# COMP 330 - Fall 2010 - Assignment 5

Due 8:00 pm Nov 19, 2010

**General rules:** In solving this you may consult books and you may also consult with each other, but you must each find and write your own solution. In each problem list the people you consulted. This list will not affect your grade. There are in total 115 points, but your grade will be considered out of 100. You should drop your solutions in the assignment drop-off box located in the Trottier Building on the 3rd floor left of the elevators.

1. Show that the following languages are undecidable.

   (a) (10 Points)
   $$\{\langle M\rangle \mid M \text{ is a TM and } L(M) = L(110\{0,1\}^*)\}.$$

   (b) (15 Points)
   $$\{\langle M\rangle \mid M \text{ is a TM and } L(M) \text{ is finite}\}.$$

   (c) (15 Points)
   $$\left\{\langle M\rangle \;\middle|\; \begin{array}{c} M \text{ is a TM which has a state } q \text{ that is never entered on any input,} \\ \text{and } q \text{ is not the accept or the reject state.} \end{array}\right\}.$$

2. (10 Points) Show that neither $L$ nor its complement $L^c$ is recursively enumerable, where
   $$L = \{\langle M, v, w\rangle \mid M \text{ is a Turing Machine which accepts } v \text{ and does not accept } w\}.$$

3. Show that the following languages are decidable:

   (a) (10 Points)
   $$\{\langle M\rangle \mid M \text{ is an LBA that accepts at least one string of length less than } 100\}.$$

   (b) (10 Points)
   $$\{\langle M, w\rangle \mid M \text{ is a TM that on } w \text{ never moves the head beyond the first } 2|w| \text{ cells of the tape}\}.$$

4. (5 Points) Show that if $A$ is decidable and $B$ is any arbitrary language satisfying $B \neq \emptyset$ and $B \neq \Sigma^*$, then $A \leq_m B$.

5. (5 Points) Show that if $A \leq_m A^c$ and $A$ is recursively enumerable, then $A$ is decidable.

6. (15 Points) Show that if $L_1$ and $L_2$ are recursively enumerable, and $L_1 \cap L_2$ and $L_1 \cup L_2$ are both decidable, then $L_1$ and $L_2$ are decidable.

7. (20 Points) A polynomial is called *non-negative on integers*, if it is non-negative for every assignment of integers to its variables. A polynomial is called *non-negative on non-negative integers*, if it is non-negative for every assignment of non-negative integers to its variables.

   Answering Hilbert's 10th problem, Matiyasevich showed that the following language is undecidable:

$$L = \left\{ p \;\middle|\; \begin{array}{c} p \text{ is a multivariate polynomial with integer coefficients} \\ \text{and p=0 for some assignment of integers to its variables} \end{array} \right\}.$$

   Use this to show that the following two languages are undecidable:

   (a)

$$K = \left\{ p \;\middle|\; \begin{array}{c} p \text{ is a multivariate polynomial with integer coefficients} \\ \text{which is } \underline{\text{non-negative on integers}}. \end{array} \right\}.$$

   (b)

$$K^+ = \left\{ p \;\middle|\; \begin{array}{c} p \text{ is a multivariate polynomial with integer coefficients} \\ \text{which is } \underline{\text{non-negative on non-negative integers}}. \end{array} \right\}.$$