COMP 330 Fall 2021
Assignment 2
Due Date: 5^{th} October 2021

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21*st* September 2021

There are **5** questions for credit. There is an extra question which is a puzzle. If you can't do it, it does not mean anything about how well you are understanding the material. If you like mathematical puzzles you might enjoy this one. **Please do not submit it.** The homework is due on myCourses at 5pm.

Question 1[20 points]
Give regular expressions for the following languages over $\{a, b\}$:

1. $\{w | w$ contains an even number of occurrences of $a\}$
2. $\{w | w$ contains an odd number of occurrences of $b\}$
3. $\{w |$ does not contain the substring $ab\}$
4. $\{w |$ does not contain the substring $aba\}$

Try to make your answers as simple as possible. We will deduct marks if your solution is *excessively* complicted.

Question 2[20 points]
Suppose that you have a DFA $M = (S, \Sigma, s_0, \delta, F)$. Consider two distinct states s_1, s_2 *i.e.* $s_1 \neq s_2$. Suppose further that *for all* $a \in \Sigma$ $\delta(s_1, a) = \delta(s_2, a)$. Show that for any *nonempty* word w over Σ we have $\delta^*(s_1, w) = \delta^*(s_2, w)$.

Question 3[20 points]
Show that the following languages are not regular by using the pumping lemma.

1. $\{a^n b^m a^{n+m} | n, m \geq 0\}$,
2. $\{x | x = x^R, x \in \Sigma^*\}$, where x^R means x reversed; these strings are called *palindromes*. An example is *abba*, a non-example is *baba*.

Question 4[20 points] Show that the following languages are not regular by using the pumping lemma.

1. $\{x \in \{a, b, c\}^* || x|$ is a square.$\}$ Here $|x|$ means the length of x.
2. $\{a^{2n}b^n\}$.

Question 5[20 points] Show that the language

$$F = \{a^i b^j c^k | i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$$

is not regular. Show, however, that it satisfies the statement of the pumping lemma as I proved it in class, *i.e.* there is a p such that all three conditions for the pumping lemma are met. Convince yourself this does not contradict the pumping lemma.

Pure puzzle[0 points] Design an **NFA** K with n states, over a one-letter alphabet, such that K rejects some strings, but the *shortest* string that it rejects has length *strictly* greater than n.