Labelled Markov Processes

Lecture 1: Labelled Transition Systems

Prakash Panangaden[1]

[1]School of Computer Science
McGill University

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Outline

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- Lecture 1: Labelled transition systems and bisimulation.
- Lecture 2: Labelled Markov processes.
- Lecture 3: Logical characterization of bisimulation.
- Lecture 4: The metric analogue of bisimulation.

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- Labelled transition systems.
- Bisimulation.
- Making sense of coinduction.
- Games for bisimulation and simulation.
- Logical characterization.

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- Probabilistic bisimulation can be defined for continuous state-space systems. [LICS97]
- Logical characterization. [LICS98,Info and Comp 2002]
- Metric analogue of bisimulation. [CONCUR99, TCS2004]
- Approximation of LMPs. [LICS00,Info and Comp 2003]
- Weak bisimulation. [LICS02,CONCUR02]
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The definition

- A set of states S,
- a set of *labels* or *actions*, L or $\mathcal{A}$ and
- a transition relation $\subseteq S \times \mathcal{A} \times S$, usually written

$$\rightarrow_a \subseteq S \times S.$$

  The transitions could be indeterminate (nondeterministic).
- We write $s \xrightarrow{a} s'$ for $(s, s') \in \rightarrow_a$.

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A simple example

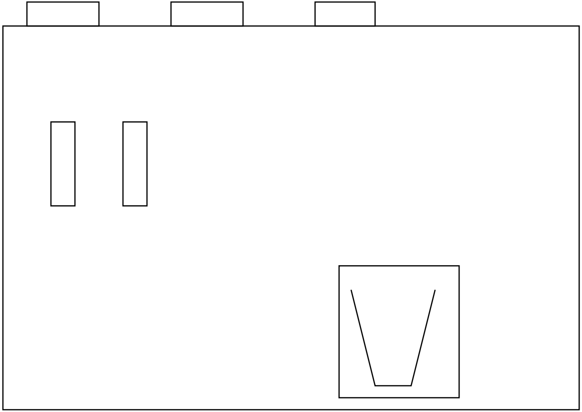
$$\begin{tikzcd}
 & s_0 \arrow[dl, "a"'] \arrow[dr, "a"] & \\
s_1 & & s_2 \arrow[d, "b"] \\
 & & s_3
\end{tikzcd}$$

A_1

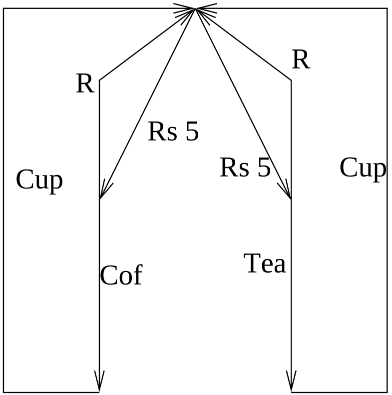
$$\begin{tikzcd}
 & s_0 \arrow[dl, "a"'] \arrow[dr, "b"] & \\
s_1 \arrow[d, "c"'] \arrow[drr, "c"] & & s_2 \arrow[d, "a"] \\
s_4 & & s_3
\end{tikzcd}$$

A_2

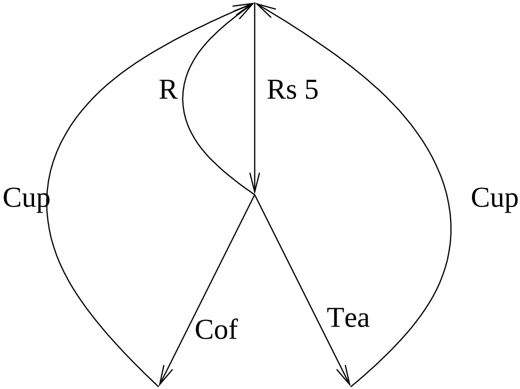
A vending machine



Vending machine LTSs



Another (?) vending machine LTSs



Are the two LTSs equivalent?

- One gives *us* the choice whereas the other makes the choice *internally*.
- The sequences that the machines can perform are identical: $[Rs.5; (Cof + Tea); Cup]^*$
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Bisimulation

s and t are states of a labelled transition system. We say s is **bisimilar** to t – written $s \sim t$ – if

$$s \xrightarrow{a} s' \Rightarrow \exists t' \text{ such that } t \xrightarrow{a} t' \text{ and } s' \sim t'$$

and

$$t \xrightarrow{a} t' \Rightarrow \exists s' \text{ such that } s \xrightarrow{a} s' \text{ and } s' \sim t'.$$

Does it make sense?

- The definition of bisimilarity seems circular.
- In fact, it is perfectly well defined.
- There are three or four ways of explaining it.

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Coinduction via induction

- Define a *family* of equivalence relations $\sim_n$ indexed by the natural numbers.
- $\sim_0$ is the universal relation: $\forall s, t \;\; s \sim_0 t$.
- $s \sim_{n+1} t$ if

$$\forall a, s \xrightarrow{a} s' \Rightarrow \exists t', t \xrightarrow{a} t' \text{ and } s' \sim_n t'$$

  and vice versa.
- $s \sim t$ if and only if $\forall n, s \sim_n t$.

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Coinduction as a greatest fixed point

- Fix a labelled transition system with state space S.
- Let $\mathcal{R}$ be the collection of equivalence relations on S ordered by inclusion.
- Define $\mathcal{F} : \mathcal{R} \longrightarrow \mathcal{R}$ by

$$s\mathcal{F}(R)t \text{ means } \forall a, s \xrightarrow{a} s' \Rightarrow \exists t', t \xrightarrow{a} t' \text{ and } s'Rt'$$

  and vice versa.
- $\mathcal{R}$ is a complete lattice partially ordered by inclusion and $\mathcal{F}$ is a monotone function.
- It is a (moderately) easy exercise to show that $\mathcal{F}$ has a greatest fixed point: this is bisimulation.

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Bisimulation relations

- Define *a* (note the indefinite article) bisimulation relation R to be an equivalence relation on S such that
- $$sRt \text{ means } \forall a, s \xrightarrow{a} s' \Rightarrow \exists t', t \xrightarrow{a} t' \text{ with } s'Rt'$$
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- This is not circular; it is a condition on R.
- We define $s \sim t$ if there is *some* bisimulation relation R with sRt.
- This is the version that is used most often.

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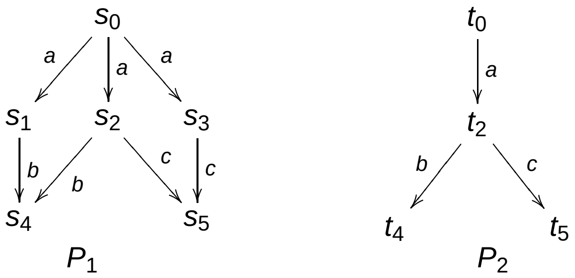
- 

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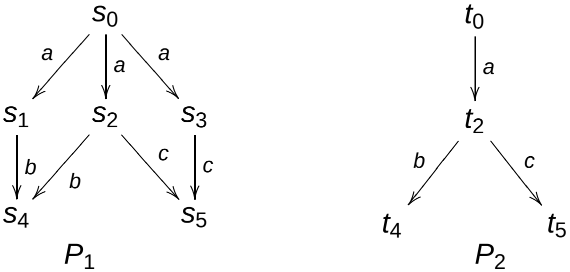
- 

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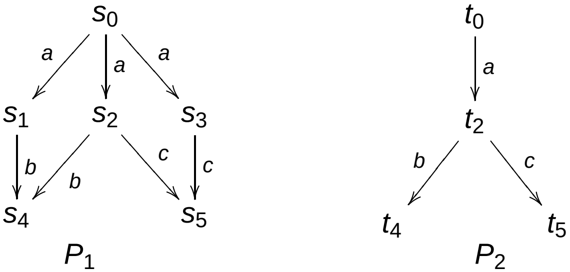
- 

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The bisimulation game

- Two players: maker (M) and spoiler (S). M wants to establish a bisimulation and S wants to spoil the bisimulation.
- S chooses a process with which to play and makes a move.
- M must match S's move.
- S chooses again which process she wants to play and makes a move which M must match.
- If M has a winning strategy then the processes are bisimilar.
- If we did not allow S to switch after the first move then a winning strategy for M implies two-way simulation: much weaker than bisimulation.

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How do we know that two processes are **not** bisimilar?

- Define a logic as follows:
- $$\phi ::== \mathrm{T} | \neg\phi | \phi_1 \wedge \phi_2 | \langle a \rangle \phi$$
- $s \models \langle a \rangle \phi$ means that $s \xrightarrow{a} s'$ and $t \models \phi$.
- We can define a dual to $\langle\rangle$ (written []) by using negation.
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Examples of HM Logic

- T is satisfied by any process, F is not satisfied by any process.
- $s \models \langle a \rangle T$ means s can do an a action.
- $s \models \neg\langle a \rangle \phi$ or $s \models [a]F$ means s cannot do an a action.
- $s \models \langle a \rangle(\langle b \rangle T)$ means that s can do an *a and then* do a b.

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The Hennessy-Milner theorem

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- Basic assumption: the processes are finitely-branching (otherwise you need infinitary conjunctions).
- To show that two processes are not bisimilar find a formula on which they disagree.

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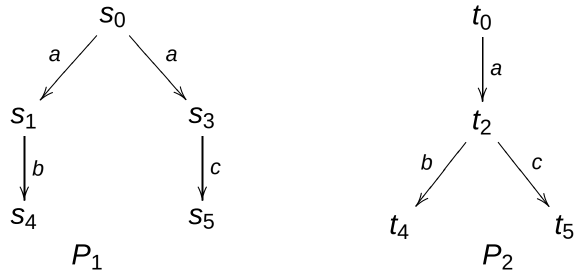
- 

- Here s_0 and t_0 are not bisimilar.
- $s_0 \models \langle a \rangle (\neg \langle b \rangle T)$ but t_0 does not satisfy this formula.
- $t_0 \models \langle a \rangle (\langle b \rangle T \wedge \langle c \rangle T)$ but s_0 does not satisfy this.
- The conjunction captures branching structure.

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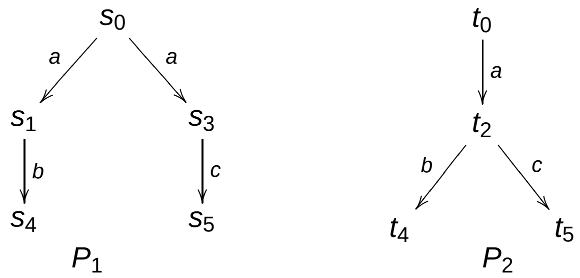
- 

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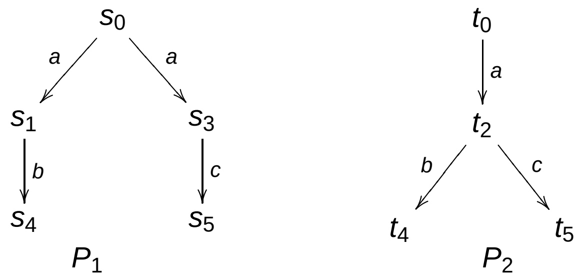
- 

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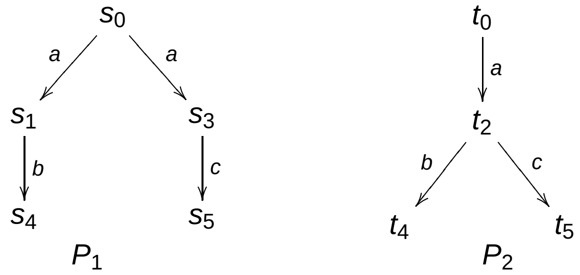
- 

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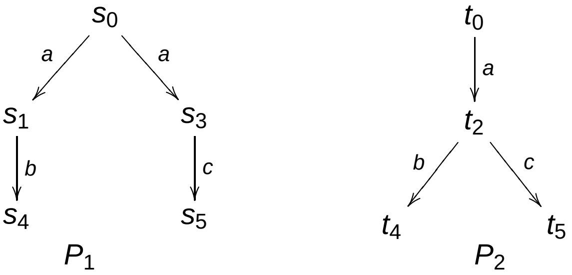
- 

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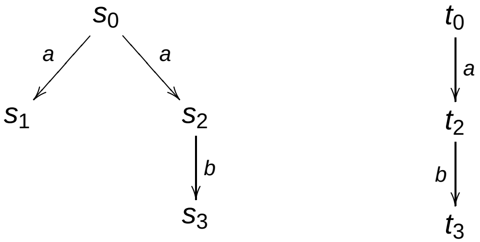

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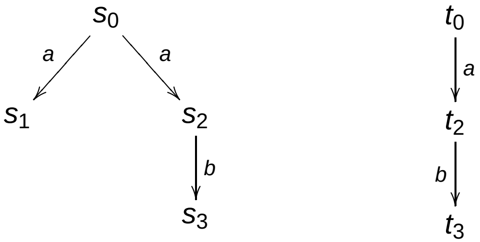

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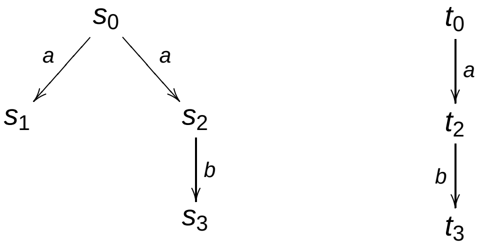

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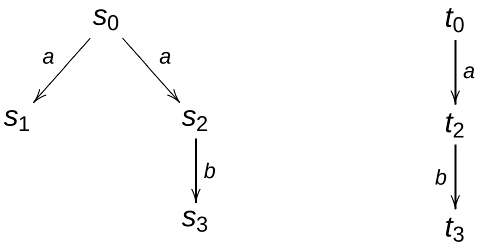

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Next Lecture

We do everything probabilistically.