COMP 330 - Fall 2010 - Assignment 6

Due 8:00 pm Dec 3, 2010

General rules: In solving this you may consult books and you may also consult with each other, but you must each find and write your own solution. In each problem list the people you consulted. This list will not affect your grade. There are in total 115 points, but your grade will be considered out of 100. You should drop your solutions in the assignment drop-off box located in the Trottier Building on the 3rd floor left of the elevators.

1. Recall that for $f, g : \mathbb{N} \to \mathbb{R}^+$, we say $f = O(g)$ if and only if

$$\exists c, n_0 > 0 \forall n > n_0 \qquad f(n) \leq cg(n). \tag{1}$$

   In each one of the following cases show that $f = O(g)$ by proving (1).

   (a) (5 Points) $f(n) = (n+5)^3$ and $g(n) = n^3$.
   (b) (10 Points) $f(n) = 10^{\sqrt{n}}$ and $g(n) = 2^n$.
   (c) (10 Points) $f(n) = \log_2 n$ and $g(n) = \sqrt{n}$.

2. Recall that for $f, g : \mathbb{N} \to \mathbb{R}^+$, we say $f = o(g)$ if and only if

$$\forall \epsilon > 0 \exists n_0 > 0 \forall n > n_0 \qquad f(n) \leq \epsilon g(n). \tag{2}$$

   In each one of the following cases show that $f = o(g)$ by proving (2).

   (a) (10 Points) $f(n) = 342n^2 + n + 10$ and $g(n) = n^2 \log_2 n$
   (b) (10 Points) $f(n) = \sqrt{n} + 10$ and $g(n) = \sqrt{n \log_2 n}$.

3. Consider the following algorithm:

   On input w which is the binary representation of a positive integer:

   - For $i = 2, 3, \ldots, w-1$
   - If w is divisible by i, reject.
   - If w was not divisible by any of the above values of i, accept.

   (a) (5 Points) What is the language of the above Turing Machine?
   (b) (5 Points) Is the running time of this Turing Machine polynomial? (Explain)

4. (15 Points) A cycle of size m in a graph G is a set of <u>distinct</u> vertices $v_1, \ldots, v_m$ such that v_1 is adjacent to v_2, v_2 is adjacent to v_3,$\ldots$,v_{m-1} is adjacent to v_m, and v_m is adjacent to v_1. Show that the following language is in P:

$$C_{100} = \{\langle G \rangle : G \text{ is a graph which contains a cycle of size at least } 100\}.$$

5. (10 Points) Show that if P=NP, then every language in NP except $\emptyset$ and Σ^* is NP-complete.

6. (10 Points) Show that if L is in NP, then L^* is also in NP.

7. Show that the following languages are NP-complete.

   (a) (10 Points)

   $$\{\langle\phi\rangle \;:\; \phi \text{ is a CNF with a solution that sets exactly half of the variables to TRUE}\}.$$

   (b) (15 Points) A d-clique in a graph G is a set of vertices such that every two of them are in distance at most d from each other. So a 1-clique is an actual clique. The language in question is

   $$\{\langle G, k\rangle \mid G \text{ contains a 2-clique with } k \text{ vertices}\}.$$