Approximation Algorithms **Homework 3**
Date: February 18, 2010 **Due:** March 4, 2010

1. (10 points) Problem 8.1 (page 209).
2. (10 points) Problem 8.3 (page 210).
3. (5 points) Problem 8.6 (page 211).
4. (10 points) Consider the greedy algorithm discussed for the multiway cut problem in the class. Find the minimum cut separating s_i and s_j for each $i \neq j$. Include the edges of the minimum such cut in the solution and recurse on both parts of the cuts. Show that this greedy algorithm is a $2 - \frac{2}{k}$-approximation for the multiway cut problem.
5. Recall the K-median problem discussed in class: given an n-point metric (V, d), and a number K, find a set C of size K that minimizes $\sum_{v \in V} d(v, C)$. In this exercise we give another way to solve the K-median problem using probabilistic embeddings.
   (a) (5 points) Suppose you are given a tree T with nonnegative edge lengths as your input to the K-median problem, show how to solve the K-median problem in polynomial time for this class of input graphs. (Hint: Use Dynamic Programming).
   (b) (10 points) To solve the K-median problem on a general metric, we use probabilistic embedding of the input metric to derive a probability distribution over tree metrics. We then sample a tree according to this distribution and apply your polynomial time algorithm from the previous part to it. We then use the same set of K nodes as the solution to the K-median problem on the original metric.
      i. Show that an optimal K-median solution C in the original metric gives a solution whose expected cost is at most $O(\log n)$ times the cost of C. Thus the optimal solution in the sampled tree has expected cost no more than this.
      ii. Show how any K-median solution of cost D in the sampled tree T has cost no more than D in the original metric.

   Mention how these two claims imply an $O(\log n)$-approximation for the K-median problem on any metric.