We have the following system:

$$\max \begin{cases} 3x_1 + x_2 \\ x_1 - x_2 \le -1 \\ -x_1 - x_2 \le -3 \\ 2x_1 - x_2 \le 2 \\ x_1, x_2 \ge 0 \end{cases}$$

We introduce x_0.

$$\begin{cases} x_1 - x_2 - x_0 \le -1 \\ -x_1 - x_2 - x_0 \le -3 \\ 2x_1 - x_2 - x_0 \le 2 \\ x_1, x_2, x_3 \ge 0 \end{cases}$$

$x_1 = 0$, $x_2 = 0$, $x_0 = 3$ is a feasible solution. We introduce x_3, x_4, x_5

$$\begin{cases} \max\ z' = -x_0 \\ x_3 = -1 - x_1 + x_2 + x_0 \\ x_4 = -3 + x_1 + x_2 + x_0 \\ x_5 = 2 - 2x_1 + x_2 + x_0 \end{cases}$$

A feasible solution is $x_3 = 2, x_4 = 0, x_5 = 5$

Pivot on x_0.

$$\begin{cases} z' = 3 - x_1 - x_2 - x_4 \\ x_3 = -1 - x_1 + x_2 + (3 - x_1 - x_2 + x_4) \\ x_0 = 3 - x_1 - x_2 + x_4 \\ x_5 = +2 - 2x_1 + x_2 + (3 - x_1 - x_2 + x_4) \end{cases}$$

$$\begin{cases} z' = 3 - x_1 - x_2 - x_4 \\ x_3 = 2 - 2x_1 + x_4 \\ x_0 = 3 - x_1 - x_2 + x_4 \\ x_5 = 5 - 3x_1 + x_4 \end{cases}$$

Now we pivot on x_1 and get.

$$\begin{cases} z' = -2 - \frac{x_4}{2} - \frac{x_3}{2} + x_2 \\ x_1 = 1 - \frac{x_3}{2} + \frac{x_4}{2} \\ x_0 = 2 + \frac{x_3}{2} + \frac{x_4}{2} - x_2 \\ x_5 = 2 + \frac{3x_3}{2} - \frac{x_4}{2} \end{cases}$$

Now we pivot on x_2.

$$\begin{cases} x_1 = 1 - \frac{x_3}{2} + \frac{x_4}{2} \\ x_2 = 2 + \frac{x_3}{2} + \frac{x_4}{2} - x_0 \\ x_5 = 2 + \frac{3x_3}{2} - \frac{x_4}{2} \end{cases}$$

$z' = -x_0$

A feasible solution is $x_1 = 1, x_2 = 2, x_3 = 0, x_4 = 0, x_5 = 2$

We can remove x_0.

$$x_1 = 1 - \frac{x_3}{2} + \frac{x_4}{2}$$
$$x_2 = 2 + \frac{x_3}{2} + \frac{x_4}{2}$$
$$x_5 = 2 + \frac{3x_3}{2} - \frac{x_4}{2}$$

$$\max\ z = 3x_1 + x_2$$

We are back to our initial optimization problem.

Pivot on x_4.

$$\begin{cases} \max\ z = 13 + 5x_3 - 4x_5 \\ x_1 = 3 + x_3 - x_5 \\ x_2 = 4 + 2x_3 - x_5 \\ x_4 = 4 + 3x_3 - 2x_5 \end{cases}$$

At this point we can conclude that this optimization problem has no finite solution. The reason is that we can take $x_3 = a$, $x_5 = 0$, $x_4 = 4 + 3a$, $x_2 = 4 + 2a$, $x_1 = 3 + a$, $z = 13 + 5a$, for any $a \geq 0$. Therefore if $a \to \infty$, then $z \to \infty$ as well.

Check the constraints.

$$x_1 - x_2 = -1 - a \leq -1 \quad \checkmark$$
$$-x_1 - x_2 = -7 - 3a \leq -3 \quad \checkmark$$
$$2x_1 - x_2 = 2 \leq 2 \quad \checkmark$$

And we are done!

Recall R. Bland's rule.

① For the entering variable, choose the smallest index j for which c_j is positive.

② If the ratio test gives a tie for leaving variable, choose the one with smallest subscript.

a) After fifth iteration, we choose X_1 instead of X_6

b)
$$\begin{cases} X_5 = 9X_6 + 4X_1 - 8X_2 - 2X_3 \\ X_4 = -X_6 - 0.5X_1 + 1.5X_2 + 0.5X_3 \\ X_7 = 1 - X_1 \\ \hline z = 24X_6 + 22X_1 - 93X_2 - 21X_3 \end{cases}$$

$$\begin{cases} X_5 = 9X_6 - 8X_2 - 2X_3 + 4\cdot(-2X_6 - 2X_4 + 3X_2 + X_3) \\ X_1 = -2X_6 - 2X_4 + 3X_2 + X_3 \\ X_7 = 1 + 2X_6 + 2X_4 - 3X_2 - X_3 \\ \hline z = 24X_6 + 22(-2X_6 - 2X_4 + 3X_2 + X_3) - 93X_2 - 21X_3 \end{cases}$$

$$\begin{cases} X_5 = X_6 + 4X_2 + 2X_3 \\ X_1 = -2X_6 - 2X_4 + 3X_2 + X_3 \\ X_7 = 1 + 2X_6 + 2X_4 - 3X_2 - X_3 \\ \hline z = -20X_6 - 44X_4 - 27X_2 + X_3 \end{cases}$$

Pivot on X_3.

$$
\begin{cases}
X_5 = 2 + 5X_6 - 2X_2 - 2X_7 \\
X_1 = 1 - X_7 \\
X_3 = 1 + 2X_6 + 2X_4 - 3X_2 - X_7 \\
\hline
z = 1 - 18X_6 - 42X_4 - 30X_2 - X_7
\end{cases}
$$

So we have the opt. solution - $X_2 = X_6 = X_4 = X_7 = 0$

$X_1 = 1$, $X_5 = 2$,

$X_3 = 1$!

$z^* = 1$.