Modal Vibrations for Character Animation

Paul G. Kry

School of Computer Science,
McGill University,
Montreal, Canada
{kry}@cs.mcgill.ca
http://www.cs.mcgill.ca/~kry

Abstract. Modal vibrations can be used as a representation for the motion of an elastic system, decoupling the dynamics into a set of independent equations, and providing a good approximation to the system behavior for small displacements from the equilibrium state. In computer animation, elastic joints are commonly used in the simulation and control of articulated characters, which naturally permits a modal representation. This paper revisits the computation of modes for a skeletal character, and surveys recent work on the use of modal vibrations for kinematic animation of locomotion and jumping, and in the creation of physically based locomotion controllers that exhibit a desired style. Examples of other applications are also presented, and possibilities for future work are discussed.

Keywords: physically based animation, modal vibrations, locomotion

1 Introduction

Character animation is an important part in the creation of video games and animated movies. Artists can laboriously create key frames to design motion in a desired style that brings a virtual character to life. Alternatively, motion capture can be an attractive option. It can be much faster to record an actor and then replay this motion to produce a required animation. However, this becomes more difficult when the motion needs to be edited or adapted to a new situation. Likewise, capturing the motion of animals can have other difficulties. A third option for animating characters is to devise algorithms or procedures that can generate motion with minimal input. Examples include the kinematic procedures, or optimization techniques that produce motion given an objective function. In this paper we look at how modal vibrations can be used in the context of this third option to produce character animation.

Using modal vibrations is interesting in the context of creating kinematic animations because the resulting motion will have a natural appearance due to the physically based nature of the basis motions. Instead of animating individual joints, a modal vibration involves a coordinated motion of all joints. For instance, instead of animating the hip joints of a human, we can animate a mode that exhibits a leg swinging motion, which will also include nuanced motion in the rest of the body, such as a spinal twist and arm swings. In the context of a dynamic simulation, the modal vibrations have the nice property of being independent for small displacements from the rest pose, and the low frequency modes provide a useful reduced basis that can be used to approximate the motion, or likewise, reduce the search space in an optimization to create a physically valid controller.

This paper revisits the computation of modes for a skeletal character, and describes a number of applications for modal vibrations in character animation. Specifically, we show how to use modal vibrations of skeletal structure for animating locomotion using heuristics [10], and building on this work, how to create physically valid locomotion controllers with optimization, for a variety of characters of different morphologies and in a style that follows a modal sketch [19]. We also discuss other interesting potential applications for modal vibrations, such as inverse kinematics, interactive puppetry, and control of balance.

2 Related Work

One of the most important aspects of character animation is animating locomotion, and this is a problem that can benefit from the use of modal vibrations as we have demonstrated in previous work [10, 19]. More generally, however, modeling and animating locomotion has attracted a vast amount of attention for many years in computer graphics, in addition to being an important topic of research in robotics and biomechanics. Locomotion controllers have been proposed for the computer animation of a variety of virtual creatures, such as fish [4] and birds [29]. Hodgins proposed controllers for human athletics [7], and later studied how to adapt controllers to new characters [6]. This later work shows that it is challenging to tune a controller to a different morphology, even within the same species. Thus, there is interest in working toward automatic generation of animation of arbitrary virtual animals, for instance, the automatic kinematic animations for user-designed creatures in the game Spore [5].

Physically based character animation, for instance locomotion control, is complex because many degrees of freedom must be managed simultaneously. The large dimensionality of the search space is a significant problem in designing a controller. Typically optimization, such as maximization of traveled distance, is done off-line due to the large space of possible controllers. Sims used genetic programming to cope with the size of the search space, and evolved both the creature's structure and the controller simultaneously [23]. Van de Panne and Fiume [26] used optimization to tune the weights of simplified neural networks representing controllers.

The automation of controller generation can be simplified through *a priori* knowledge or example data to speed up or guide results. For instance, general momentum parametric templates inspired by biomechanics can be used to create complex dynamic motion [14]. This idea of motion-pattern templates is also illustrated in the work of Wu and Popovic [29] to represent the motor control of the beat of a bird wing. In addition, Wampler and Popovic [27] optimize cyclic motion to create a variety of running creatures, while including morphological parameters such as limb size and length in the optimization.

Many optimization based animation methods can be seen as example applications of *space-time constraints* [28, 17], a technique which is also known as *collocation* methods in engineering. However, in contrast to using the physical equations of motion as constraints, optimization can use forward simulation from specified initial conditions. This alternative, also known as *shooting*, is used by Nunes et al. [19]. This avoids the problem of dealing with non-physical sample trajectories in the search space and frees us from the tricky problem of specifying ground contact constraints within the optimization. The forward simulation is implicitly constrained to its dynamics, and standard contact formulations can be employed.

Generation of optimization based animation can be accelerated by restricting the problem to a reduced search space. Safonova et al. [22] exploit such low-dimensional spaces to create solutions with a specific behavior or style. Their reduced space comes from a principal component analysis of motion capture. Alternatively, Fang and Pollard [3] use a set of physics constraints based on aggregate quantities to simplify the optimization problem for efficient synthesis of motion through a set of sketched poses. Faloutsos et al. [2] use a reduced number of deformation modes, though user designed, to simplify the control problem for deformable models. The lower frequency modal vibrations a natural choice for a low-dimensional space, and are used by Nunes et al. [19] to simplify the optimization problem.

Much like other work on locomotion [15, 21], we note that a skeletal system stores energy during locomotion through the compression of elastic joints. While the idea that this is a key component of locomotion is well known in biomechanics [1], it is most commonly illustrated with simple models since these simple models are still a powerful tool for answering important questions about different gaits. An interesting contribution in the use of modal vibrations is that it reduces the complexity of a 3D model while preserving important characteristics and behaviour of the original model, such as mass distribution and spinal flexion. Finally, the model of locomotion as a superposition of natural vibrations is also related to work on coupled oscillators in neural models for controlling human locomotion [25]. The use of central pattern generators for modeling and controlling locomotion in robots and simulation has been and is still an active area or research; see the recent survey of Ijspeert [8].

3 Computing Modes of Skeletal Characters

Skeletal structure, deformation of tissues, and stiffness of muscles and tendons all play an important role in the dynamics of real animals. An appealing strategy for modelling the dynamics of a virtual animal is to simplify the biomechanical system as a set of rigid bones connected by compliant joints, where the stiffness of the joints approximates the combination of tendons, muscles, and the deformation of the surrounding tissues.

We must select the stiffness of each joint due to the muscles and tendons, the masses associated with each of the rigid links, and a rest pose where the elastic joints are in equilibrium. Given this information, we can compute a set of modal vibrations. In the following section we revisit the computation of modes for a skeletal character using the constrained full coordinate rigid body formulation described by Kry et al. [10], and we provide additional clarity and discussion. Furthermore, some issues and limitations will be described, such as the difficulty of using a single basis for larger motions, ground contact, and gravity.

3.1 Notation and Rigid Body Dynamics

We use a standard method for modelling and simulating a constrained system of rigid bodies (we follow a formulation similar to that of Murray et al. [16]). We place the local frame for body a at the body's center of mass and with axes aligned with the principle axes of inertia. The homogeneous coordinates of frame a with respect to frame b is given by a 3×3 rotation matrix Θ, and a 3×1 displacement p. We write the velocity of body a with respect to the world frame W as a column vector ${}^{a}\phi_{a-W} = (\omega^T, v^T)^T$, where ω is the angular velocity and v is the linear velocity. The leading superscript denotes that the vector is expressed in the coordinates of body a. The combination of a force and torque, called a wrench, is written ${}^{a}w = (\tau^T, f^T)^T$ where τ is the rotational torque and f is the translational force. Velocities transform according to the adjoint transformation ${}^{b}_{a}\mathsf{A}d$, while wrenches transform with the inverse transpose, that is, ${}^{b}w = {}^{a}_{b}\mathsf{A}d^{T}\,{}^{a}w$. Letting $[p]$ denote the skew symmetric matrix equivalent of the cross product $p\times$, the 6×6 adjoint matrix is defined by

$$ {}^{b}_{a}\mathsf{A}d = \begin{pmatrix} \Theta & 0 \\ [p]\Theta & \Theta \end{pmatrix}. $$

The state of the system is given by the position and orientation of each body, and the 6 dimensional body velocities. Writing the body velocities and accelerations in body coordinates, the motion of a set of rigid bodies is given by the Newton-Euler equation $M\dot{\phi} = w_{\phi} + w$. Here, M is a diagonal mass matrix assembled for all bodies, and we let ϕ and w (without leading superscripts) refer to the block column vectors containing velocities and wrenches of all bodies. The wrench w_{ϕ} is the Coriolis and centrifugal term due to the current body velocities. Finally, w consists of wrenches due to gravity, muscle stiffness, and damping.

3.2 Constraints

Because we are using a full coordinate rigid body formulation to write the equations of motion, we must use constraints to ensure that the bones in the skeleton of our animal models stay connected and can only rotate about certain axes. These constraints, are a function of the current state, but can actually be easier to write at the velocity level as we will see shortly, and are of the form we need to permit a modal analysis.

We define a frame j for each joint, located at the joint, and with the x axis purposely aligned with the free rotation for rotary joints. The velocity-level constraint for joint j connecting body a and body b has a very simple form when the relative velocity of a and b is written in the coordinates of the frame j. For a single axis rotary joint, it states that the velocity must be zero in all directions exept for rotation about the x axis,

$$ I_{2:6}\,{}^{j}\phi_{a-b} = 0, \tag{1} $$

where $I_{2:6}$ is the 5×6 matrix consisting of rows 2 through 6 (the bottom 5 rows) of the size 6 identity matrix and ${}^{j}\phi_{a-b} = {}^{j}_{a}\mathsf{A}d\,{}^{a}\phi_{a-W} - {}^{j}_{b}\mathsf{A}d\,{}^{b}\phi_{b-W}$. For a spherical joint we use $I_{4:6}$. Written in matrix form the constraints require $G\phi$ to be zero, where the sparse matrix G has 3 rows for every spherical joint, and 5 rows for every rotary joint.

3.3 Compliant Joints

The muscles and tendons of an animal produce torques at its joints, which we model linear springs and dampers. The torque at a compliant single axis joint, index j, with stiffness k_j is ${}^{j}\tau_{jx} = k_j(\theta_j - \theta_{j0})$, where θ_j is the current joint angle and θ_{j0} is the desired angle. The torque acting upon each of the connected bodies is equal but opposite: ${}^{a}w_j = {}^{j}_{a}Ad^T I_{1:1}^T\, {}^{j}\tau_{jx}$ and ${}^{b}w_j = -{}^{j}_{b}Ad^T I_{1:1}^T\, {}^{j}\tau_{jx}$. Here the 1×6 matrix $I_{1:1}$ is the first row of the identity matrix and corresponds with the degrees of freedom that were not constrained in Equation 1. In a similar manner to G, we construct a matrix R which allows the block vector of wrenches to be written in matrix form, $w = R^T K(\theta - \theta_0)$. Here K is the diagonal stiffness matrix containing the stiffness of each joint. Note that R also transforms the spatial velocities of the bodies to angular velocities at the joints, $\dot{\theta} = R\phi$. This formulation changes very little for a 3-axis spherical joint. Using the logarithmic map, the displacement from rest of spherical joint of index j can be written as 3 component vector ω_j, analogous to an angular velocity over a time interval. The 3 component torque can be written ${}^{j}\tau_j = k_j \omega_j$ (though stiffness can also differ for each axis).

We can view a small displacement from the rest pose as a twist ξ, a block vector containing the twist for each of the bodies. The twist can be interpreted as the displacement from rest by moving at a constant velocity ϕ over some small time step h, that is, $\xi = \phi h$. As such we can write the elastic forces as $w = R^T KR\xi$. Damping wrenches are also important in modal analysis, and can be computed as $w_c = R^T CR\phi$, where C is the diagonal matrix of damping coefficients.

3.4 Modal Analysis with Constraints

Given this physical model consisting of the equilibrium pose, the stiffness of the joints, and the mass of the links, we have sufficient information to compute the modal vibrations. The analysis shown here differs from what is typically done in graphics. We deal with constraints and rigid articulated systems connected by elastic joints, in contrast to the analysis of jelly-like objects, trees, and other elastic materials [20, 24].

The constrained equations of motion for the multibody system is given by

$$M\dot{\phi} = w_\phi + w + G^T \lambda. \tag{2}$$

where $G^T\lambda$ are constraint forces. The constraints also require the solution to satisfy $G\phi = 0$, or at the acceleration level, $\dot{G}\phi + G\dot{\phi} = 0$. Suppose that the compliant joints in our virtual animal's skeleton are in equilibrium. Given a small twist away from equilibrium, ϕh, the wrenches due to the compliant joints will be $R^T KR\phi h$. We can drop the damping forces form the analysis because the modal directions are unchanged provided we use Rayleigh damping. We also drop the w_ϕ term since it is quadratic in velocity. Replacing w with this spring force above gives $M\dot{\phi} = R^T KR\phi h + G^T\lambda$. When calculating the vibration modes, we must only consider the *kinematically admissible* directions of the constrained system. These degrees of freedom are described by the null space N of G, which we compute with a singular value decomposition. Thus, $\phi = N\dot{x}$, where $\dot{x}$ is the velocity in admissible coordinates. Replacing $\dot{x}h$ with x, and letting $\dot{\phi} = N\ddot{x}$ (since we are only looking at small vibrations we let $\dot{N} = 0$), we project into these coordinates to obtain $\mathsf{M}\ddot{x} = \mathsf{K}x$, with $\mathsf{M} = N^T MN$ and $\mathsf{K} = N^T R^T KRN$. The stiffness K

is be rank deficient because we can apply any rigid motion to the entire structure without producing a bend at any joint. Thus, K has 6 degrees of freedom in its null space corresponding to rigid motions. However, the mass matrix M is always invertible so we can easily solve for the eigenvalues and eigenvectors as either a generalized eigenvalue problem or via $\mathsf{M}^{-1}\mathsf{K}$. With rank deficient K, the eigenvectors include rigid motions with corresponding eigenvalues equal to zero.

The eigenvectors u_i of $\mathsf{M}^{-1}\mathsf{K}$ are the modal shapes, and can be seen as involving coordinated joint motions RNu_i about the equilibrium pose. Figure 1 depicts the first four non-rigid modes for a dog model, which can be described as bounding, back twisting, stretching, and alternating legs, respectively. The first 6 modes are rigid motions, and all but one of these next four modes are parallel to the sagittal plane of the dog and are relevant for locomotion. Mode u_7, involves a twist of the spine and may instead be useful for balance.

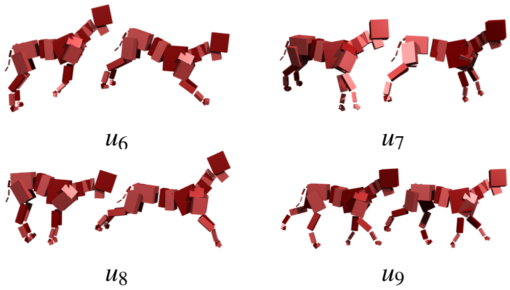

Fig. 1. The first four non rigid vibration modes of a dog model with 80 degrees of freedom.

The eigenvalues λ_i give the corresponding frequency, $f_i = \sqrt{\lambda_i}/2\pi$ Hz. In this basis, the behaviour of the system separates into a set of independent differential equations. The solution of the entire system is equivalent to the superposition of the solutions to these independent equations. Adding Rayleigh damping to the system (that is, a damping matrix equal to a combination of the mass and stiffness matrices) does not change the shapes of the modes, but does alter the frequencies of the resulting modes. See work of James and Pai [9] for a concise and intuitive formulation of damped modal frequencies in the presence of Rayleigh damping.

3.5 Discussion and Limitations

A joint coordinate formulation will lead to the same vibration shapes. In some implementations this will be simpler provided a dense reduced mass matrix is already available. The mass matrix will only depend on the equilibrium pose, and adjusting the stiffness of joints is easy with a diagonal matrix. However, our formulation with constraints has the advantage that we can easily add additional constraints in the same framework. For instance, we can easily model a virtual human pushing a cart, or holding an object in two hands; we do this by adding constraints to the hands (forming a closed loop). We can also add biological constraints, such as a constraint to fix the orientation of the head so that it remains stable during locomotion. There are likewise situations where we may want to set a constraint for a foot in contact with the ground. Adding these constraints has subtle yet important effects on the resulting mode shapes.

The masses of the mechanical links that make up the system have an important influence on the resulting modal vibrations. These values can be assigned based on those reported in the biomechanics literature. They can also be computed automatically from the geometry of the given model assuming a uniform density. The slightly more challenging problem is to set the stiffness of each joint. A reasonable set of values can be obtained by choosing a minimal stiffness that allows the virtual animal to maintain an upright natural rest posture in a simulation with normal gravity. Using lower stiffness values will result in a relaxed style in the modal vibrations; changing the stiffness and the equilibrium pose will alter the style. In previous work, stiffness has been identified as encapsulating style, and can be estimated from motion capture [13]. Stiffness has also been estimated by analysis of motion and force capture at the time of contact [11].

During locomotion, a character typically modulates the stiffness of different joints. However, the modal vibrations only capture the dynamic behavior of a system with fixed stiffness and only for small displacements about the equilibrium pose. Thus, the analysis above only serves as a coarse approximation of the true system, though it could be possible to generate a collection of linearized systems to model the behavior of the system under different stiffness values. Large displacements from the rest pose also present a problem for the modal vibration model. We can also consider using multiple linearized systems to model the dynamics at a collection of configurations in pose space. Making use of multiple linearized models for computer animation and control presents a variety of new problems that will be interesting to investigate in future work.

Ground contacts can only be included in the analysis as bilateral constraints. In our work on physically valid controllers, we try to account for ground contacts by computing modal vibrations for the character in the air, and with a foot constrained to the ground. We use the appropriate model depending on the state of the character. Joint limits are also unilateral constraints, and we have not taken these into account in our physically valid controllers. For kinematic animation, the joint limits can be applied by softly clamping joint angles. Finally, an important limitation with the analysis above is that it does not take into account bounding motions under gravity. To produce a physically valid motion, there is an important connection between the period of a bounding motion and the frequency of the modes used by the controller to produce this motion. It would be interesting to explore analytical solutions to this problem in future work.

4 Applications of Skeletal Modal Vibrations

In this section we briefly describe a number of the applications for modal vibrations, including kinematic animation of locomotion and jumping, the creation of physically valid controllers, controllers which respect an artist created modal sketch, inverse kinematics, interactive puppetry, and balance.

4.1 Kinematic Locomotion and Jumping

The work of Kry et al. [10] describes how to use modal vibrations to produce kinematic animations of locomotion and jumping. The observation is that sinusoidal mixes of low-frequency modes produce motions that resemble different locomotion gaits. Figure 2 outlines the overall approach. The amplitudes and phases of different gaits are identified from a collection of heuristics that look for knee bends and leg swings. These animations focus on the low-frequency modes since they require the least energy to produce and maintain large deformations of the structure. Furthermore, we observe that the

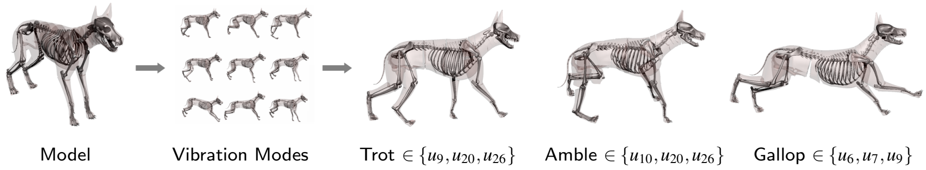

Fig. 2. Example kinematic gaits for a dog model. Small collections of modal vibrations can be combined to produce animations that resemble different gaits.

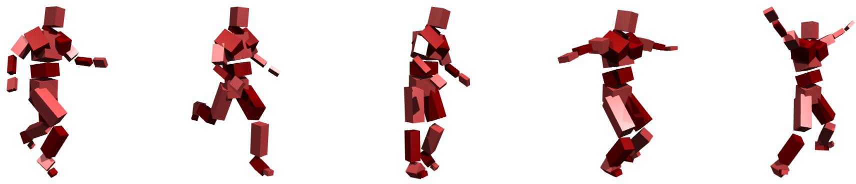

Fig. 3. Examples kinematic animations of cyclic human motion.

lower modal frequencies of the different virtual animals typically correspond well with the period of real gaits for those animals. Figure 3 shows additional examples of kinematic animations using a human character. These examples include, from left to right, running, running with a head orientation constraint, walking with hands constrained to push a cart, jumping with arms extended, and jumping jacks.

4.2 Inverse Kinematics

A potentially interesting application for modal vibrations is inverse kinematics (IK). In a traditional IK solver, a set of joint angles is found such that an end effector, such as a hand or foot, takes on a desired position. There are typically many solutions to a given IK problem, but a unique solution can be found by selecting the minimum norm joint displacement that satisfies the end effector position constraint. The norm may need to be weighted in order to produce a natural looking solution (for instance, to favor using an elbow over a wrist). An interesting alternative is to use the modal vibration basis to represent the joint displacements. The eigenvalues can be used as the natural weighting allowing low frequency modal displacements to be favored over higher frequencies. Furthermore, the modal vibration space can be truncated to force a solution that uses a limited number of degrees of freedom. The result of using modal vibrations in this way will produce IK solutions that consist of poses that are physically plausible with respect to dynamics. Instead of moving a single arm to achieve a reaching motion, a modal basis IK solution will involve a coordinated motion of all joints, and could naturally include a symmetric swing of the other arm in the opposite direction.

4.3 Optimized Physically Valid Control

Physically valid locomotion controllers based on modal vibration can be created by hand for very simple creatures, such as the worm in Figure 4. However, for more com-

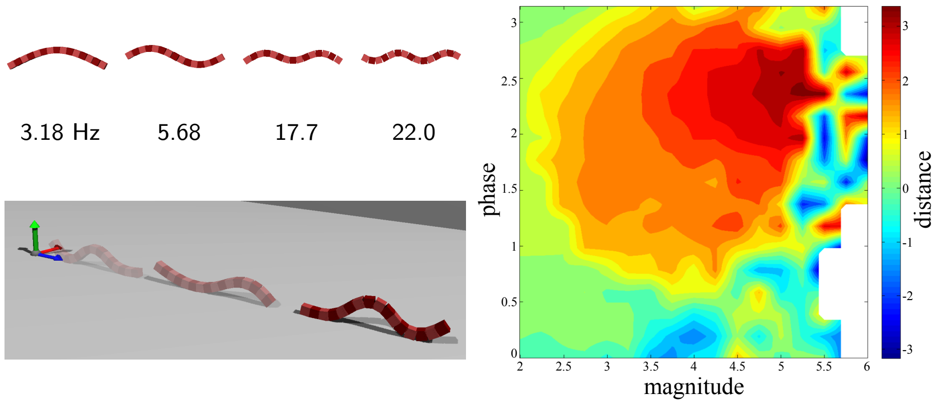

Fig. 4. At left, the modes of a simple worm creature with rigid links, and below a locomotion produced by activating the first two modes out of phase. At right, distances traveled during 5 seconds for different phase settings with different mode activation magnitudes.

plicated creatures, this quickly becomes intractable. Even in this simplest case, the landscape of optimal control parameters shown in the figure is not smooth with various local minima and maxima. Here, the objective is for the worm to maximize the distance traveled over 5 seconds, where the control parameters are a magnitude used to equally *activate* the first two modes using a sinusoidal displacement of the joint rest positions, and a phase difference between the two modes. Note that a $\pi/2$ phase difference is not optimal for this locomotion controller. The white areas on the right of the landscape are parameter settings where the simulation produces poor behaviour; in these energetic motions the behaviour is non-cyclic and involve large jumps. There are also lots of local maxima in these areas. To better pose the problem we can fix the magnitude as a constraint to limit the energy expenditure and then search for the optimal phase.

4.4 Interactive Control

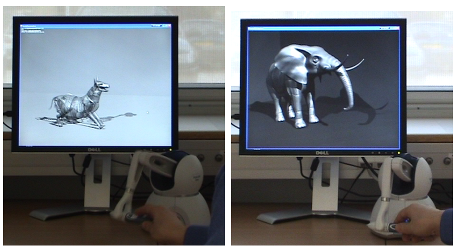

Fig. 5. Interactive control by mapping modal displacements to different axes of an input device.

One of the interesting benefits of modal vibrations is that by choosing the right degrees of freedom (vibration modes) the control problem is expressed in a remarkably small space. The control of these degrees of freedom can be given directly to a user to produce animation. The user effectively becomes a puppeteer, interactively controlling two or three degrees of freedom. This is similar in spirit to the interactive control of physically based systems proposed by Laszlo et al. [12]. Much like the creation of a kinematic animation, the first step involves selecting which modes the user will control. Subsequently, we let the user control the coefficients of these modes with the 2D movement of a mouse, or

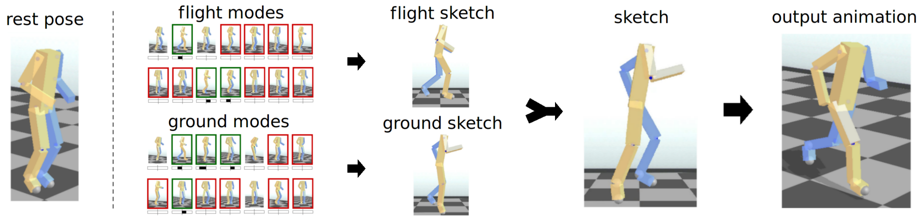

Fig. 6. In an artist guided controller creation, the animator sketches air and ground phases giving priority to certain modes (green), while marking others to avoid (red). Optimization transforms the sketch into a physically based locomotion controller.

the 3D motion of a stylus (see Figure 5). This defines the *desired* pose for a controller of the virtual animal in a physical simulation. Puppetry could also be purely kinematic, but using simulation allows interesting contact interactions and secondary dynamics.

4.5 Sketch Guided Locomotion Control

In nature, restorative rebound is a common strategy that appears in natural gaits [1]. This effect is produced by exploiting the musculoskeletal structure to reuse energy stored in opposing muscles, tendons and ligaments. Muscles and tendons can be thought to work like springs and dampers [18], and under this model the restorative energy at its core is derived from the passive return of a muscle "spring" under the influence of its passive stiffness. The modal vibrations can be seen as storing energy in muscles through changes in the overall shape of the skeletal structure. The Low energy vibrational modes of a specific structure efficiently use this effect, treating the body holistically as a restorative energy device. Motivated by the economy of the energy savings afforded by proper coordinations within the body, it is possible that animals exploit natural vibrations in the production of their dynamics motions. Under this assumption, the work of Nunes et al. [19] looks at modal vibrations as a useful basis from which to build physically valid locomotion controllers.

Figure 6 shows an overview of how a physically valid controller can be created. The modes are used as a palette of natural coordinations, and using an interface, the animator builds a "sketch" of a stylized motion from this palette. This directs the optimization, but also provides a starting point from which to search for successful control parameters. These control parameters consist of durations and amplitudes of modal impulses and their timing, the period of the cycle, and the initial conditions (position and velocity) of the character. The objective function consists of a measure of performance, specifically, energy expenditure divided by speed. The solver uses a shooting method, combined with a constraint to ensure that the motion is cyclic. This is solved with the combination of Covariance Matrix Adaptation, a gradient free sample based optimization technique that can deal with the many local minima in the optimization problem, and the method of multipliers, which allows the solution to satisfy the cyclic motion constraint. Finally, a collection of additional barrier constraints are used to guide the optimization to a desirable solution.

4.6 Control of Balance with Modal Displacements

As seen previously, some modes are naturally useful for creating kinematic animations of locomotion. Likewise, these same modes are those that an animator would use to create a sketch to guide an optimization in finding a physically valid controller, as described in the previous section. However, this optimization is only concerned with creating an efficient cyclic motion. Some of the other modal vibrations will be useful in a feedback controller to maintain balance. In preliminary experiments, we have seen that a standing balance controller can be created by using a simple linear feedback on center of mass position and velocity. We believe this approach can also be applied to locomotion, and that this will be an interesting topic to investigate in future work.

5 Conclusion

The method for computing mode shapes discussed in this paper is specially adapted to the case of skeletal animals; the bones are rigid while the joints are elastic, and we allow for the creation of additional constraints as desired or necessary. We observe that in contrast to many other techniques for character animation the techniques presented here do not use any reference motion data, either key-framed or captured. In the case of dynamic locomotion controllers, the final outcome is derived from only a single rest pose and the input mode-based sketch. We have also presented other possible applications for modal vibrations, including interactive puppetry, inverse kinematics, and balance control. We believe that with all the applications of modal vibrations in character animation, they will soon become common in animation software suites.

Acknowledgements

This work was supported by NSERC, and GRAND NCE Project MOTION.

References

1. Alexander, R.M.: Elastic Mechanisms in Animal Movement. Cambridge Univ. Press (1988)
2. Faloutsos, P., van de Panne, M., Terzopoulos, D.: Dynamic free-form deformations for animation synthesis. IEEE Trans. on Visualization and Computer Graphics **3**(3) (1997) 201–214
3. Fang, A.C., Pollard, N.S.: Efficient synthesis of physically valid human motion. ACM Transactions on Graphics **22**(3) (2003) 417–426
4. Grzeszczuk, R., Terzopoulos, D.: Automated learning of muscle-actuated locomotion through control abstraction. In: SIGGRAPH '95: Proceedings of the 22nd annual conference on Computer graphics and interactive techniques. (1995) 63–70
5. Hecker, C., Raabe, B., Enslow, R.W., DeWeese, J., Maynard, J., van Prooijen, K.: Real-time motion retargeting to highly varied user-created morphologies. ACM Transacitons on Graphics **27**(3) (2008) 27:1–27:11
6. Hodgins, J.K., Pollard, N.S.: Adapting simulated behaviors for new characters. In: SIGGRAPH '97: Proceedings of the 24th annual conference on Computer graphics and interactive techniques. (1997) 153–162

7. Hodgins, J.K., Wooten, W.L., Brogan, D.C., O'Brien, J.F.: Animating human athletics. In: SIGGRAPH '95: Proceedings of the 22nd annual conference on Computer graphics and interactive techniques. (1995) 71–78
8. Ijspeert, A.J.: Central pattern generators for locomotion control in animals and robots: a review. Neural Networks **21**(4) (2008) 642–653
9. James, D.L., Pai, D.K.: DyRT: dynamic response textures for real time deformation simulation with graphics hardware. ACM Transactions on Graphics **21** (July 2002) 582–585
10. Kry, P.G., Reveret, L., Faure, F., Cani, M.P.: Modal locomotion: Animating virtual characters with natural vibrations. Computer Graphics Forum **28**(2) (2009) 289–298
11. Kry, P.G., Pai, D.K.: Interaction capture and synthesis. ACM Transactions on Graphics **25**(3) (2006) 872–880
12. Laszlo, J., van de Panne, M., Fiume, E.: Interactive control for physically-based animation. In: SIGGRAPH '00: Proceedings of the 27th annual conference on Computer graphics and interactive techniques. (2000) 201–208
13. Liu, C.K., Hertzmann, A., Popovic, Z.: Learning physics-based motion style with nonlinear inverse optimization. ACM Transactions on Graphics **24**(3) (2005) 1071–1081
14. Liu, C.K., Popovic, Z.: Synthesis of complex dynamic character motion from simple animations. In: SIGGRAPH '02: Proceedings of the 29th annual conference on Computer graphics and interactive techniques. (2002) 408–416
15. McGeer, T., Alexander, R.: Passive bipedal running. Proceedings of the Royal Society of London, Series B, Biological Sciences **240**(1297) (1990) 107–134
16. Murray, R., Li, Z., Sastry, S.S.: A mathematical introduction to robotic manipulation. CRC Press (1994)
17. Ngo, J.T., Marks, J.: Spacetime constraints revisited. In: SIGGRAPH '93: Proceedings of the 20th annual conference on Computer graphics and interactive techniques. (1993) 343–350
18. Novacheck, T.F.: The biomechanics of running. Gait and Posture **7**(1) (1998) 77–95
19. Nunes, R.F., Cavalcante-Neto, J.B., Vidal, C.A., Kry, P.G., Zordan, V.B.: Using natural vibrations to guide control for locomotion. In: Proceedings of the ACM SIGGRAPH Symposium on Interactive 3D Graphics and Games. (2012) 87–94
20. Pentland, A., Williams, J.: Good vibrations: modal dynamics for graphics and animation. In: SIGGRAPH '89: Proceedings of the 16th annual conference on Computer graphics and interactive techniques. (1989) 215–222
21. Raibert, M.H., Hodgins, J.K.: Animation of dynamic legged locomotion. In: SIGGRAPH '91: Proceedings of the 18th annual conference on Computer graphics and interactive techniques. (1991) 349–358
22. Safonova, A., Hodgins, J.K., Pollard, N.S.: Synthesizing physically realistic human motion in low-dimensional, behavior-specific spaces. ACM Trans. Graph. **23**(3) (2004) 514–521
23. Sims, K.: Evolving virtual creatures. In: SIGGRAPH '94: Proceedings of the 21st annual conference on Computer graphics and interactive techniques. (1994) 15–22
24. Stam, J.: Stochastic dynamics: Simulating the effects of turbulence on flexible structures. Computer Graphics Forum **16**(3) (1997) C159–C164
25. Taga, G.: A model of the neuro-musculo-skeletal system for human locomotion. Biological Cybernetics **2**(73) (1995) 113–121
26. van de Panne, M., Fiume, E.: Sensor-actuator networks. In: SIGGRAPH '93: Proc. 20th annual conference on Computer graphics and interactive techniques. (1993) 335–342
27. Wampler, K., Popović, Z.: Optimal gait and form for animal locomotion. ACM Transactions on Graphics **28**(3) (2009) 60:1–60:8
28. Witkin, A., Kass, M.: Spacetime constraints. In: SIGGRAPH '88: Proceedings of the 15th annual conference on Computer graphics and interactive techniques. (1988) 159–168
29. Wu, J.c., Popović, Z.: Realistic modeling of bird flight animations. ACM Transactions on Graphics **22**(3) (2003) 888–895