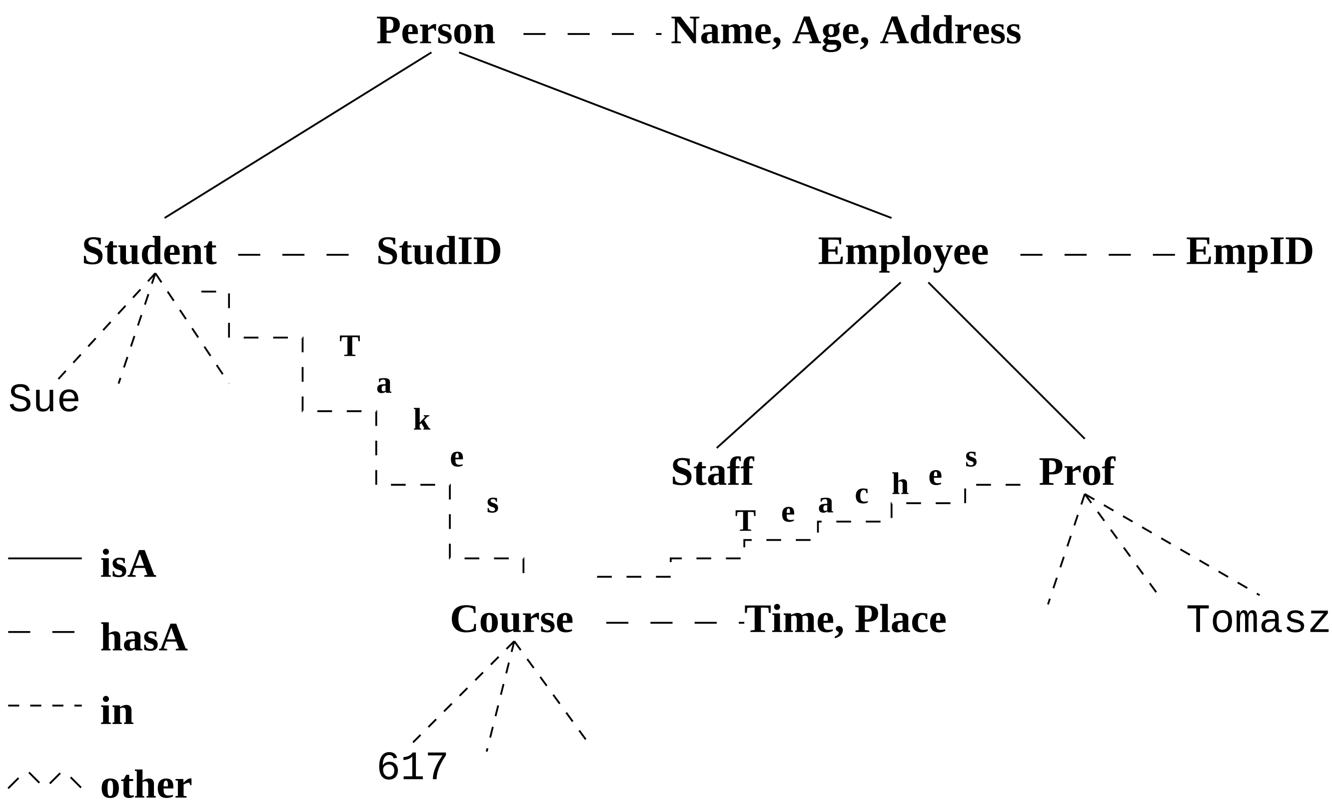
The author gratefully acknowledges support from the taxpayers of Québec and of Canada who have paid his salary and research grants while this work was developed at McGill University, and from his students (who built the implementations and investigated the data structures and algorithms) and their funding agencies.

Recursion, Knowledge, and Logic

- Hierarchies
  - Ancestor
  - Part-Of
- Syllogisms
  - ISA
  - LAPS
- Horn Clauses

Semantic Nets



Compute the *closure* of this under inheritance (isA).

Semantic Nets

SemNet(*First*	*Rel*	*Second*)
Person	hasA	Name
Person	hasA	Age
Person	hasA	Address
Employee	isA	Person
Employee	hasA	EmpID
Student	isA	Person
Student	hasA	StudID
Student	takes	Course
Sue	in	Student
Prof	isA	Employee
Prof	teaches	Course
Staff	isA	Employee
Tomasz	in	Prof
Course	hasA	Time
Course	hasA	Place
617	in	Course

Semantic Nets

Closure should give attribute inheritance, etc:

*SemNet**(*First*	*Rel*	*Second*)
:	:	:
`Prof`	`isA`	`Person`
`Student`	`hasA`	`Name`
:	:	:
`Employee`	`hasA`	`Name`
:	:	:
`Staff`	`hasA`	`Name`
:	:	:
`Prof`	`hasA`	`Name`
`Prof`	`hasA`	`EmpID`
:	:	:

What about

*SemNet**(*First*	*Rel*	*Second*)	
:	:	:	
`Sue`	`hasA`	`Name`	
:	:	:	
`Tomasz`	`in`	`Employees`	
:	:	:	
`Tomasz`	`teaches`	`Course`	
:	:	:	
`Prof`	`teaches`	`617`	
:	:	:	
`Employee`	`teaches`	`Course`	**no!**
:	:	:	
`Tomasz`	`teaches`	`617`	**no!**
:	:	:	

We also don't know

`Tomasz teaches Sue` in 617:

need a separate (ternary) relation.

In fact, *SemNet* and *SemNet** could be seen as *schemas*: relations containing other relation names, as *metadata*.

Syllogisms

A, E	Universally quantified			
A	every X is Y	$X \subseteq Y$	$Y' \subseteq X'$	
E	no X is Y	$Y \subseteq X'$	$X \subseteq Y'$	
I, O	Existentially quantified			
I	some X is Y	$X \not\pitchfork Y$	$Y \not\pitchfork X$	
O	some X is not Y	$X \not\pitchfork Y'$	$Y' \not\pitchfork X$	

- **A, E** are just the *isA* hierarchy rules:
  - antisymmetric, transitive: $\therefore$ closure

- **I, O** combine with **A, E**: call $\not\pitchfork$ *laps* rules:
  - symmetric
  - *X laps Y & Y isA Z* $\Rightarrow$ *X laps Z* (*laps* is closed under **icomp** with *isA*)
  - *X isA Y & X isA Z* $\Rightarrow$ *Y laps Z*
  - *X isA Y* $\Rightarrow$ *X laps Y*

Some syllogisms

Barbara *isA* YZ *isA* XY —— *isA* XZ	Darii *isA* YZ *laps* XY —— *laps* XZ	Darapti *isA* YZ *isA* YX —— *laps* XZ	Disamis *laps* YZ *isA* YX —— *laps* XZ	Barbari *isA* YZ *isA* XY —— *laps* XZ
Celarent *isA* ZY´ *isA* XY —— *isA* ZX´	Ferio *isA* ZY´ *laps* XY —— *laps* XZ´	Felapton *isA* ZY´ *isA* YX —— *laps* XZ´	Datisti *isA* YZ *laps* YX —— *laps* XZ	Celaront *isA* ZY´ *isA* XY —— *laps* ZX´
Cesare *isA* YZ´ *isA* XY —— *isA* ZX´	Festino *isA* YZ´ *laps* XY —— *laps* XZ´		Bocardo *laps* YZ´ *isA* YX —— *laps* XZ´	*etc.*
Camestres *isA* ZY *isA* YX´ —— *isA* ZX´	Baroco *isA* ZY *laps* XY´ —— *laps* XZ´		Ferison *isA* ZY´ *laps* YZ —— *laps* XZ´	

Special (*isA*) syllogisms: logical closure

	Isa (*Subj*	*Obj*)
1. No shark ever doubts that it is well fitted out.	S	F
2. A fish, that cannot dance a minuet, is contemptible.	M'	C
3. No fish is quite certain that it is well fited out, unless it has three rows of teeth.	F	T
4. All fishes, except sharks, are kind to children.	S'	K
5. No heavy fish can dance a minuet.	H	M'
6. A fish with three rows of teeth is not to be despised.	T	C'
∴ No heavy fish is unkind to children.	H	K

(Lewis Carroll, 1896, *Symbolic Logic*)

Horn Clauses An Inference Engine

[*New*]*Facts*	*Horn*		
(*Concl*)	(*Rule#*	*Ante*	*Concl*)
lays eggs	1	lays eggs	is bird
has feathers	1	has feathers	is bird
swims	2	flies	is bird
———	2	is not mammal	is bird
is bird	3	is bird	is duck
———	3	swims	is duck
is duck	3	is brown	is duck
	4	is bird	is duck
	4	swims	is duck
	4	is green	is duck
	4	is red	is duck
	5	is duck	migrates
	5	is not tame	migrates

NewFacts **is** *Facts* **ujoin**
[*Concl*] **in** (*NewFacts*[*Concl* $\supseteq$ *Ante*]*Horn*)

Relixpert expands this 1-line inference engine to 50, in a 200-line expert system shell: TDKE **6** (1991) 151