Simplex Method Example

$$\max \quad z = 2x_1 - x_2 + 2x_3$$

$$\text{s.t.} \quad 2x_1 + x_2 \leq 10$$

$$x_1 + 2x_2 - 2x_3 \leq 20$$

$$x_2 + 2x_3 \leq 5$$

$$x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0$$

Introduce slack (basic) variables, s_1, s_2, s_3

$$2x_1 + x_2 + \quad + s_1 = 10$$

$$x_1 + 2x_2 - 2x_3 \quad + s_2 = 20$$

$$x_2 + 2x_3 \quad + s_3 = 5$$

Constraints.

Simplex Tableau

	x_1	x_2	x_3	s_1	s_2	s_3	b	
s_1	2	1	0	1	0	0	10	
s_2	1	2	-2	0	1	0	20	$\frac{20}{-2} = -10$
s_3	0	1	2	0	0	1	5	$\frac{5}{2}$ ← Departing row
	-2	1	-2	0	0	0	0	

pivot

Entering

current z value.

Current solution is $(0, 0, 0, 10, 20, 5)$

and $z^* = 0$.

1) Locate the most negative entry in the bottom row (it is -2 in this case, there is a tie,

2) Form the ~~matri~~ ratios of the entries in the "b-column" with their corresponding positive entries in the entering column. The departing row corresponds to the smallest non-negative ratio $\frac{b_i}{a_{ij}}$.

Remark: If all entries in the entering column are 0 or negative, then there is no maximum solution.

The entry in the departing row and entering column is called the pivot.

3) If all entries in the bottom row are zero or positive, stop! This is the final tableau. If not, start from 1.

Now we make row operations to make pivot to become 1.

$R_3 \leftarrow \frac{1}{2} R_3$

We get $\Longrightarrow$

	x_1	x_2	x_3	s_1	s_2	s_3	b
s_1	2	1	0	1	0	0	10
s_2	1	2	$\boxed{-2}$	0	1	0	20
x_3	0	$\frac{1}{2}$	1	0	0	$\frac{1}{2}$	$\frac{5}{2}$
	-2	1	$\boxed{-2}$	0	0	0	0

We want to make $\boxed{-2}$ and $\boxed{-2}$ to become 0's.

$R_2 \longleftarrow 2 \cdot R_3 + R_2$

$R_4 \longleftarrow 2 \cdot R_3 + R_4$

We get

	X_1	X_2	X_3	S_1	S_2	S_3	b	
S_1	2	1	0	1	0	0	10	$\frac{10}{2}$
S_2	$\boxed{1}$	3	0	0	1	1	25	$\frac{25}{1}$
X_3	0	$\frac{1}{2}$	1	0	0	$\frac{1}{2}$	$\frac{5}{2}$	
	$\boxed{-2}$	2	0	0	0	1	5	

pivot

Entering

We make $\boxed{1}$ and $\boxed{-2}$ to become

0. First let us make pivot to become 1.

$R_1 \longleftarrow \frac{1}{2} R_1$

	X_1	X_2	X_3	S_1	S_2	S_3	b
X_1	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	5
S_2	$\boxed{1}$	3	0	0	1	1	25
X_3	0	$\frac{1}{2}$	1	0	0	$\frac{1}{2}$	$\frac{5}{2}$
	$\boxed{-2}$	2	0	0	0	1	5

Now let $R_2 \leftarrow R_2 + (-1) \cdot R_1$

$R_4 \leftarrow R_4 + 2 R_1$

We get.

	x_1	x_2	x_3	s_1	s_2	s_3	b
x_1	1	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	5
s_2	0	$\frac{5}{2}$	0	$-\frac{1}{2}$	1	1	20
x_3	0	$\frac{1}{2}$	1	0	0	$\frac{1}{2}$	$\frac{5}{2}$
	0	3	0	1	0	1	15

all other variables are zeros!

z^*, maximum value.

So the optimal solution is

$$(x_1, x_2, x_3, s_1, s_2, s_3) = (5, 0, \tfrac{5}{2}, 0, 20, 0).$$

And the max. value is $z^* = 15$.