Workshop
on
Causal Structure
in
Physics and Computer Science
Introductory Remarks

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What is all this about?

Serendipity!

- Partially ordered sets have emerged as a key mathematical ingredient in the causal set approach to quantum gravity (Sorkin) but also as a fundamental notion in the semantics of programming languages: domain theory. *It is possible* that ideas (especially topological and sheaf theoretic ideas) in the two fields are common. Causal structure in computer science was also captured in the order-theoretic framework (Winskel).

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- This workshop is a first step towards exploring these connections and making contacts between researchers in the two fields.

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- Newtonian: Absolute space, absolute time. No bound on propagation of effects. Topology is $\mathbf{R}^4 = \mathbf{R} \times \mathbf{R}^3$.

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- General Relativistic: Absolute light cones as above; curved spacetime geometry. Topology can be nontrivial; in particular *global topology* is important.
- Quantum Gravity: Discrete spacetime structure; causal structure given by a *partially ordered* set. Can we recover the topology from the causal structure?

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- **Affine structure**: parallel transport
- **Metric structure**: distance

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Gravity as Spacetime Geometry

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Einstein's Idea:

- there is no such thing as gravitational "force".
- The presence of matter curves the ambient spacetime.
- Other material objects (even light) follow the straight lines (geodesics) of the curved geometry.
- Geometry becomes a dynamical entity: expanding universes and gravity waves.

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Posets in Computer Science

- Posets model *partial* information.

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- Posets representing data types are called **domains**.
- Computable functions between domains are order preserving (monotone).

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- There is a topology - the Scott topology - which captures the above notion.

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- Samson Abramsky: topology of domains
- Fotini Markopoulou Kalamara: quantum causal evolution

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