COMP 360 Algorithm Design Techniques *Due October 24th*

Assignment 3: Strong Components and Shortest Paths

(1) *Strongly Connected Components*

Apply the DFS based algorithm seen in class (or in the textbook) for finding strongly connected components on the graph shown in Figure 1. (For the proof of correctness of this algorithm, please study the class handout, which gives a shorter proof than the one in the text.) Begin with root vertex A and use the usual ordering convention $A \prec B \prec C \prec \cdots$ whenever a choice of arc or vertex needs to be made.

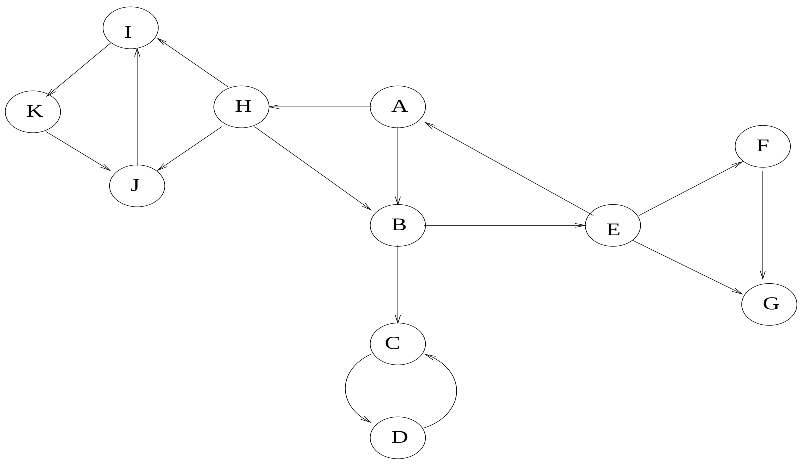


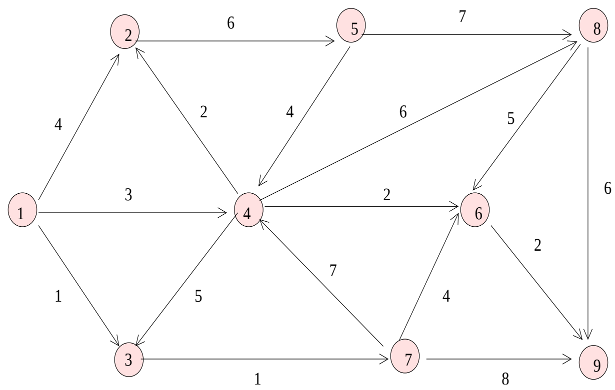
FIGURE 1.

(2) *Dijkstra's Algorithm*

Run Dijkstra's algorithm to obtain a shortest path tree (from vertex 1) in the graph shown in Figure 2.

(3) *Coin Changing.*

Suppose that a country has coins of denominations $c_1, c_2, \ldots, c_k$ cents. Given a value t cents, the coin changing problem is to determine whether or not we can "change" t exactly into this country's coins, and to use the minimum number of coins possible if we can do so. For example, if our coin denominations are $c_1 = 2$, $c_2 = 5$ and $c_3 = 11$ then we can change $t = 24$ cents into four 5 cent coins plus two 2 cent coins. An optimal solution, though, uses three



coins, i.e. two 11 cent coins plus one 2 cent coin. Observe that it is impossible to change $t = 3$ cents exactly in to coins.

(a) Create an acyclic graph with vertices labelled $0, 1, 2, 3, \ldots$ for which there is a directed path from 0 to t if and only if t cents can be changed exactly into coins with denominations $c_1, c_2, \ldots, c_k$.

(b) How can we use this graph to find how to change t cents into the minimum possible number of coins?

(4) *True or False*

Consider a directed graph G with a given source vertex s, and non-negative edge lengths. For each of the following statements, prove the statement is true or give a counter-example to show that it is false. Assume that the value of k below is equal to or less than the minimum edge weight.

(a) If every edge in a graph has a different cost then the graph has a unique shortest path tree.

(b) Take a shortest path problem in a graph. If each edge length decreases by k units then the shortest path distances decrease by a multiple of k.

(c) Increasing by k units the length of each edge out of the source node s does not alter the shortest path tree.

(d) Increasing by k units the length of each edge out of some non-source node v does not alter the shortest path tree.

(5) *Dijkstra's algorithm*

Find an example to show that Dijkstra's algorithm can give incorrect distance information if the edge weights can be negative. Your example should *not* contain any negative weight cycles.

(6) *Floyd-Warshall*
How can the output of the Floyd-Warshall algorithm (Ch. 26.2 of the white book) be used to detect the presence of a negative-weight cycle?