COMP 648: Algorithmic Motion Planning
Fall 2005-06

Homework II

due Tues. Sept. 20 in class. (The first homework was to turn in your preferences for the WAFR 04 papers, available for viewing in the SOCS office, McConnell 318.)

NOTE: Prof. Whitesides will be attending a conference Sept. 12-14, and Svetlana Stolpner will give a guest lecture on Tues. Sept. 13. There will be no office hours on Tues. Sept. 13. . The t.a. is Michel Langlois, who may be contacted by email to mlangl5@cs.mcgill.ca – note the el-5 at the end, it's not a 15. Michel will be happy to answer any questions about the homework. You may discuss these problems with classmates, but your write-ups must be in your own words.

1. Configuration Space

In the two parts below, consider an obstacle consisting of a rod of length 2 positioned in the plane. The rod occupies the line segment between the origin and the point $(0, 2)$. A moving object consisting of a rod of length 1 moves in the plane, avoiding intersection with the interior of the fixed rod. Take one endpoint of the moving rod to be the reference point P. Take the reference line to be the half-line originating at P and passing along the rod.

Suppose that the moving rod moves by translation only (no rotation).

a) What is the configuration space?

b) If the reference angle θ made by the reference line with respect to the horizontal is 0, what is the configuration space obstacle $CO_A(B)$?

c) What is $CO_A(B)$ if $\theta = 45\,\text{deg}$? $= 90\,\text{deg}$?

2. Configuration Space Obstacles

In the simple case of a convex polygonal object moving in the plane by translation only past convex polygonal obstacles, we said that $CO_A(B) = B + (-A)$ – and sketched a proof.

a) Does the proof apply to more general situations than this? What can you *prove* about $CO_A(B)$ when A moves by translation only? Make as general a statement as you can, and then give a proof of it.

b) Suppose the moving object A can rotate as well as translate. What can you say about the formula $B + (-A)$? Does it still apply? Why or why not? Give a proof that the formula still holds, or a counter-example showing that sometimes, the formula does not hold.

3. Convexity

Let $w_1, \ldots, w_n$ be non-negative real numbers that sum to 1. A *convex linear combination* of a set of n points in the plane, say $(x_1, y_1), \ldots, (x_n, y_n)$, is obtained by taking their weighted linear combination $w_1(x_1, y_1) + \ldots + w_n(x_n, y_n)$.

Prove that every point inside a convex polygon is a convex linear combination of the *vertices* of the polygon. (*hint*: One possible approach would be to draw a line through the point, see where the line hits the boundary of the polygon, then somehow use those points of intersection.)