1

Example:

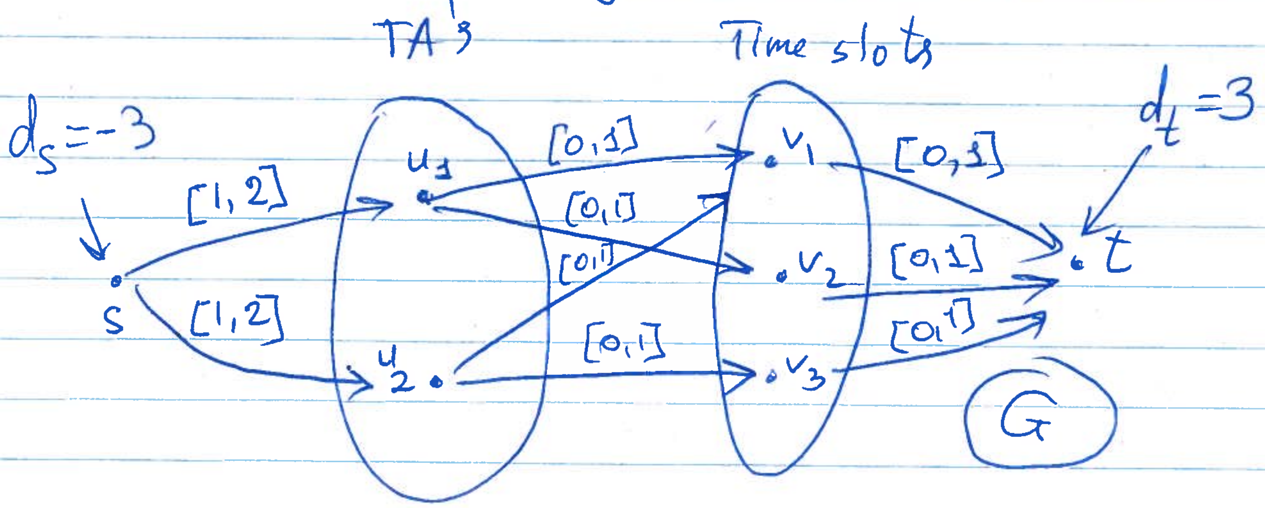
A special case of 7.28a.

The instructor wants three office hours per week. Suppose there are two TA's and three available time slots I_1, I_2 and I_3. Suppose first TA is available for time slots I_1 and I_2, and the second one is available for time slots I_1 and I_3. Suppose both must work at least one and at most two hours. We want to know whether there is a proper scheduling?

One can easily notice that, yes! Let first TA work for I_1 and I_2 and the second one to work for I_3.

Let us show how this could have been found if we used flow network (the one we setup during the class).

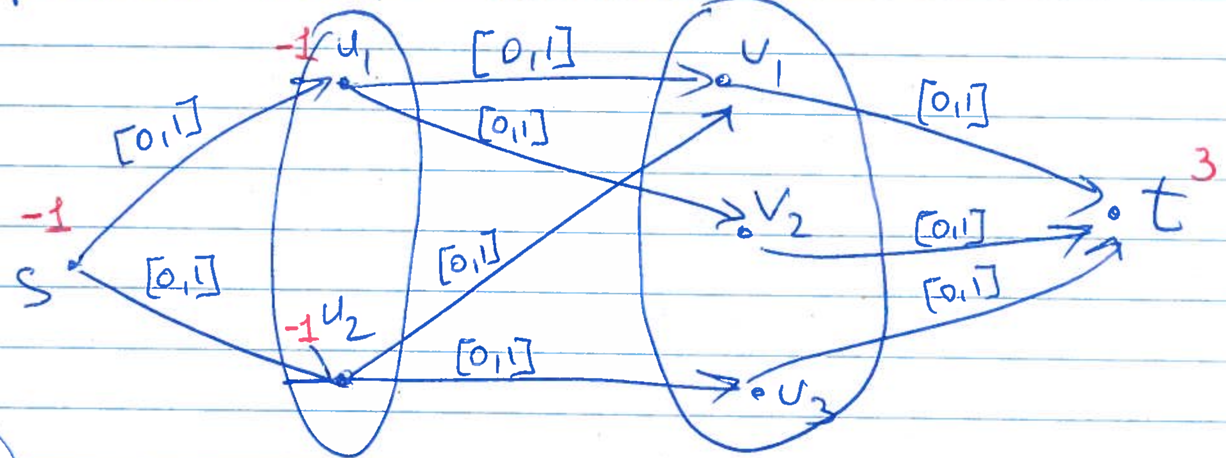
In our setting $a=1$, $b=2$, $c=3$.



2

Note* The capacities for edges (v_i, t) for $1 \le i \le 3$ must be $[0,1]$, since there cannot be two TA's working at the same time slot.

Now, let us first construct G' network flow without lower bounds. For that we set G' to be on the same set of vertices and edges, but we assign new capacities and demands.



We assign $c'_e := c_e - l_e \quad \forall e \in E$

We update demands as well:

$$d'_v := d_v - \left(\sum_{e \text{ into } v} l_e - \sum_{e \text{ out of } v} l_e\right)$$

$d'_s := -3 - (0 - 2) = -1$

$d'_{u_1} = d'_{u_2} = 0 - (1 - 0) = -1$

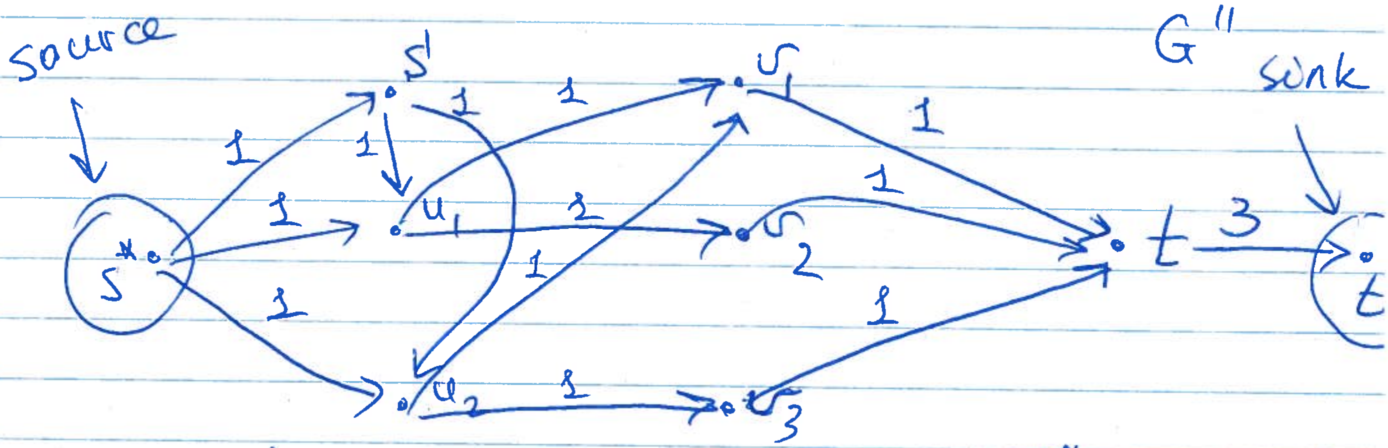
$d'_{v_1} = d'_{v_2} = d'_{v_3} = 0 - (0 - 0) = 0$

$d'_t = 3 - (0 - 0) = 3$

③

So in this new flow network G', there are no non-zero lower bounds, it has three sources — $\{s, u_1, u_2\}$ since $d(s) < 0, d(u_1) < 0, d(u_2) < 0$, G' has one sink, that is t.

Now from G' we construct a new flow network G'' which has no demands.



We attach new vertices s^* and t^* as specified

We assign

$$c''(s^*, s) := -d'_s = 1$$

$$c''(s^*, u_1) = -d'_{u_1} = 1$$

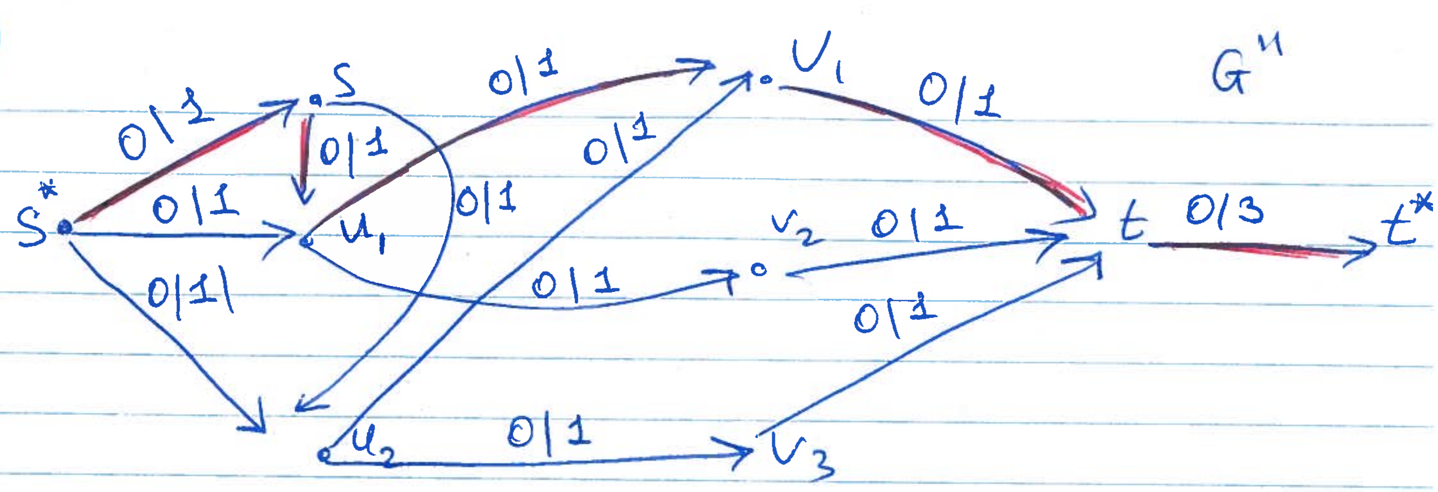
$$c''(s^*, u_2) = -d'_{u_2} = 1$$

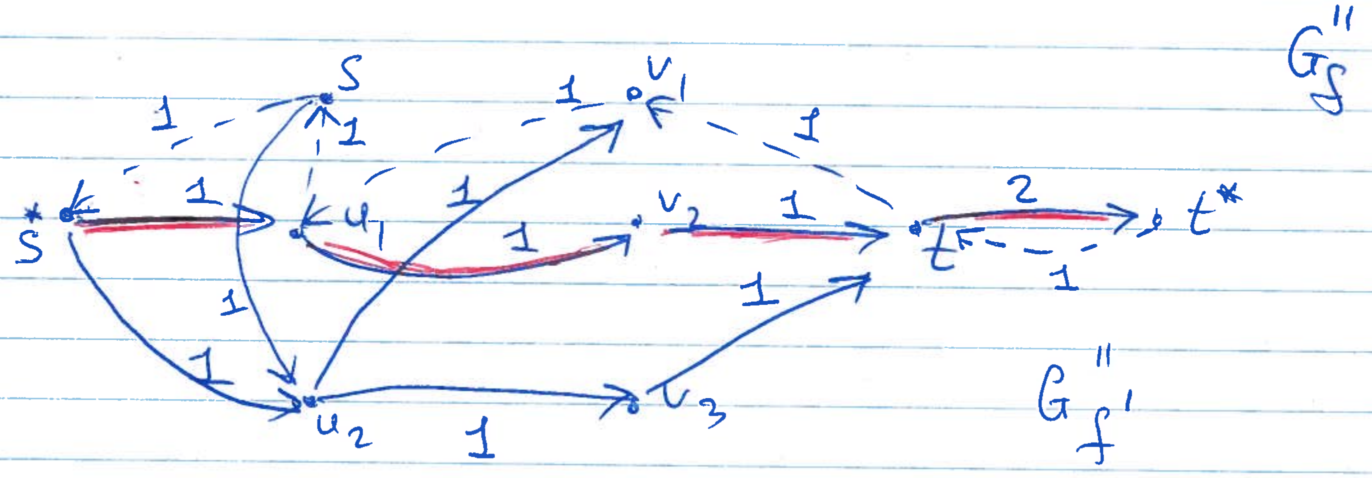
$$c''(t, t^*) = d'_t = 3$$

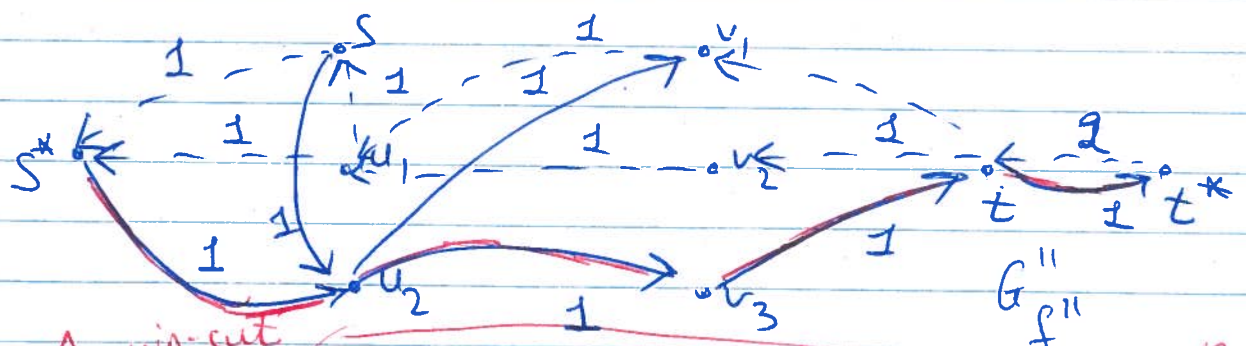
The rest stays the same as in G'.

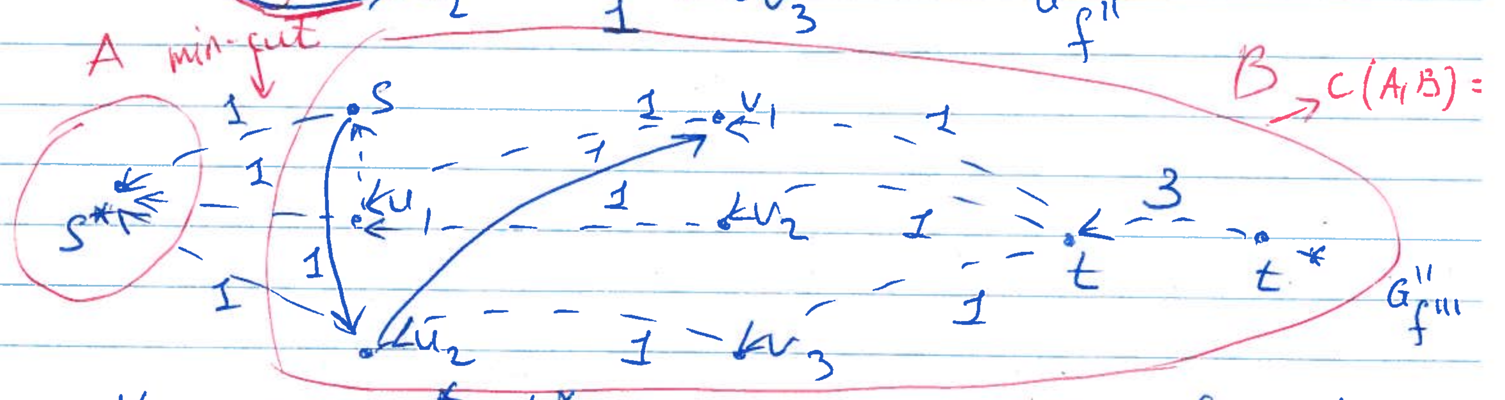
Now, we want to show that there exists a max-flow of value $v = 3$ in G''

4









No more s^*-t^* paths, hence we found the max flow and it has value $\underline{\nu = 3}$, what we were looking for. Note that