

Assignment 5

Due February 14 at the beginning of lecture

The work you submit must be your own. You may discuss problems with each others; however, you should prepare written solutions alone. Copying assignments is a serious academic offense, and will be dealt with accordingly.

Question 1 (8pt) When the edge costs are not distinct, there may be more than one minimum spanning tree. The execution, and hence the output, of Kruskal's algorithm depend on the ordering of edges that have the same cost—recall that Kruskal's algorithm works by first sorting the edges in nondecreasing order of cost.

The question here is, for any graph G and cost function c , and any minimum spanning tree T of G , is there an execution of Kruskal's algorithm that outputs T ? Either prove this, or give a counterexample (i.e., exhibit a graph G with a cost function c and a minimum spanning tree T such that no matter how edges of G are ordered, Kruskal's algorithm does not produce T as the output).

Question 2 (10pt) Consider the following problem, called Weighted Job Scheduling (WJS) here. In this problem there is a single processor that can only process one job at a time. There is a set of n jobs to be processed, the i -th job requires a contiguous period of duration t_i and it has weight w_i . All t_i and w_i are positive integers. Timing starts at 0. For a schedule of the jobs, let d_i denotes the finish time of job i . Then the “weighted duration” of the schedule is defined to be

$$\sum_{i=1}^n w_i d_i$$

The WJS problem is to obtain a schedule of all jobs with smallest possible weighted duration.

Give a greedy algorithm to solve this problem. Clearly state the criterion that you optimize for each step, give the pseudo-code for the algorithm, and prove its optimality.