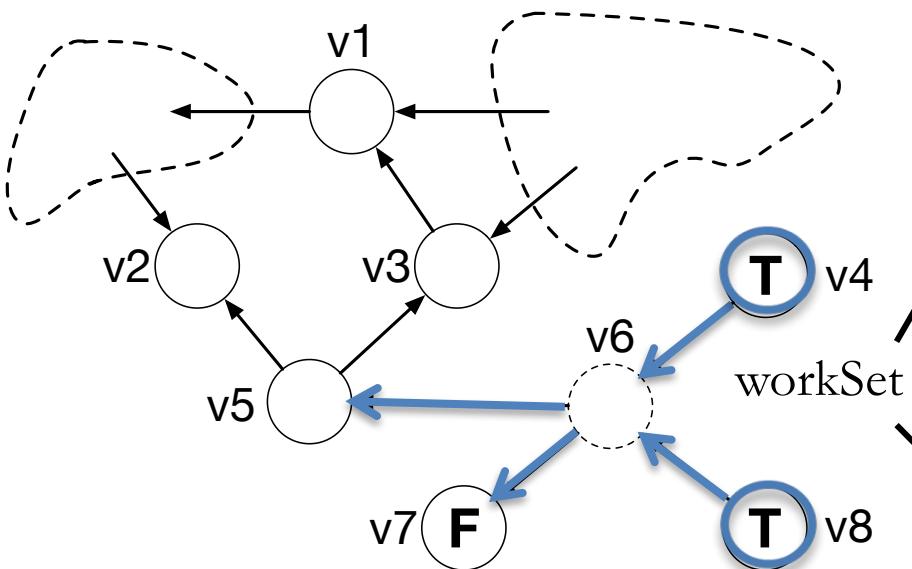
Queries =  $\{v6\}$ , formula =

v4	weight 100	$\wedge$
v8	weight 100	$\wedge$
¬ v7	weight 100	$\wedge$
¬ v3 $\vee$ v1	weight 5	$\wedge$
¬ v5 $\vee$ v2	weight 5	$\wedge$
¬ v5 $\vee$ v3	weight 5	$\wedge$
¬ v6 $\vee$ v5	weight 5	$\wedge$
¬ v6 $\vee$ v7	weight 5	$\wedge$
¬ v4 $\vee$ v6	weight 5	$\wedge$
¬ v8 $\vee$ v6	weight 5	$\wedge$

v4 = true, v5 = true, v6 = true,  
 v7 = true, v8 = true



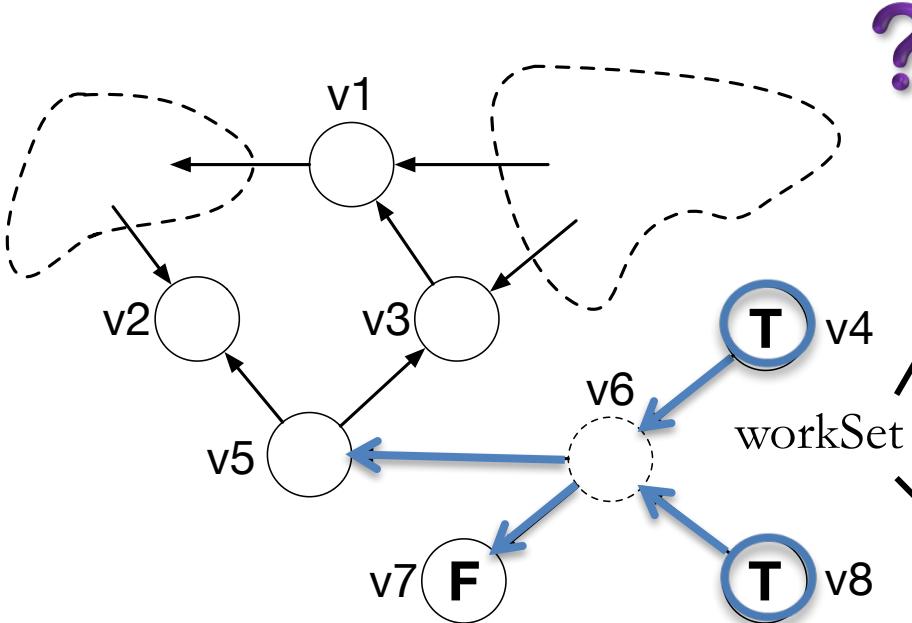
# Example: Iteration 2 (blue = true, red = false)



Queries = {v6}, formula =

v4	weight 100	$\wedge$
v8	weight 100	$\wedge$
└ v7	weight 100	$\wedge$
└ v3 $\vee$ v1	weight 5	$\wedge$
└ v5 $\vee$ v2	weight 5	$\wedge$
└ v5 $\vee$ v3	weight 5	$\wedge$
└ v6 $\vee$ v5	weight 5	$\wedge$
└ v6 $\vee$ v7	weight 5	$\wedge$
└ v4 $\vee$ v6	weight 5	$\wedge$
└ v8 $\vee$ v6	weight 5	$\wedge$
...		

# Example: Iteration 2 (blue = true, red = false)

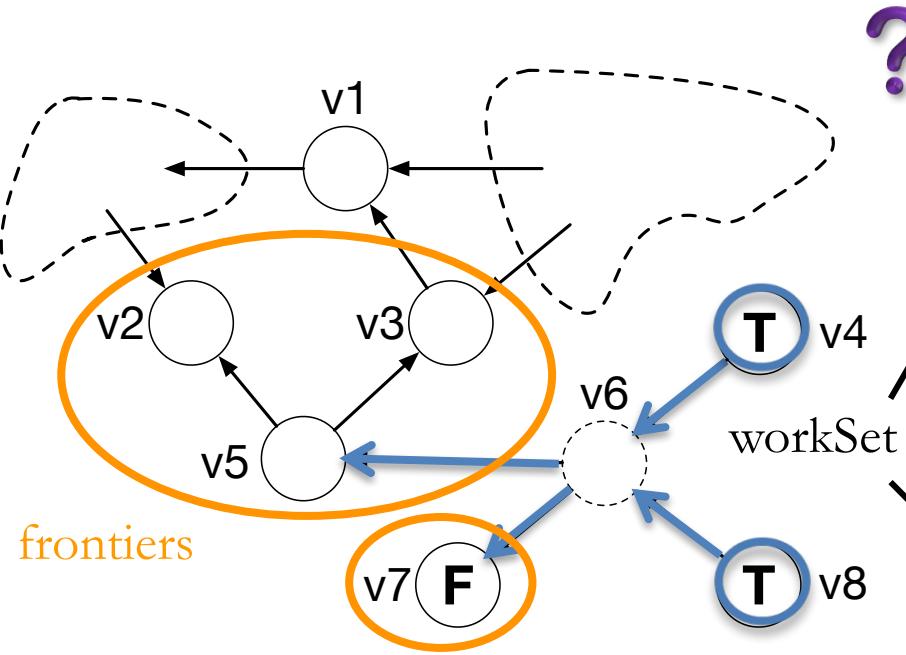


?

Queries = {v6}, formula =

v4	weight 100	$\wedge$
v8	weight 100	$\wedge$
└ v7	weight 100	$\wedge$
└ v3 $\vee$ v1	weight 5	$\wedge$
└ v5 $\vee$ v2	weight 5	$\wedge$
└ v5 $\vee$ v3	weight 5	$\wedge$
└ v6 $\vee$ v5	weight 5	$\wedge$
└ v6 $\vee$ v7	weight 5	$\wedge$
└ v4 $\vee$ v6	weight 5	$\wedge$
└ v8 $\vee$ v6	weight 5	$\wedge$
...		

# Example: Iteration 2 (blue = true, red = false)



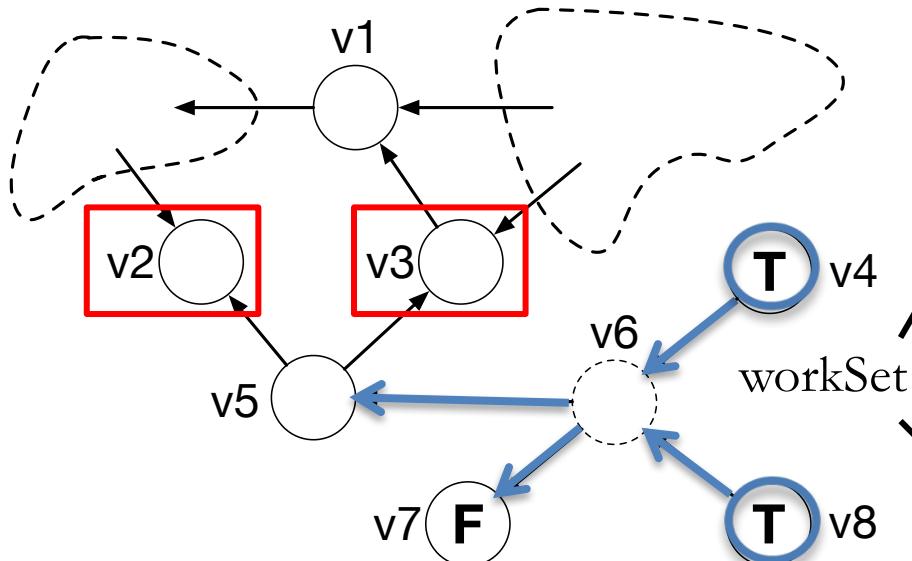
summarySet =  $\{(100, \neg v7), (5, \neg v5 \vee v2), (5, \neg v5 \vee v3)\}$



Queries = {v6}, formula =

v4	weight 100	$\wedge$
v8	weight 100	$\wedge$
└ v7	weight 100	$\wedge$
└ v3 $\vee$ v1	weight 5	$\wedge$
└ v5 $\vee$ v2	weight 5	$\wedge$
└ v5 $\vee$ v3	weight 5	$\wedge$
└ v6 $\vee$ v5	weight 5	$\wedge$
└ v6 $\vee$ v7	weight 5	$\wedge$
└ v4 $\vee$ v6	weight 5	$\wedge$
└ v8 $\vee$ v6	weight 5	$\wedge$
...		

# Example: Iteration 2 (blue = true, red = false)



summarySet =  $\{(100, \neg v7), (5, \neg v5 \vee v2), (5, \neg v5 \vee v3)\}$

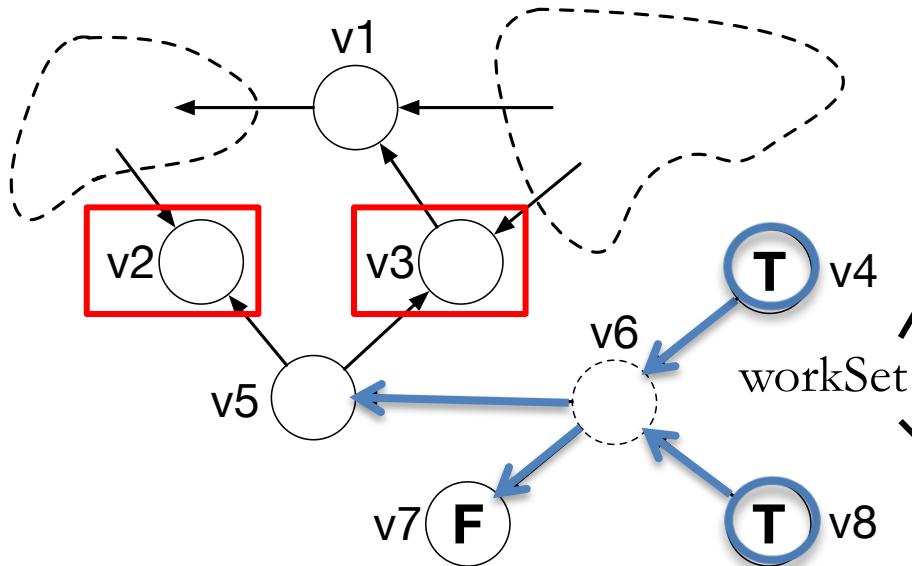
Queries =  $\{v6\}$ , formula =

$v4$	$\text{weight } 100 \wedge$
$v8$	$\text{weight } 100 \wedge$
$\neg v7$	$\text{weight } 100 \wedge$
$\neg v3 \vee v1$	$\text{weight } 5 \wedge$
$\neg v5 \vee v2$	$\text{weight } 5 \wedge$
$\neg v5 \vee v3$	$\text{weight } 5 \wedge$
$\neg v6 \vee v5$	$\text{weight } 5 \wedge$
$\neg v6 \vee v7$	$\text{weight } 5 \wedge$
$\neg v4 \vee v6$	$\text{weight } 5 \wedge$
$\neg v8 \vee v6$	$\text{weight } 5 \wedge$
...	

$\max(\text{workSet} \cup \text{summarySet}) - \max(\text{workSet}) = 0$

?

# Example: Iteration 2 (blue = true, red = false)



summarySet =  $\{(100, \neg v7), (5, \neg v5), (5, \neg v5)\}$

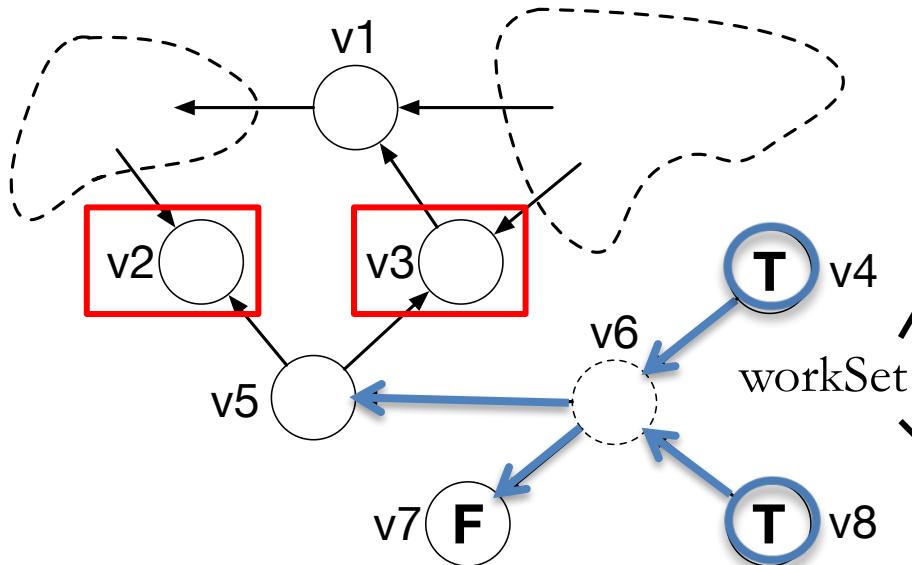
Queries =  $\{v6\}$ , formula =

v4	weight 100	$\wedge$
v8	weight 100	$\wedge$
$\neg v7$	weight 100	$\wedge$
$\neg v3 \vee v1$	weight 5	$\wedge$
$\neg v5 \vee v2$	weight 5	$\wedge$
$\neg v5 \vee v3$	weight 5	$\wedge$
$\neg v6 \vee v5$	weight 5	$\wedge$
$\neg v6 \vee v7$	weight 5	$\wedge$
$\neg v4 \vee v6$	weight 5	$\wedge$
$\neg v8 \vee v6$	weight 5	$\wedge$
...		

$\max(\text{workSet} \cup \text{summarySet}) - \max(\text{workSet}) = 0$

?

# Example: Iteration 2 (blue = true, red = false)



summarySet =  $\{(100, \neg v7), (5, \neg v5), (5, \neg v5)\}$

Queries =  $\{v6\}$ , formula =

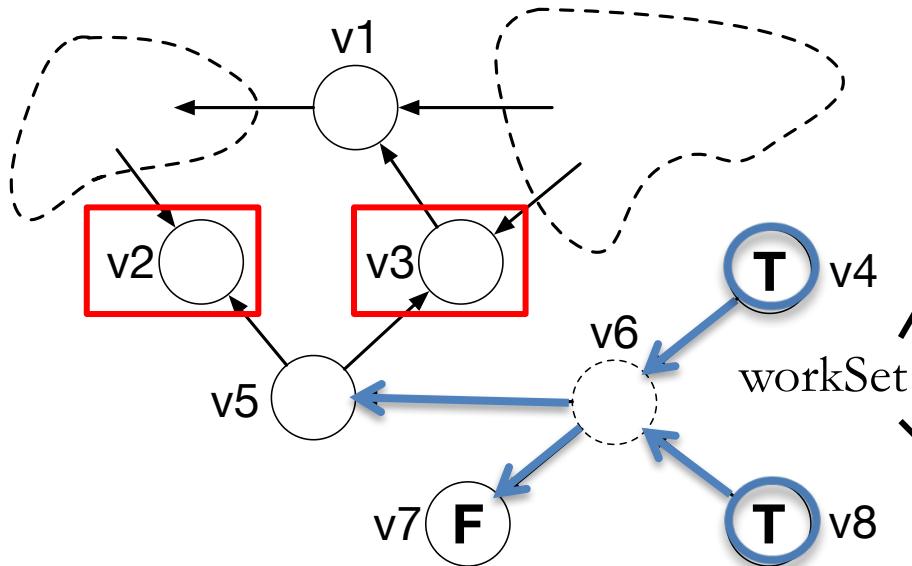
v4	weight 100	$\wedge$
v8	weight 100	$\wedge$
$\neg v7$	weight 100	$\wedge$
$\neg v3 \vee v1$	weight 5	$\wedge$
$\neg v5 \vee v2$	weight 5	$\wedge$
$\neg v5 \vee v3$	weight 5	$\wedge$
$\neg v6 \vee v5$	weight 5	$\wedge$
$\neg v6 \vee v7$	weight 5	$\wedge$
$\neg v4 \vee v6$	weight 5	$\wedge$
$\neg v8 \vee v6$	weight 5	$\wedge$
...		

220

320  $\Rightarrow \max(\text{workSet} \cup \text{summarySet}) - \max(\text{workSet}) = 0$



# Example: Iteration 2 (blue = true, red = false)



summarySet =  $\{(100, \neg v_7), (5, \neg v_5), (5, \neg v_5)\}$

320

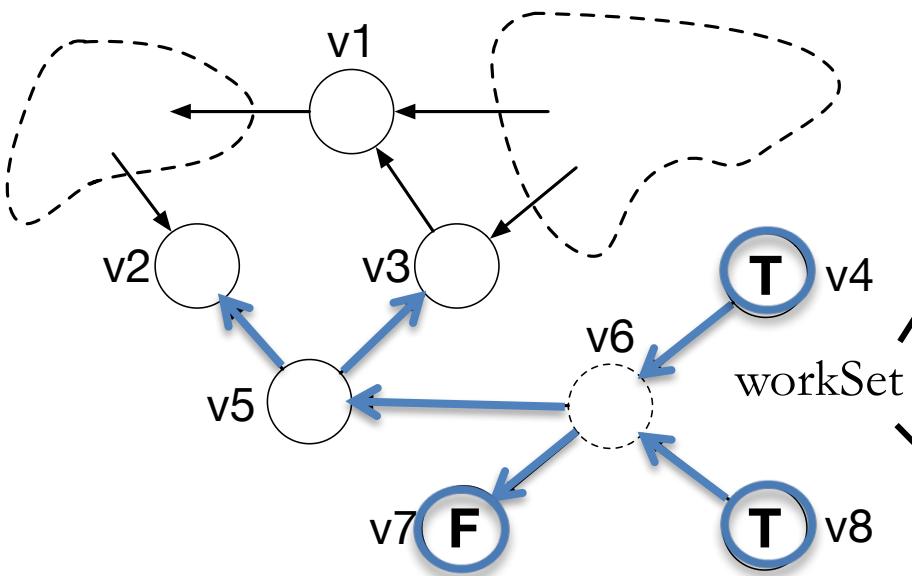
**max(workSet  $\cup$  summarySet)**

Queries =  $\{v_6\}$ , formula =

v4	weight 100	$\wedge$
v8	weight 100	$\wedge$
$\neg v_7$	weight 100	$\wedge$
$\neg v_3 \vee v_1$	weight 5	$\wedge$
$\neg v_5 \vee v_2$	weight 5	$\wedge$
$\neg v_5 \vee v_3$	weight 5	$\wedge$
$\neg v_6 \vee v_5$	weight 5	$\wedge$
$\neg v_6 \vee v_7$	weight 5	$\wedge$
$\neg v_4 \vee v_6$	weight 5	$\wedge$
$\neg v_8 \vee v_6$	weight 5	$\wedge$
...		

v4 = true, v5 = false, v6 = true,  
v7 = true, v8 = true

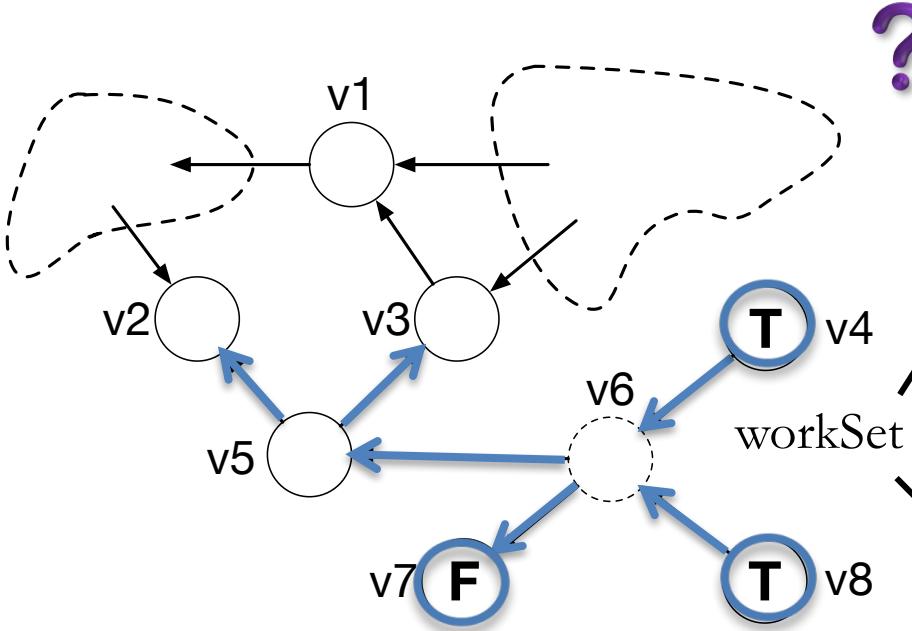
# Example: Iteration 3 (blue = true, red = false)



Queries = { $v_6$ }, formula =

$v_4$	$\text{weight}$	100	$\wedge$
$v_8$	$\text{weight}$	100	$\wedge$
$\neg v_7$	$\text{weight}$	100	$\wedge$
$\neg v_3 \vee v_1$	$\text{weight}$	5	$\wedge$
$\neg v_5 \vee v_2$	$\text{weight}$	5	$\wedge$
$\neg v_5 \vee v_3$	$\text{weight}$	5	$\wedge$
$\neg v_6 \vee v_5$	$\text{weight}$	5	$\wedge$
$\neg v_6 \vee v_7$	$\text{weight}$	5	$\wedge$
$\neg v_4 \vee v_6$	$\text{weight}$	5	$\wedge$
$\neg v_8 \vee v_6$	$\text{weight}$	5	$\wedge$
...			

# Example: Iteration 3 (blue = true, red = false)

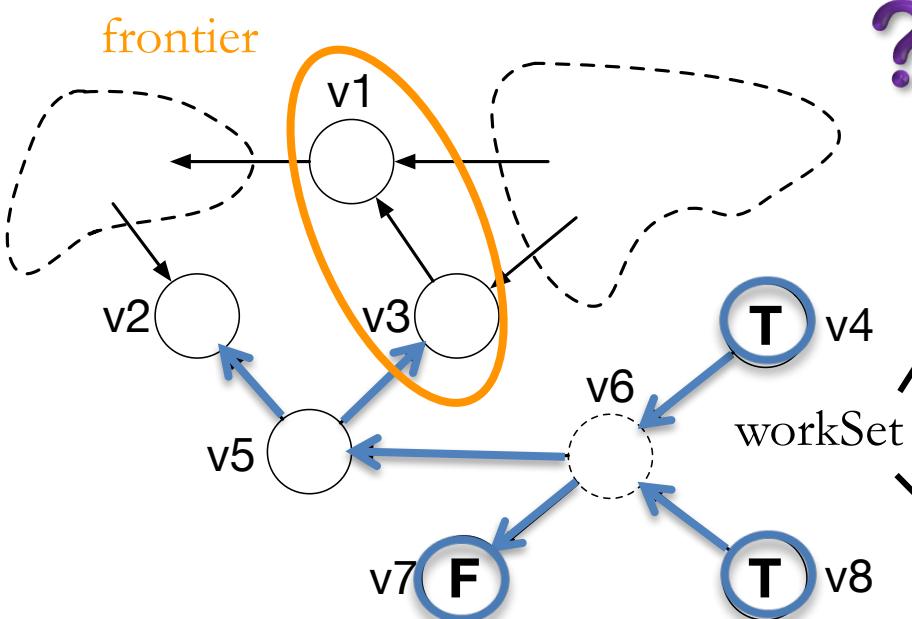


?

Queries = {v6}, formula =

v4	weight 100	$\wedge$
v8	weight 100	$\wedge$
¬ v7	weight 100	$\wedge$
¬ v3 $\vee$ v1	weight 5	$\wedge$
¬ v5 $\vee$ v2	weight 5	$\wedge$
¬ v5 $\vee$ v3	weight 5	$\wedge$
¬ v6 $\vee$ v5	weight 5	$\wedge$
¬ v6 $\vee$ v7	weight 5	$\wedge$
¬ v4 $\vee$ v6	weight 5	$\wedge$
¬ v8 $\vee$ v6	weight 5	$\wedge$
...		

# Example: Iteration 3 (blue = true, red = false)



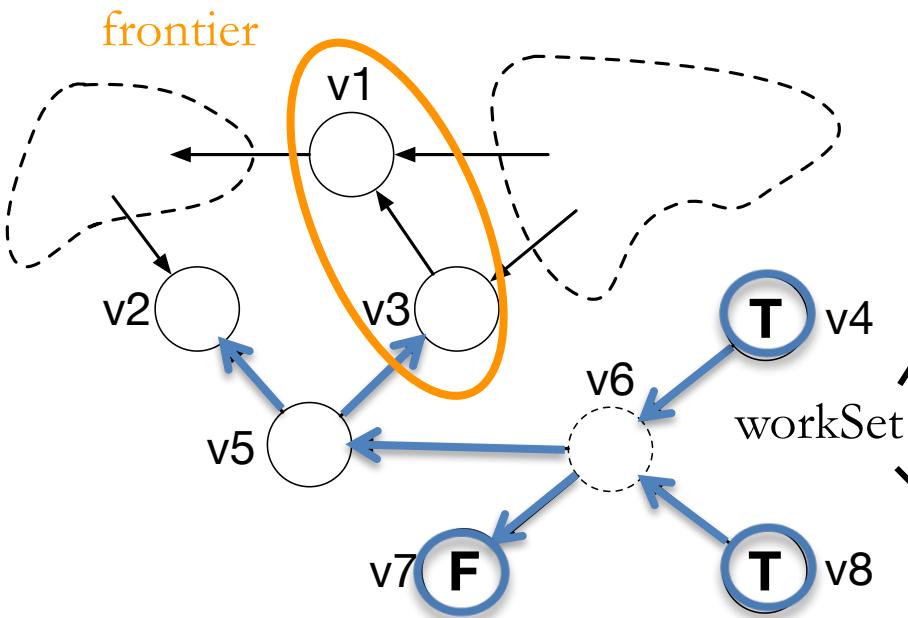
summarySet =  $\{(5, \neg v3 \vee v1)\}$



Queries =  $\{v6\}$ , formula =

v4	weight 100	$\wedge$
v8	weight 100	$\wedge$
$\neg v7$	weight 100	$\wedge$
$\neg v3 \vee v1$	weight 5	$\wedge$
$\neg v5 \vee v2$	weight 5	$\wedge$
$\neg v5 \vee v3$	weight 5	$\wedge$
$\neg v6 \vee v5$	weight 5	$\wedge$
$\neg v6 \vee v7$	weight 5	$\wedge$
$\neg v4 \vee v6$	weight 5	$\wedge$
$\neg v8 \vee v6$	weight 5	$\wedge$
...		

# Example: Iteration 3 (blue = true, red = false)



summarySet =  $\{(5, \neg v3 \vee v1)\}$

Queries =  $\{v6\}$ , formula =

v4	weight 100	$\wedge$
v8	weight 100	$\wedge$
$\neg v7$	weight 100	$\wedge$
$\neg v3 \vee v1$	weight 5	$\wedge$
$\neg v5 \vee v2$	weight 5	$\wedge$
$\neg v5 \vee v3$	weight 5	$\wedge$
$\neg v6 \vee v5$	weight 5	$\wedge$
$\neg v6 \vee v7$	weight 5	$\wedge$
$\neg v4 \vee v6$	weight 5	$\wedge$
$\neg v8 \vee v6$	weight 5	$\wedge$

...

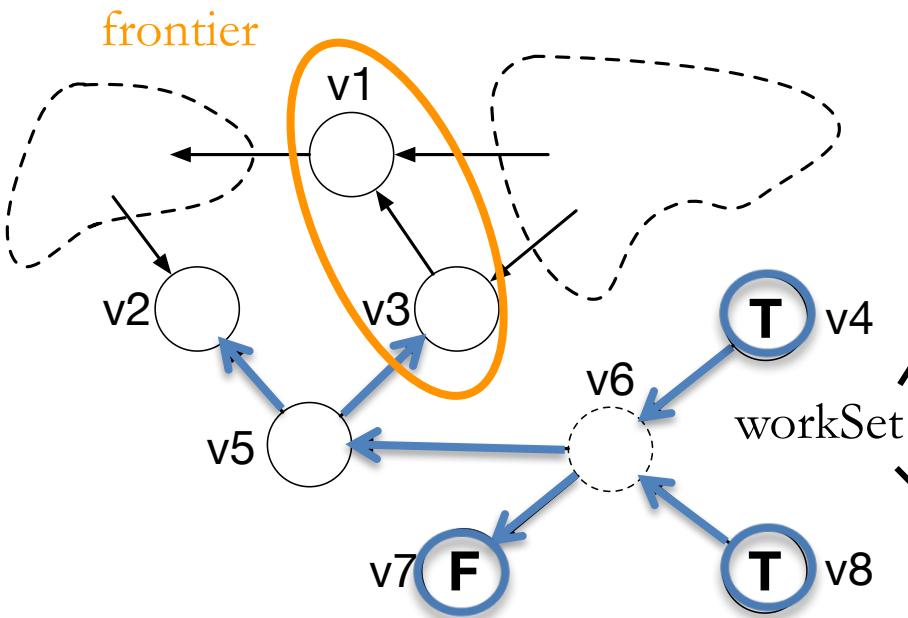
325

330

max(workSet  $\cup$  summarySet) - max(workSet) = 0



# Example: Iteration 3 (blue = true, red = false)



summarySet =  $\{(5, \neg v_3 \vee v_1)\}$

Queries =  $\{v_6\}$ , formula =

v4	weight 100	$\wedge$
v8	weight 100	$\wedge$
$\neg v_7$	weight 100	$\wedge$
$\neg v_3 \vee v_1$	weight 5	$\wedge$
$\neg v_5 \vee v_2$	weight 5	$\wedge$
$\neg v_5 \vee v_3$	weight 5	$\wedge$
$\neg v_6 \vee v_5$	weight 5	$\wedge$
$\neg v_6 \vee v_7$	weight 5	$\wedge$
$\neg v_4 \vee v_6$	weight 5	$\wedge$
$\neg v_8 \vee v_6$	weight 5	$\wedge$

...

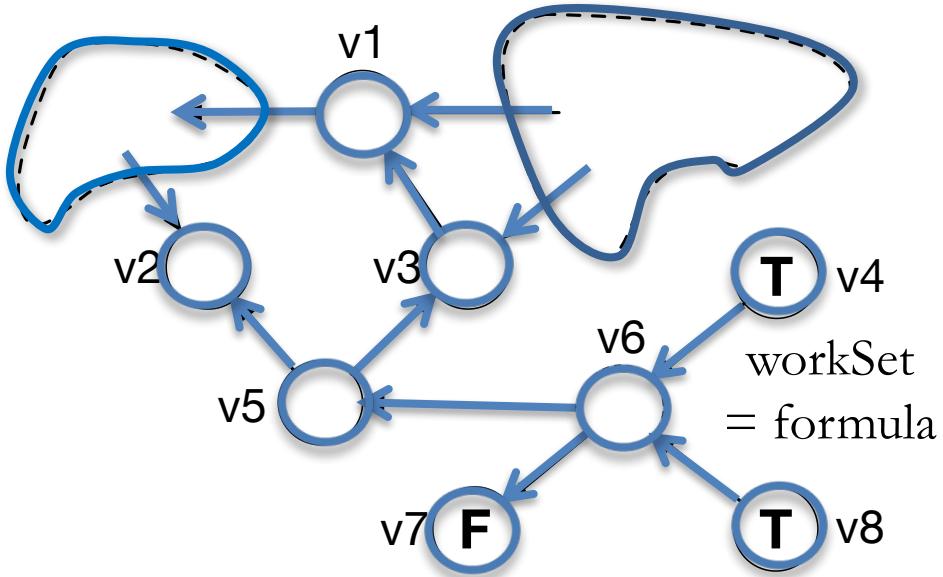
325

325

max(workSet  $\cup$  summarySet) - max(workSet) = 0



# Example



Queries = {v6}, formula =

v4	weight 100	$\wedge$
v8	weight 100	$\wedge$
└ v7	weight 100	$\wedge$
└ v3 $\vee$ v1	weight 5	$\wedge$
└ v5 $\vee$ v2	weight 5	$\wedge$
└ v5 $\vee$ v3	weight 5	$\wedge$
└ v6 $\vee$ v5	weight 5	$\wedge$
└ v6 $\vee$ v7	weight 5	$\wedge$
└ v4 $\vee$ v6	weight 5	$\wedge$
└ v8 $\vee$ v6	weight 5	$\wedge$
...		

# Benchmark Characteristics

	# queries	# variables	# clauses
ftp	55	2.3M	3M
hedc	36	3.8M	4.8M
weblech	25	5.8M	8.4M
lusearch	248	7.8M	10.9M
luindex	109	8.5M	11.9M
avrora	151	11.7M	16.3M
IE	6	47K	0.9M
ER	25	3K	4.8M
AR	10	0.3M	7.9M

K = thousands, M = millions

# Performance Results

	running time (seconds)		peak memory (MB)		# clauses (M=million)	
	current	baseline	current	baseline	current	baseline
ftp	<b>16</b>	11	<b>16</b>	1,262	<b>0.03M</b>	3.0M
hedc	<b>23</b>	21	<b>181</b>	1,918	<b>0.4M</b>	4.8M
weblech	<b>4</b>	timeout	<b>363</b>	timeout	<b>0.9M</b>	8.4M
lusearch	<b>115</b>	timeout	<b>659</b>	timeout	<b>1.5M</b>	10.9M
luindex	<b>169</b>	timeout	<b>944</b>	timeout	<b>2.2M</b>	11.9M
avrona	<b>178</b>	timeout	<b>1,095</b>	timeout	<b>2.6M</b>	16.3M
IE	<b>2</b>	2,760	<b>13</b>	335	<b>27K</b>	0.9M
ER	<b>13</b>	2	<b>6</b>	44	<b>9K</b>	4.8M
AE	<b>4</b>	timeout	<b>4</b>	timeout	<b>2K</b>	7.9M

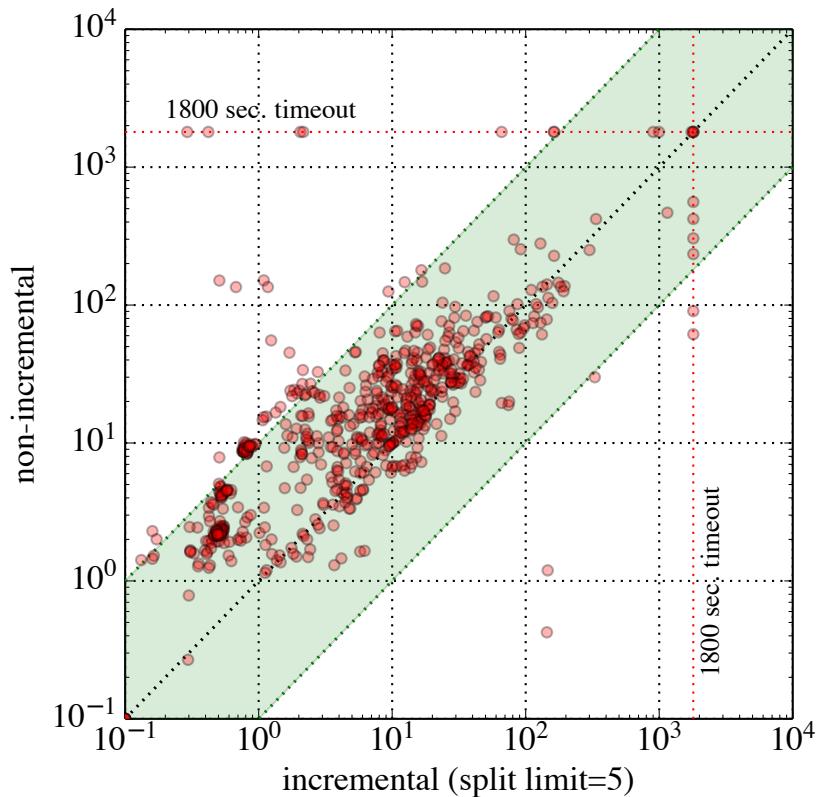
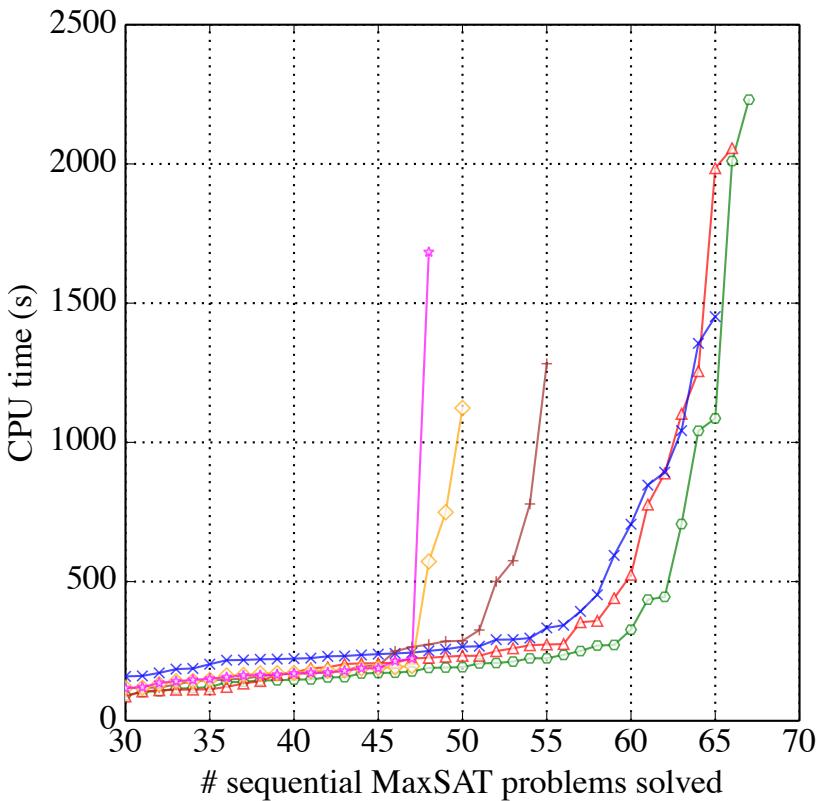
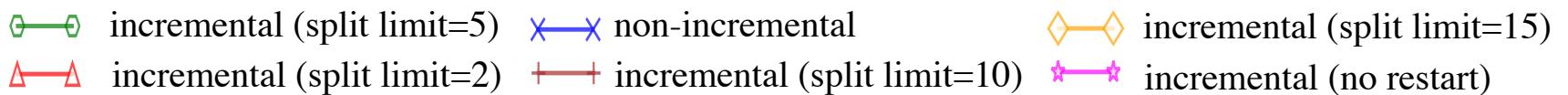
# Incremental Solving [CP 2016]

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$$\begin{array}{ccccccc} \varphi_1 & \xrightarrow{\hspace{1cm}} & \varphi_2 & \xrightarrow{\hspace{1cm}} & \varphi_3 & \xrightarrow{\hspace{1cm}} & \dots \\ & & = \varphi_1 \cup \Delta_1 & & = \varphi_2 \cup \Delta_2 & & \end{array}$$

- ▶ Two levels of incrementality
  - ▶ MaxSAT level
    - ▶ Application solves a sequence of MaxSAT instances
    - ▶ Re-use unsat cores (for core-guided solver)
  - ▶ SAT level
    - ▶ Each MaxSAT solves a sequence of SAT instances
    - ▶ Leverage standard incremental SAT solving
- ▶ Key insight: sporadic restarts
  - ▶ Heuristically detect and avoid reusing bad unsat cores based on split-limit (max. # times a soft clause is split)

# Performance Results



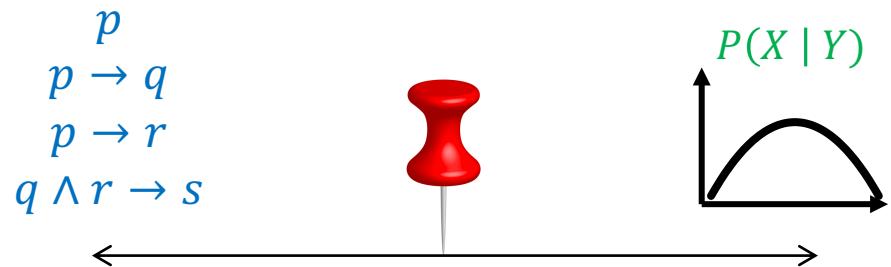
74 sequential MaxSAT problems (669 individual MaxSAT instances)

# Future Directions

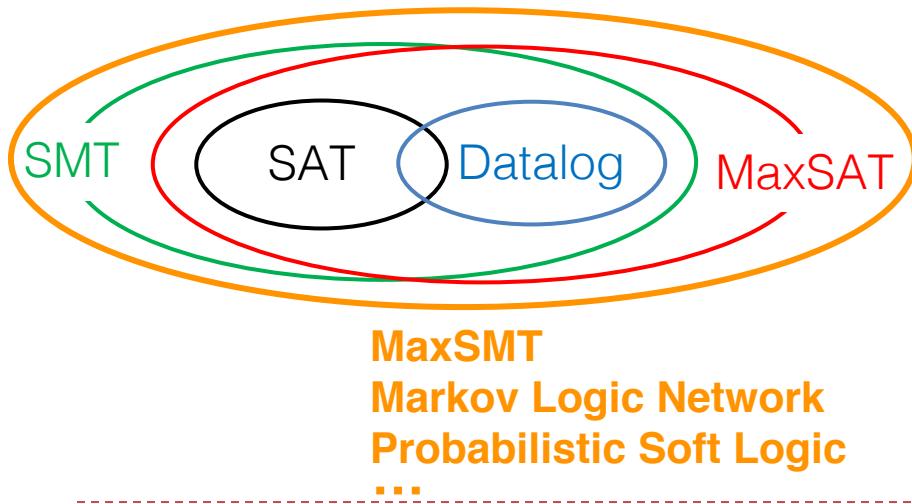
## Humans In the Loop



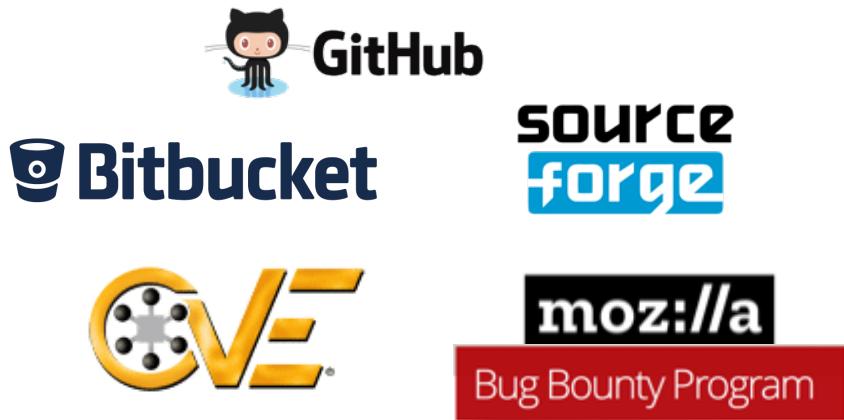
## Combining Logic With Probability



## Optimization Solvers

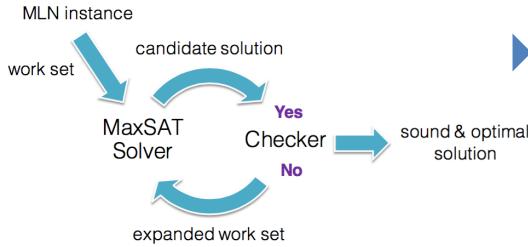


## Data-Driven



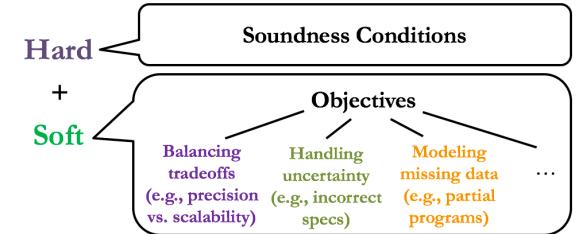
# Conclusions

- ▶ New methodology to incorporate **objectives** into **constraint-based** software analyses



- ▶ General framework to solve weighted constraints that is **sound, optimal and scalable**

- ▶ Showed practical effectiveness for **three dominant applications** of software analyses



Abstraction Selection in Automated Verification  
[PLDI 2014, POPL 2016]

