Probabilistic Graphical Models

Relationship between the directed & undirected models

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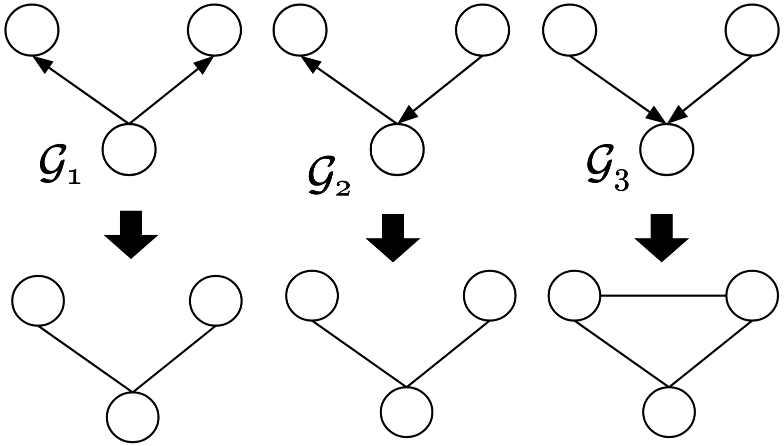
Learning Objective

understand the relationship between CIs
in directed and undirected models.

convert
- Markov network $\Rightarrow$ Bayes-net
- Markov network $\Leftarrow$ Bayes-net

1. From Bayesian to Markov networks

build an I-map for the following



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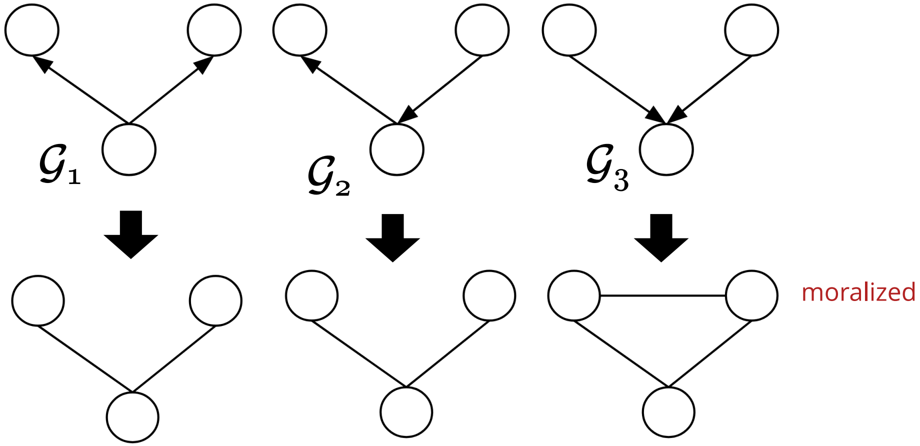

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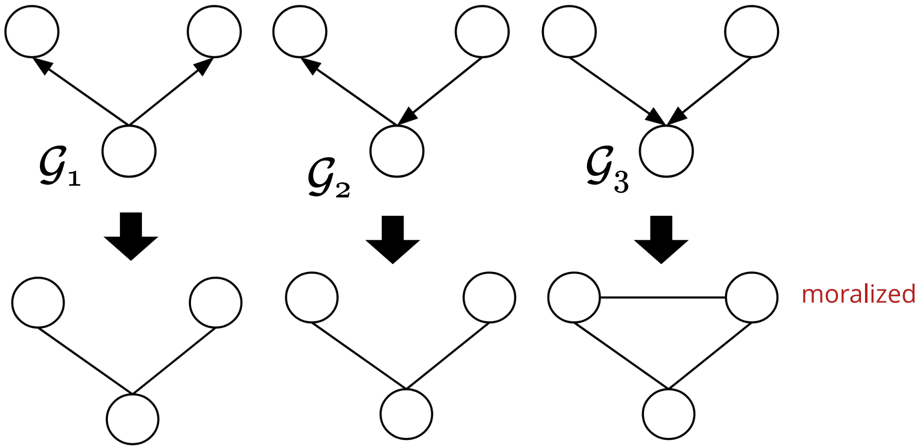

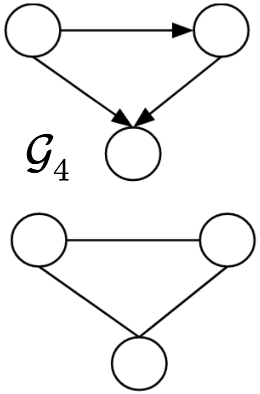

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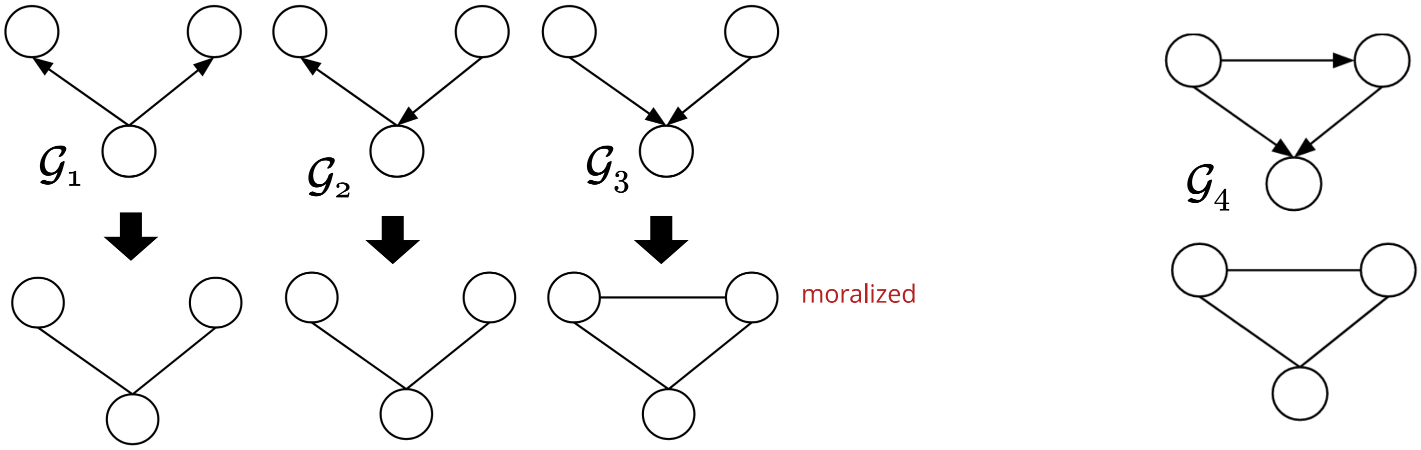

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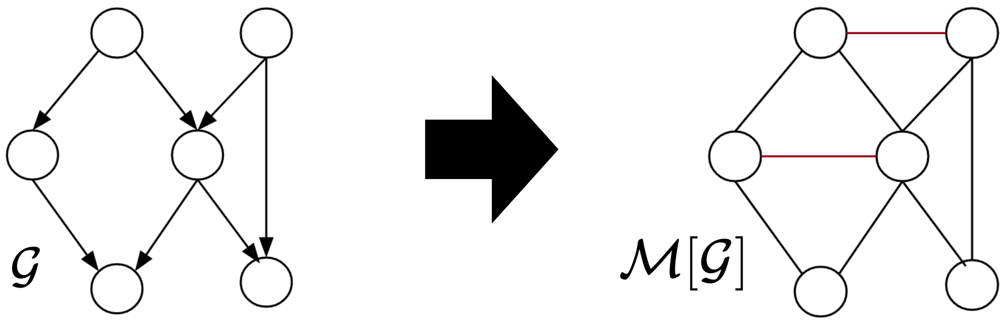
Moralize $\mathcal{G} \to \mathcal{M}(\mathcal{G})$:connect parents keep the skeleton

From Bayesian to Markov networks

moralize & keep the skeleton



for moral $\mathcal{G}$, we get a perfect map $\mathcal{I}(\mathcal{M}[\mathcal{G}]) = \mathcal{I}(\mathcal{G})$

- *directed and undirected CI tests are equivalent*

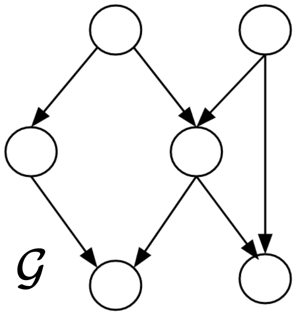
From Bayesian to Markov networks

alternative approach

- in both directed and undirected models

$$X_i \perp \text{every other var.} \mid MB(X_i)$$

- connect each node to its **Markov blanket**



children +
parents +
parents of children

From Bayesian to Markov networks

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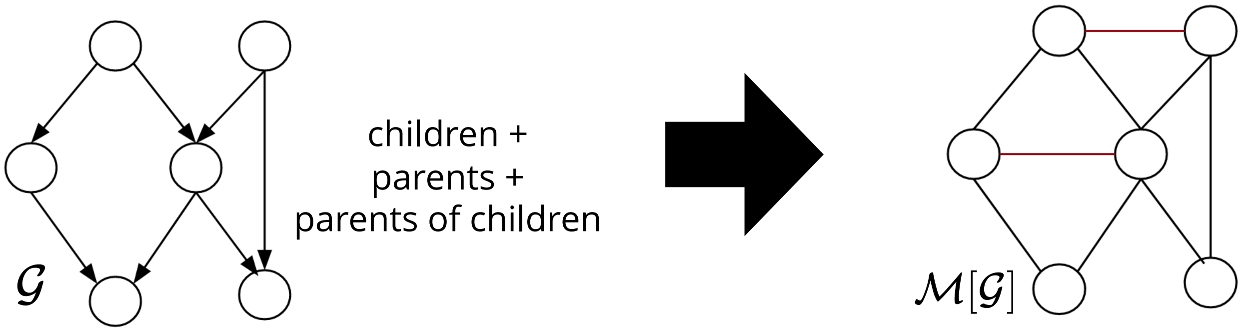
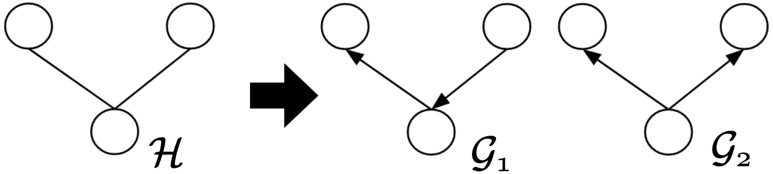


- gives the same moralized graph

2. From Markov to Bayesian networks

minimal examples 1.



$$\mathcal{I}(\mathcal{G}_1) = \mathcal{I}(\mathcal{G}_2) = \mathcal{I}(\mathcal{H})$$

2. From Markov to Bayesian networks

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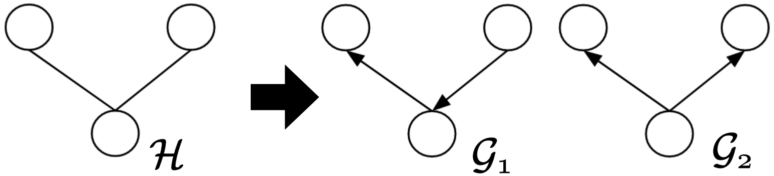

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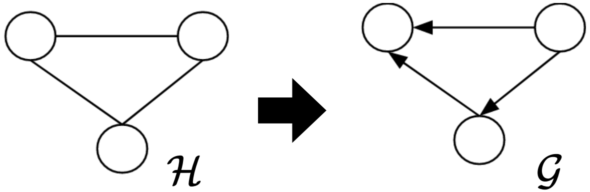
minimal examples 2.



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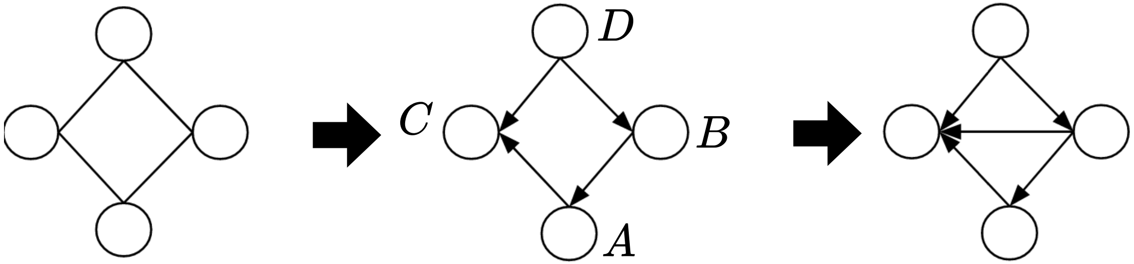
From Markov to Bayesian networks

minimal examples 3.



$B \perp C \mid A$

$\mathcal{I}(\mathcal{G}) \subset \mathcal{I}(\mathcal{H})$

From Markov to Bayesian networks

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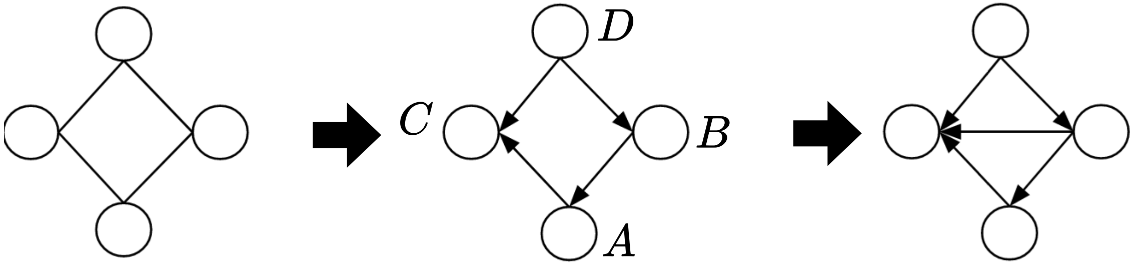

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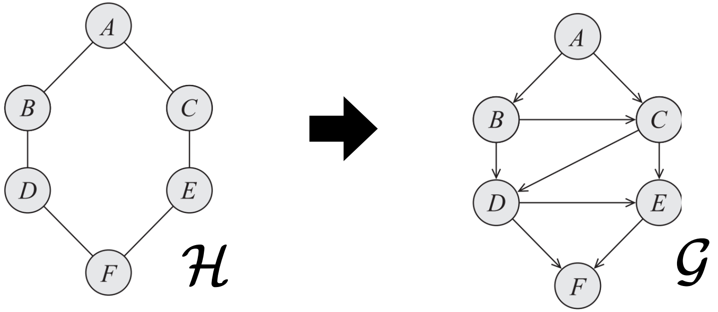

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From Markov to Bayesian networks

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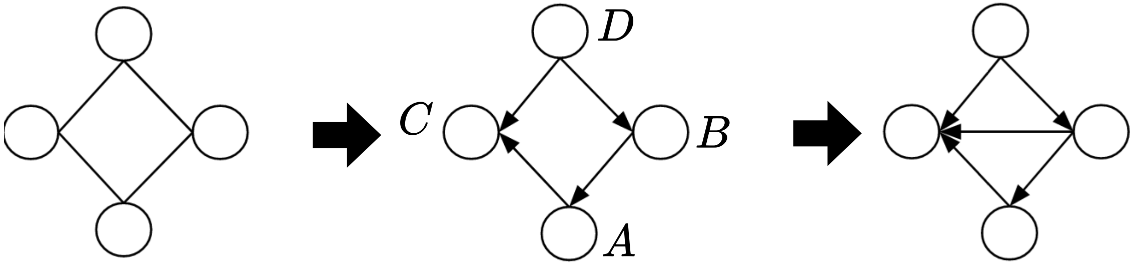

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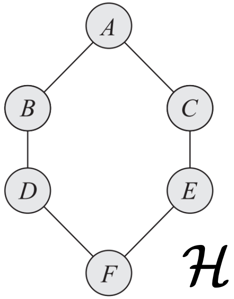
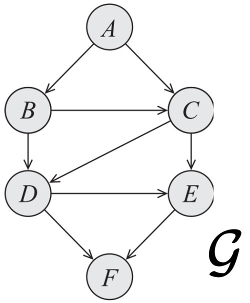


how?



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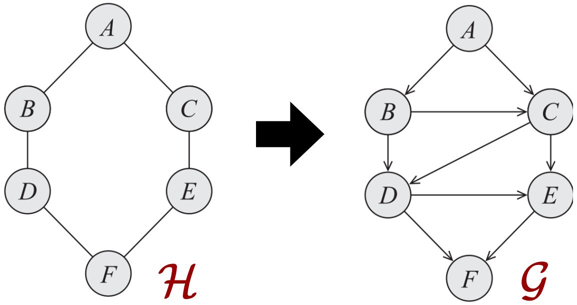

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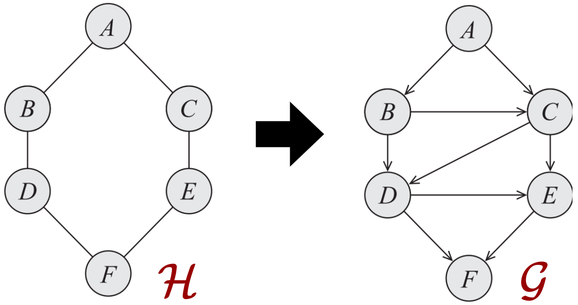

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build a **minimal** I-map from CIs in $\mathcal{H}$:

- pick an ordering - e.g., A,B,C,D,E,F
- select a minimal parent set s.t.
  - local CI (CI from non-descendents given parents)

From Markov to Bayesian networks

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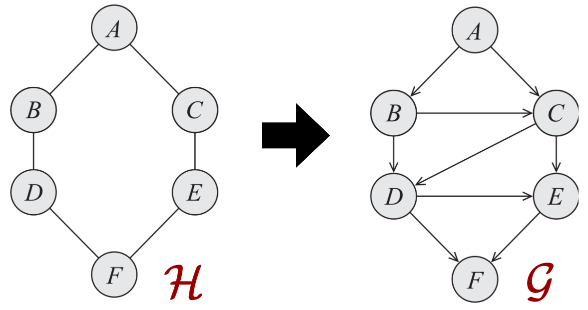

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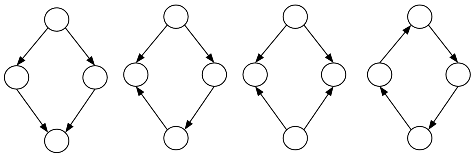

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From Markov to Bayesian networks

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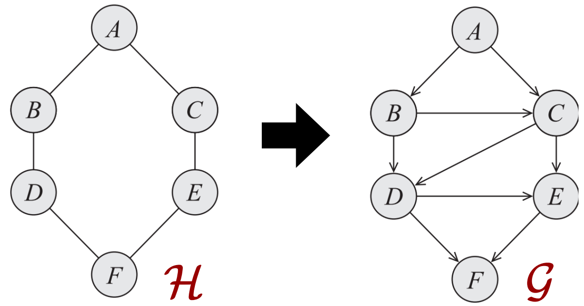

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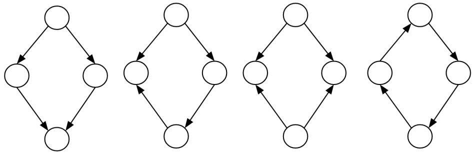


- have to triangulate the loops

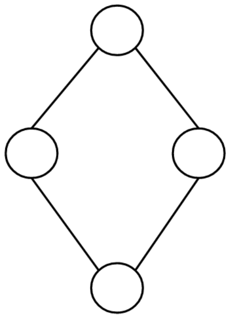
chordal $\mathcal{G}$

loops of size >3 have *chords*

Chordal = Markov $\cap$ Bayesian networks

$\mathcal{H}$ is not chordal, then $\mathcal{I}(\mathcal{G}) \neq \mathcal{I}(\mathcal{H})$ for **every** $\mathcal{G}$

- no *perfect MAP* in the form of Bayes-net



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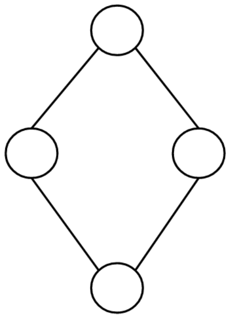

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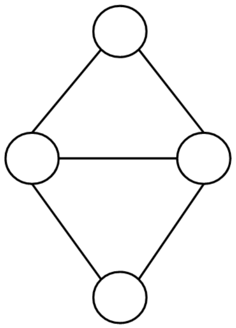

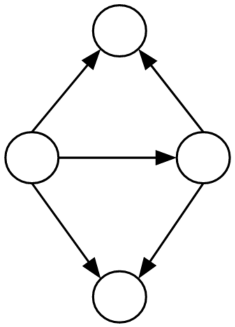

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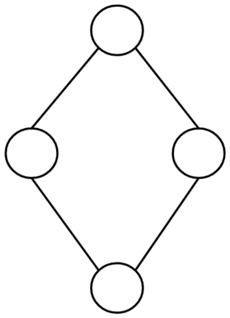

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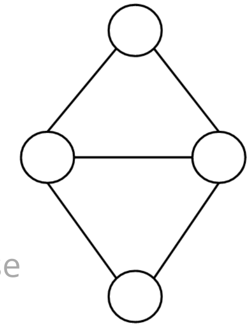
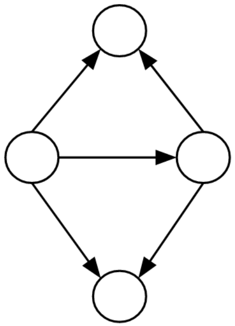
need clique-trees to build these





directed

- parameter-estimation is easy
- can represent causal relations
- better for encoding expert domain knowledge

undirected

- simpler CI semantics
- less interpretable form for local factors
- less restrictive in structural form (loops)

Summary

- directed to undirected:
  - moralize
- undirected to directed:
  - triangulate
- Chordal graphs = Markov $\cap$ Bayesian networks
  - p-maps in both directions