Milner Lecture
From bisimulation to representation learning via metrics

Prakash Panangaden
School of Computer Science
McGill University
and
Montreal Institute of Learning Algorithms

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Outline

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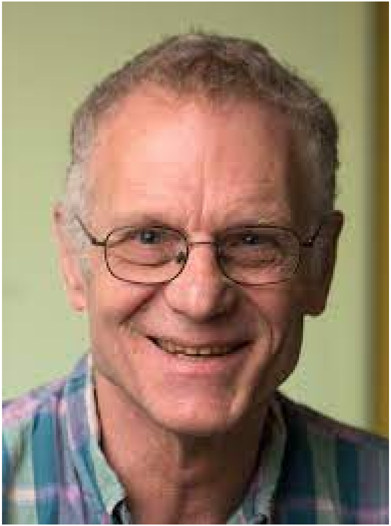
- When do two states have exactly the same behaviour?
- What can one observe of the behaviour?
- What should be guaranteed?
- (i) If two states are equivalent we should not be able to “see” any differences in observable behaviour.
- (ii) If two states are equivalent they should stay equivalent as they evolve.

Heros of concurrency theory: Milner and Park

Inspiration for my work I: Dexter Kozen



Inspiration for my work II: Lawvere and Giry

Special thanks I




Special thanks II

A bit of history

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- Representation learning using "metrics": Castro, Kastner, P. Rowland 2021

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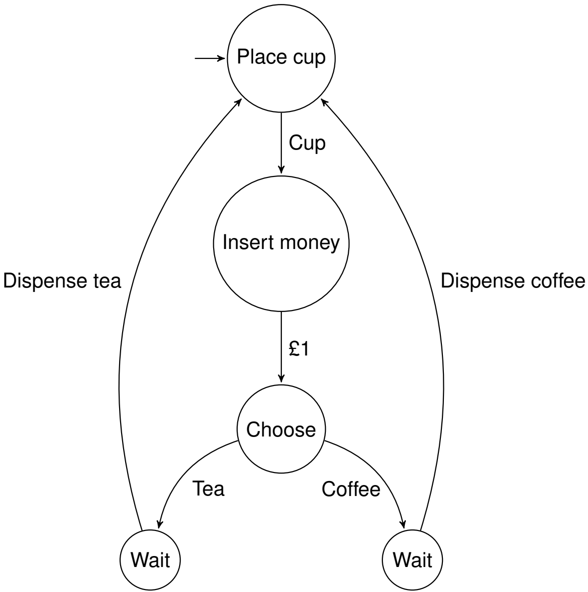
  The transitions could be indeterminate (nondeterministic).
- We write $s \xrightarrow{a} s'$ for $(s, s') \in \rightarrow_a$.

Vending machine LTSs



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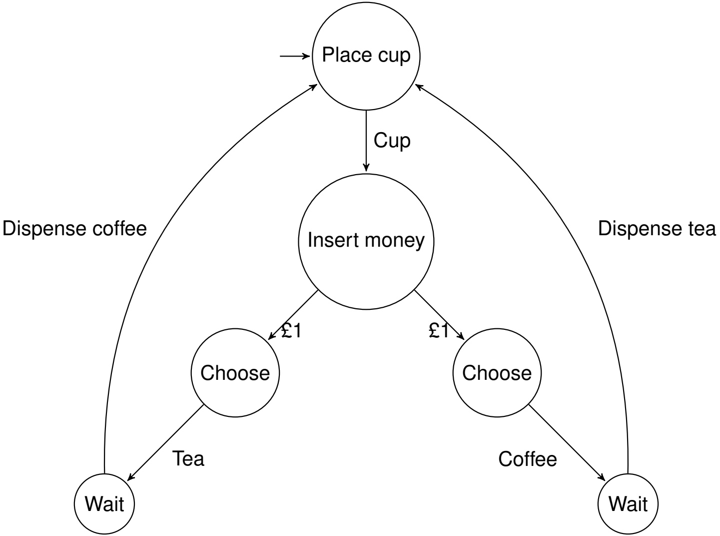

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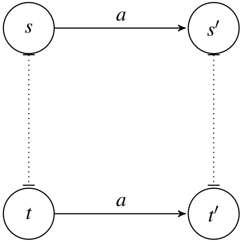
- We need to go beyond language equivalence.



[Bisimulation definition]

If $s \sim t$ then

$$\forall s \in S, \forall a \in \mathcal{A}, s \xrightarrow{a} s' \Rightarrow \exists t', t \xrightarrow{a} t' \text{ with } s' \sim t'$$

and *vice versa* with s and t interchanged.

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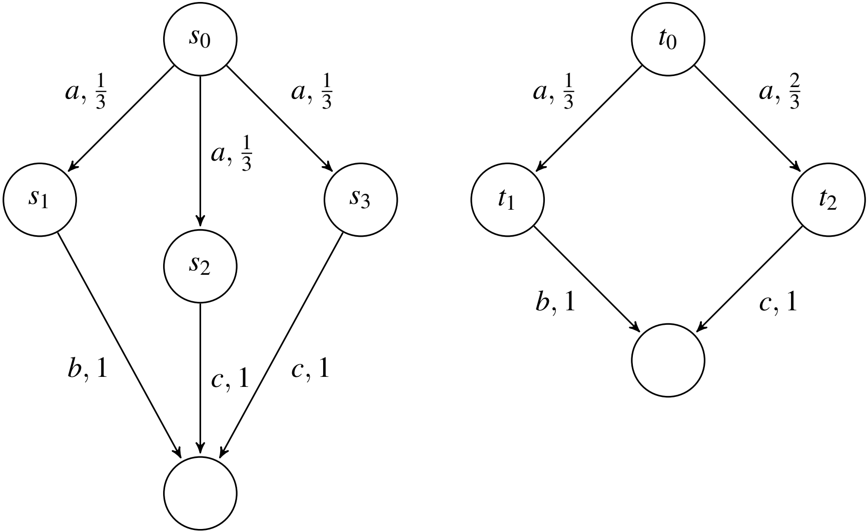
- $$(S, \mathcal{A}, \forall a \in \mathcal{A}\ T_a : S \times S \longrightarrow [0, 1])$$
- The model is *reactive*: All probabilistic data is *internal* - no probabilities associated with environment behaviour.

Probabilistic bisimulation : Larsen and Skou



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Definition

R is a bisimulation relation if whenever sRt and C is an equivalence class of R then $T_a(s, C) = T_a(t, C)$.

Markov decision processes?

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- There is a *reward* associated with each transition.
- We observe the interactions and the rewards - not the internal states.

Markov decision processes: formal definition

$$(S, \mathcal{A}, \forall a \in \mathcal{A}, P^a : S \longrightarrow \mathcal{D}(S), \mathcal{R} : \mathcal{A} \times S \longrightarrow \mathbf{R})$$

where
S : the state space, we will take it to be a finite set.
$\mathcal{A}$: the actions, a finite set
P^a : the transition function; $\mathcal{D}(S)$ denotes distributions over S
$\mathcal{R}$: the reward, could readily make it stochastic.
Will write $P^a(s, C)$ for $P^a(s)(C)$.

Policies

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Policy

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The goal is **choose** the best policy: numerous algorithms to find or approximate the optimal policy.

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- s, t are bisimilar if there is a bisimulation relation R with sRt them.
- Basic pattern: immediate rewards match (initiation), stay related after the transition (coinduction).
- Bisimulation can be defined as the *greatest fixed point* of a relation transformer.

Continuous state spaces: why?

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- Applications to probabilistic programming languages.

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- Why not discretize right away and never worry about the continuous case?
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- We lose the ability to *refine* the model later.

The Need for Measure Theory

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- More precisely, there is no translation-invariant measure defined on all the subsets of the reals.

Logical Characterization

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- but it needs disjunction.
- The proof uses tools from descriptive set theory and measure theory.
- Such a theorem originally proved for LTS with finite-branching restrictions by Hennessy and Milner in 1977 and van Benthem in 1976

The proof “engine” Josée Desharnais

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- We say “no”. A small change in the probability distributions may result in bisimilar processes no longer being bisimilar though they may be very “close” in behaviour.
- Instead one should have a (pseudo)metric for probabilistic processes.

A metric-based approximate viewpoint

- Move from equality between processes to distances between processes (Jou and Smolka 1990).

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- Move from equality between processes to distances between processes (Jou and Smolka 1990).
- Quantitative measurement of the distinction between processes.

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- All this can be formalized and was originally done by Desharnais et al. and later with a beautiful fixed-point construction by van Breugel and Worrell.
- Ferns et al. added rewards and showed that the bisimulation metric bounds the difference in optimal value functions.

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- Much of the theory of equational logic carries over to this setting.
- Algebras for such equations are naturally equipped with metrics and give a way of reasoning about bisimulation metrics.
- Mardare, P., Plotkin LICS 2016, 2017, 2021; Bacci, Mardare, P., Plotkin LICS 2018, CALCO 2021.

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- We seek optimal policies for extracting the largest possible reward in expectation.
- A plethora of algorithms and techniques, but the cost depends on the size of the state space.
- Can we *learn* representations of the state space that accelerate the learning process?

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- Representation learning means learning such a ϕ.
- The elements of M are the "features" that are chosen. They can be based on any kind of knowledge or experience about the task at hand.

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- and is not even technically a metric!

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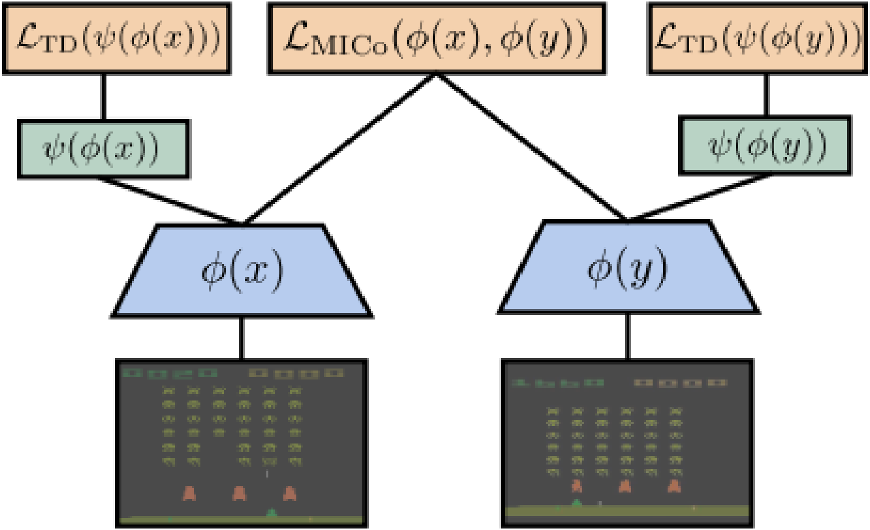
4. Do not require $d(x, x) = 0$

MICo loss

- Nearly all machine learning algorithms are optimization algorithms.
- One often introduces extra terms into the objective function that push the solution in a desired direction.
- We defined a loss term based on the MICo distance.
- For details read
  `https://psc-g.github.io/posts/research/rl/mico/`

Experimental setup



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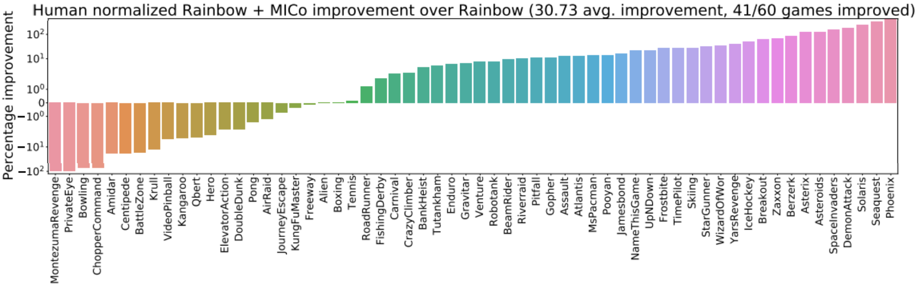
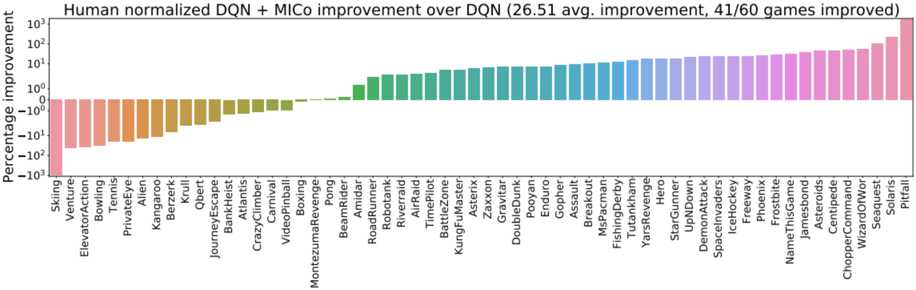
- *Every* agent performed better on about $\frac{2}{3}$ of the games.

Results for Rainbow



Results for DQN



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- The metric analogue holds promise for quantitative reasoning and approximation.
- Perhaps a fruitful line of research would be equation solving in quantitative algebras and automating equational reasoning in the quantitative setting.
- Research is alive and well and there are new areas where bisimulation is being “discovered”.