COMP 330 - Fall 2010 - Assignment 2

Due 8:00 pm Oct 15, 2010

General rules: In solving this you may consult books and you may also consult with each other, but you must each find and write your own solution. In each problem list the people you consulted. This list will not affect your grade. There are in total 115 points, but your grade will be considered out of 100. You should drop your solutions in the assignment drop-off box located in the Trottier Building on the 3rd floor left of the elevators. Check the website of the course, "http://www.cs.mcgill.ca/~hatami/comp330" for possible corrections.

1. (a) (5 points) Find a left-most derivation for $aaabbabbba$ in the following context-free grammar:

$$\begin{array}{rcl} S & \to & aB \mid bA \\ A & \to & a \mid aS \mid bAA \\ B & \to & b \mid bS \mid aBB \end{array}$$

   (b) (5 points) Draw the corresponding parse-tree of your left-most derivation.

2. (5 Points) Show that each prefix[1] of every word in the language of the following context-free grammar has at least as many 0's as 1's.

$$S \quad \to \quad 0S \mid 0S1S \mid \varepsilon$$

3. (30 points) For each one of the following languages give a proof that it is or is not regular.

   (a)
$$\{0^m1^n | m \geq 5 \text{ and } n \geq 0\}.$$

   (b)
$$\{0^m1^n | m \geq n^2\}.$$

   (c) The set of strings in $\{0,1\}^*$ which are <u>not</u> of the form ww for some $w \in \{0,1\}^*$.

   (d)
$$\left\{ 0^{\lfloor \sqrt{n} \rfloor} \middle| n = 0, 1, 2, \ldots \right\},$$

   where for a real number x, $\lfloor x \rfloor$ denotes the largest integer that is less than or equal to x.

   (e) The set of all strings $w \in \{0,1\}^*$ such that the number of occurrences of the substring 01 in w is equal to the number of occurrences of 10 as a substring.

   (f) The first two Fibonacci numbers are 0 and 1, and each subsequent number is the sum of the previous two: $0, 1, 1, 2, 3, 5, 8, 13, \ldots$. Now the language in question is

$$\{0^n | n \text{ is a Fibonacci number}\}.$$

[1] A prefix is a substring that starts from the beginning of the word.

4. (20 points) For each one of the following languages construct a context-free grammar that generates that language:

   (a)
   $$\{0,1\}^*.$$

   (b)
   $$\{0^m 1^n \mid m \geq n \text{ and } m-n \text{ is even}\}.$$

   (c) The complement of $\{0^n 1^n \mid n \geq 0\}$ over the alphabet $\{0,1\}$.

   (d) The set of strings in $\{0,1\}^*$ which are <u>not</u> palindromes:
   $$\{w \in \{0,1\}^* \mid w \neq w^R\}.$$

5. (10 points) Show that the language of the grammar $S \to 0S1 \mid 1S0 \mid SS \mid \varepsilon$ is
   $$\{w \in \{0,1\}^* \mid w \text{ contains the same number of zeros and ones}\}.$$

6. (10 Points) Use the equivalence of context-free grammars and push-down automata to show that if A and B are regular languages, then $\{xy | x \in A, y \in B, |x| = |y|\}$ is context-free.

7. (10 Points) Let L be an *infinite* regular language over the single letter alphabet $\Sigma = \{0\}$. For every integer m, let
   $$L_m = \{w \in L \mid |w| \leq m\}$$
   be the set of the strings of length at most m in L. Show that there is a real number $c > 0$ and an integer $M > 0$ such that for every $m \geq M$, we have $\frac{|L_m|}{m} > c$.

8. For a positive integer m, a languages L over $\{0,1\}$ is called *m-bounded*, if the length of every word in L is at most m.

   (a) (5 Points) How many m-bounded languages are there?

   (b) (15 Points) For $m \geq 100$, show that for more than half of the $2m$-bounded languages, there is no DFA with 2^m states that recognizes them. [Hint: Count the number of DFA's with 2^m states.]