COMP 531: Advanced Theory of Computation (Winter 2014)

Assignment 1

Due January 29th

Instructions

Follow these instructions closely.

You will benefit most if you seriously try solving each problem yourself. You may work with each other but you must write up your own solutions. For each question, you should clearly acknowledge the people you have worked with. You are not allowed to use any resources that contain the solution to an assignment question. However, we value honesty above all. You will get full marks if you happen to find the solution to a question and you write your own solution, **as long as you properly acknowledge your source**. Failure to acknowledge your source can result in 0 points.

Clarity and conciseness of your solutions are as important as correctness. It is important to learn how to write your ideas and solutions clearly and rigorously. You will lose marks for correct solutions that are poorly explained/presented. When writing your solutions, assume that your audience is your class mates rather than the instructor of the course. The high level ideas and an overview of your argument should be presented before any technical details, and all non-trivial claims have to be proven.

If you do not know how to solve a problem, do not answer it. This will earn you 20% of the points. Do not make yourself believe in a wrong proof, this is bad for you. **And definitely do not try to sell it!** If you don't know how to solve a problem but you have some non-trivial ideas, write them down. If you have a solution with gaps, write your argument and clearly indicate the gaps.

Submit your assignments in class or send a copy to `aada@cs.mcgill.ca` before midnight of the due date.

Questions

1. (**10 points**) Suppose that f and g are computable in logarithmic space. Show that $f \circ g$ is computable in logarithmic space.

2. - (**5 points**) Explain why there is a $\log T(n)$ factor entering the statement of the Time Hierarchy Theorem.
   - (**5 points**) In the Space Hierarchy Theorem seen in class, we required that $S_1(n), S_2(n) \geq \log n$. Explain why this condition is needed.

3. (**10 points**) Let A be the language of properly nested parentheses and brackets, e.g. ([[]]([])) is in A but (([)]) is not. Show that $A \in \mathsf{L}$.

4. (**10 points**) Let $PRIME$ be the language of all prime numbers. Prove that $PRIME$ is in $\mathsf{NP} \cap co\mathsf{NP}$.
   You can use the following fact without proof:
   An integer $n > 2$ is prime if and only if there is an integer $1 < r < n$ such that $r^{n-1} \equiv 1 \mod n$ and for all prime divisors q of $n-1$, $r^{\frac{n-1}{q}} \not\equiv 1 \mod n$.

5. (**10 points**) Give an example of a non-regular language which can be decided in space $O(\log \log n)$.

6. (**Extra Credit: 5 points**) Colour the edges of the complete undirected graph on n vertices (i.e. K_n) with c colors. Show that if $n > r^c$ there must be a monochromatic path of length r. Your proof must be self-contained.