# COMP 330 - Fall 2010 - Assignment 1

Due 8:00 pm Sept 27, 2010

**General rules:** In solving these questions you may consult books but you may not consult with each other. There are in total 110 points, but your grade will be considered out of 100. You should drop your solutions in the assignment drop-off box located in the Trottier Building on the 3rd floor left of the elevators.

1. (20 points) Give DFA's accepting the following languages over the alphabet $\{0, 1\}$: (*For this question drawing the state diagram is sufficient, and you are not requited to describe the corresponding* 5*-tuples* $(Q, \Sigma, \delta, q_0, F)$).

   (a) The set of all strings $w$ such that the number of ones in $w$ is divisible by 3 and not divisible by 2 (*for example* 1011 *is accepted but* 10 *and* 111111 *are not*).

   (b) The set of all strings of length at least two that start and end with the same symbol (*for example* 00, 1011 *are accepted but* 0, 011011 *and* 10 *are not accepted*).

   (c) The set of all strings that contain 0110 as a substring (*for example* 10110 *is accepted but* 01010001 *is not*).

   (d) The set of all strings of length at least three such that every three consecutive symbols contain at least two 1's (*for example* 1101 *is accepted but* 1010111 *is not*).

2. (10 points) Give an NFA $M = (Q, \Sigma, \delta, q_0, F)$ accepting the following language over the alphabet $\{a, b, c, d\}$: The set of all strings that contain at least two $a$'s with no $c$ appearing between them (For example *bcabbdaca* and *aa* are accepted but *abccdabcb* and *bd* are not accepted). *In this question you are required to describe each one of* $Q$, $\Sigma$, $\delta$, $q_0$ *and* $F$ *in your construction. In particular give the table of* $\delta$.

3. (15 points) Let $L$ be a *finite* language over $\{0,1\}$. Describe a DFA that accepts $L$. *For this question you are not allowed to use any theorems or lemmas from the lecture or the book.*

4. (20 points) Let $M$ be a DFA accepting a language $A$ over the alphabet $\{0,1\}$. For each one of the following languages design an NFA accepting that language.

   (a) $A \setminus \{\varepsilon\}$.

   (b) $\{w^R | w \in A\}$, where $w^R$ is the reverse of the string $w$. (For example the reverse of 01101 is 10110).

   (c)
   $$\{x_1x_2\dots x_n | x_1, x_2, \dots, x_n \in A \text{ and } n \text{ is divisible by } 3\}.$$

5. (15 points) Let $r, s$ and $t$ be regular expressions. Prove or disprove each one of the following equalities.

   (a) $(rs \cup r)^* = r(sr \cup r)^*$

   (b) $s(rs \cup s)^* = rr^*s(rr^*s)^*$

   (c) $(r \cup s)^* = r^* \cup s^*$

6. (5 points) Let $r$ and $s$ be regular expressions. Give a regular expression $X$ such that always
   $$L(X) = L(rX \cup s).$$

7. (10 points) Express the following set as a regular expression: The set of all strings of length at least three over $\{0,1\}$ such that every three consecutive symbols contain at least two 1's.

8. (15 points) Show that if $A$ is a regular language, then so is $\{w | ww^R \in A\}$.