McGill University COMP531 Winter 2010 **Instructor: Phuong Nguyen**

Assignment 3

Due March 15 in lecture

The work you submit must be your own. You may discuss problems with each others; however, you should prepare written solutions alone. Copying assignments is a serious academic offense, and will be dealt with accordingly.

(It is helpful to give a high level description of a proof or an algorithm before giving the details.)

Question 1 (10pt) A language A is said to be *sparse* if there is a polynomial p so that for all n, the number of elements of A of length exactly n is at most $p(n)$. That is, for all n:

$$\#\{x \ : \ x \in A, |x| = n\} \leq p(n)$$

(Here $\#S$ stands for the cardinality of a set S.) Prove that a language is in $\mathbf{P}/poly$ if and only if it is polytime Turing reducible to a sparse language; in other words, prove that

$$\mathbf{P}/poly = \{\mathbf{P}^A \ : \ A \text{ is sparse}\}$$

Question 2 (modified Exercise 6.3 in the text) (10pt) In class we have seen that the undecidable language UniHALT is in $\mathbf{P}/poly$ (in fact, in $\mathbf{P}/1$) but not in $\mathbf{P}$. In this question you are asked to show that there is a *decidable* language that is in $\mathbf{P}/1$ but that is not in $\mathbf{P}$.

Question 3 (10pt) Prove that $PATH \in$ logspace-uniform $\mathbf{AC}^1$. In other words, show that there is a family of $\mathbf{AC}^1$ circuits that computes $PATH$, and furthermore this family can be described using a logspace Turing machine. (This shows that $\mathbf{NL} \subseteq$ logspace-uniform $\mathbf{AC}^1$.)

Question 4 (Exercise 14.3 in the text) (10pt) Prove that if all the max-terms of a Boolean function f are of size at most s, then f is expressible as an s-CNF.

Question 5 (10pt) Using the fact that Parity is not in non-uniform $\mathbf{AC}^0$, show that the function MULT is not in non-uniform $\mathbf{AC}^0$ (by reducing Parity to MULT).