COMP 360 Algorithm Design Techniques *Due Friday, November 28, noon*

Assignment 5: NP-Completeness

Part of the goal of this homework is to gain experience with using *problem transformation* to show that a problem apparently has no polynomial time solution. This is the other use of problem transformation. (Recall that in Homework IV, problem transformation was used to solve problems, rather than to prove that problems are apparently hard.)

For your reference in this homework, here are the definitions of the decision problems you will be working with.

The Three Colourability Problem (3-COL) is defined as follows. *Given:* a graph $G = (V, E)$ *Question:* Can the vertices of G be coloured with three colours so that each vertex gets exactly one colour, and so that for each edge $e \in E$, the endpoints of e receive different colours?

The Integer Linear Programming Problem (ILP) is as follows. *Given:* a set of m linear constraints in n variables $x_1, \ldots, x_n$ (as for linear programming) with the additional constraint that each variable x_i must be *integer* valued. *Question:* Can the constraints be satisfied?

The 4-colourability problem (4-COL) is exactly the same as 3-COL, except that 4 colours may be used.

The Independent Set Problem (IS) is defined as follows. *Given:* graph $G = (V, E)$ and positive integer k *Question:* Is there a subset $V' \subseteq V$ such that no edge of E has both endpoints in V' and such that $|V'| \geq k$?

The Vertex Cover Problem (VC) is defined as follows. *Given:* a graph $G = (V, E)$ and a positive integer k *Question:* Is there a subset $V' \subset V$ such that $|V'| \leq k$ and such that for each $e \in E$, at least one endpoint of e is in V'?

Now here are your homework problems to solve (continued on second sheet).

(1) *Three Colourability (3-COL)*
  - (a) Prove 3-COL $\in NP$.
  - (b) Give a polynomial time algorithm for transforming 3-COL to ILP. Prove your transformation works correctly and that the running time is polynomial in the length of the input to 3-COL.
  - (c) Same as above, but transform 3-COL to SAT instead of to ILP.

(d) Suppose we know 3-COL is NP-Complete. Use this fact to prove that 4-COL is NP-complete.

(2) *Independent Set (IS) and Vertex Cover (VC)*
Prove that Independent Set (IS) is NP-complete. You may assume that Vertex Cover is NP-complete.