McGill University COMP360 Winter 2011 **Instructor: Phuong Nguyen**

Assignment 3 Solution

Question 1 Show that the problem (called A1P2) from Assignment 1 is NP-complete by many-one reducing one of the following problem to it:

3CNF-SAT, CLIQUE, 3-COL, IND (Independent set), SubsetSum

Clearly specify which problem you use, describe the transformation, and prove the correctness of the reduction.

Solution

Specify the problem: We will reduce 3-COL to A1P2.

The reduction: On input G, the output is (G, K_3) where K_3 is the complete graph on 3 vertices (i.e., the triangle).

Correctness of reduction: We name the three vertices of K_3 by R, B and Y, respectively. There are two directions.

For the first direction, suppose that G is 3-colorable. So suppose that the vertices of G can be colored by Red, Blue, and Yellow, so that the endpoints of every edge of G have different colors. Then there is a mapping f from G to K_3 defined by

$$f(v) = \begin{cases} R & \text{if } v \text{ is colored Red} \\ B & \text{if } v \text{ is colored Blue} \\ Y & \text{if } v \text{ is colored Yellow} \end{cases}$$

Now if (u, v) is an edge in G, then u and v have different colors, so $f(u) \neq f(v)$ and hence there is an edge between $f(u)$ and $f(v)$ in K_3.

For the second direction, suppose that there is a mapping f from G to K_3 that satisfies the condition of the problem A1P2. We color the vertices of G by coloring v Red if $f(v) = R$, Blue if $f(v) = B$, and Yellow if $f(v) = Y$. We need to show that this is a valid coloring. So let (u, v) be any edge of G. The mapping f satisfies the condition that $(f(u), f(v))$ is an edge of K_3. Hence u and v must have different colors. QED.

Question 2 Show that the problem Knapsack (see below) is NP-complete by many-one reducing one of the following problem to it:

3CNF-SAT, CLIQUE, 3-COL, IND, SubsetSum, A1P2

Clearly specify which problem you use, describe the transformation, and prove the correctness of the reduction.

For sake of completeness, here is our version of the Knapsack problem.

Input: A weighted set where the i-th element (or item) has weight w_i and value v_i:

$$S = \{(w_1, v_1), (w_2, v_2), \ldots, (w_n, v_n)\}$$

and an upper bound W for the total weight and a target V for the total value.

All numbers are written in binary.

Question: The question is whether there is a set of items in S with total weight $\leq W$ and total value $\geq V$.

Solution

Solution

Specify the problem to reduce: Subsetsum.

The reduction: On input (S, t) to Subsetsum, where $S = \{x_1, x_2, \ldots, x_n\}$ we create the following input to Knapsack: The weighted set S' is

$$\{(x_1, x_1), (x_2, x_2), \ldots, (x_n, x_n)\}$$

(i.e., the i-th element has weight x_i and value x_i), and $W = V = t$.

Correctness of reduction: Because the weight and value of each item of S' are the same, S' contains a subset of total weight at most t and total value at least t if and only if S contains a subset of total exactly t. QED.