Solutions to Assignment#2

Liana Yepremyan

1 4-SAT problem

Claim 1.1. *4-SAT is NP-complete.*

Proof. One way to show this is by reduction from 3-SAT. Let ϕ be a 3-SAT formula with variable set $X = \{x_1, \ldots, x_n\}$ and clause set $C = \{c_1, \ldots, c_m\}$. We transform ϕ into a 4-SAT instance ψ with variables $X = \{x_1, \ldots, x_n, x\}$, where x is a new variable. For each $c \in C$, where $c = \{l_1, l_2, l_3\}$ we define the following two clauses:

$$c_1 = \{l_1, l_2, l_3, x\},$$
$$c_2 = \{l_1, l_2, l_3, \neg x\}.$$

Let $C_i = \{c_i^j | j = 1, ..., m\}$, for $i = 1, 2$. We construct ψ with clause set $C = C_1 \cup C_2$. An easy claim follows.

Claim 1.2. *ϕ is satisfiable if and only if ψ is satisfiable.*

We observe that any assignment that satisfies ϕ also satisfies ψ, regardless of how we set the value of x. On the other hand, any assignment satisfying ψ can only satisfy one of the clauses only using the variable x, and therefore the same assignment restricted to variables $x_1, \ldots, x_n$ must also satisfy ϕ. $\square$