COMP 102: Excursions in Computer Science

Lecture 9: Sorting Data

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Acknowledgement: Some of today's slides were taken from:
http://www.cs.rutgers.edu/~mlittman/courses/cs442-06/

Sock Matching

- We've got a basketful of mixed up pairs of socks.
- We want to pair them up reaching into the basket as few times as we can.

Sock Sorter A

- Strategy: Repeat until basket is empty
  - Grab a sock.
  - Grab another.
  - If they don't match, toss them back in the basket.
- Will this procedure ever work?
- Will it *always* work?

Measuring Performance

- Let's say we have 8 pairs of socks.
- How many times does this strategy reach into the basket?
  - Min?
  - Max?
  - Average?
- How do these values change with increasing numbers of pairs of socks?

Sock Sorter B

- Strategy: Repeat until basket is empty
  - Grab a sock.
  - Is its match already on the bed?
  - If yes, make a pair.
  - If no, put it on the bed.

Measuring Performance

- Once again, assume we have 8 pairs of socks.
- How many times does this strategy reach into the basket?
  - Min?
  - Max?
  - Average?
- How do these values grow with increasing numbers of pairs of socks?
- How does this compare with Sock Sorter A?

Comparing Algorithms

Repeat For Each Sock

sockA

- Do you have a matching pair? Set it aside.
- Do you have a non-matching pair? Put them back in the basket.

sockB

- Is there a match on the table? Pair them and set the pair aside.
- Otherwise, find an empty place on the table and set the sock down.

Notable if No Table

- Sock Sorter B seems like it is faster.
- One disadvantage of Sock Sorter B is that you must have a big empty space.
- What if you can only hold 2 socks at a time?

Sock Sorter C

- Strategy: Repeat until basket empty
  - Grab a sock.
  - Grab another.
  - Repeat until they match:
    - Toss second sock into the basket.
    - Grab a replacement.

Measuring Performance

- Once again, let's imagine we have 8 pairs of socks.
- How many times does this strategy reach into the basket?
  - Min?
  - Max?
  - Average?
- How do these values grow with increasing numbers of pairs of socks?

Comparing Algorithms

Round #2

sockA

- Do you have a matching pair? Set it aside.
- Do you have a non-matching pair? Put them both back in the basket.

sockC

- Do you have a matching pair? Set it aside.
- Do you have a non-matching pair? Put **one** back in the basket.

Analysis of Sock Sorter C

- Roughly the same number of matching operations as Sock Sorter A, but since it always holds one sock, roughly half the number of socks taken out of the basket.

Algorithms

- Sock Sorter A, Sock Sorter B and Sock Sorter C are three different algorithms for solving the problem of sock sorting.

- Different algorithms can be better or worse in different ways.
  - Number of operations
    E.g. total # of times reaching into basket, total # of comparisons.
  - Amount of memory
    E.g. # of socks on the bed (or in the hand) at any given time.

Lessons Learned

- Given notion of time (# instructions to execute) and space (amount of memory), we can compare different algorithms.

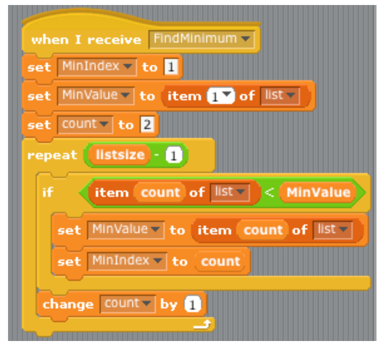
- It's important to use a good algorithm!

- It's especially important to think how time and space change, as a function of the size of the problem (i.e. # pairs of socks).

On the usefulness of sorting

- Recall last class's example about finding the minimum in an array.

- How many times is MinValue assigned?
  - Case 1: List is in increasing order.
    - Only once!
  - Case 2: List is in decreasing order.
    - MinValue gets assigned K times.
  - Case 3: List is in random order.
    - A bit harder to estimate…



If we are going to use the list **many** times, better to sort it first!

Sorting Lists

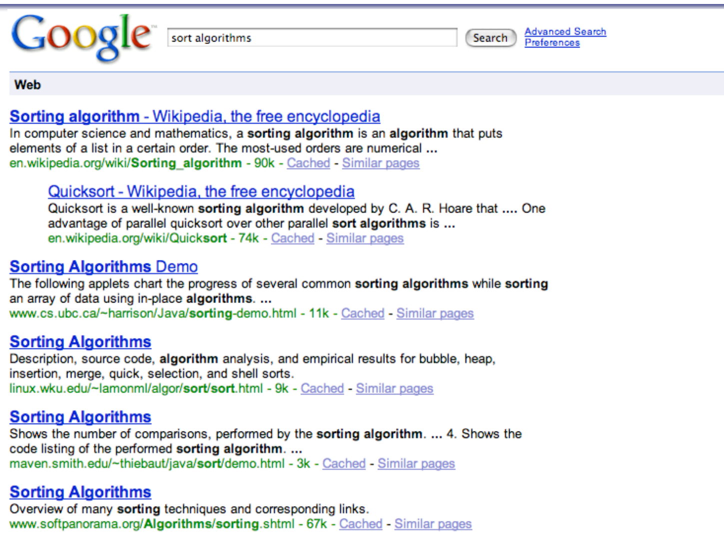
- Many problems of this type! This is an important topic in CS.
  - Sorting words in alphabetical order.
  - Ranking objects according to some numerical value (price, size, …)

Unsorted / Sorted

262, 201, 918, 301, 187, 762, 397, 277, 645, 306, 765, 798, 689, 867, 276, 402, 124, 545, 907, 569, 259, 152, 399, 481, 977, 947, 774, 727, 292, 285, 173, 599, 464, 212, 147, 696, 242, 559, 155, 569, 806, 784, 415, 321, 820, 126, 469, 225, 646, 438

124, 126, 147, 152, 155, 173, 187, 201, 212, 225, 242, 259, 262, 276, 277, 285, 292, 301, 306, 321, 397, 399, 402, 415, 438, 464, 469, 481, 545, 559, 569, 569, 599, 645, 646, 689, 696, 727, 762, 765, 774, 784, 798, 806, 820, 867, 907, 918, 947, 977

Sorting web pages



Sorting arrays

- Consider an array containing a list of names:

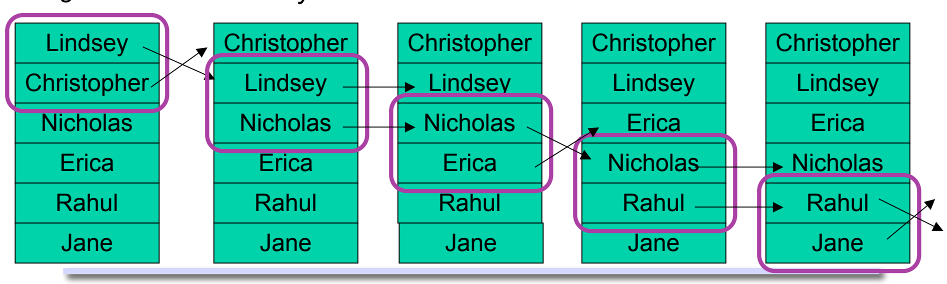
Lindsey
Christopher
Nicholas
Erica
Rahul
Jane

- How can we arrange them in alphabetical order?

A simple way to sort: Bubble sort

- Compare the first two values. If the second is larger, then swap.
- Continue with the 2nd and 3rd values, and so on.
- When you get to the end, start again.
- Repeat until no values are swapped.

Original list: *Partially sorted list:*



Let's think about Bubble sort

- Is this a good way to sort items?
  - Simple to implement. This is good!
  - Guaranteed to find a fully sorted list. This is good too!
- How do we decide whether it's a good method?

Useful things to consider

- How long will it take?
- How much memory will it take?
- Is there a way we can measure this?
- Best criteria:
  - number of basic machine operations: *move*, *read, write data*
  - amount of machine memory
- Can we think of something similar, at a higher level?

Predicting the "cost" of a sorting program

- Number of pairwise comparisons

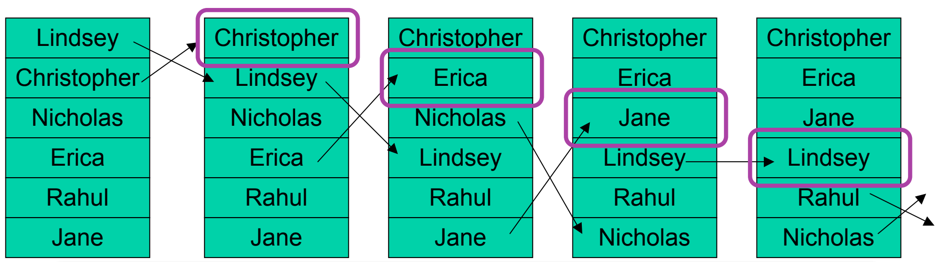
  For Bubble sort:
  - Let's say *n* is the number of items in the array.
  - Need *n-1* comparisons on every pass through the array.
  - Need *n* passes in total (at most).
  - So ***n*(n-1)* pairwise comparisons**.
- Amount of memory we need (in addition to the original array)

  For Bubble sort:
  - Everything happens within the original array.
  - Need to keep track of the index of the current item being compared.
  - Need to keep track, during each pass, of whether a swap was done.
  - So **only 1 integer and 1 bit of memory**.

A more intuitive sort method: Selection sort

- Scan the full array to find the first element, and put it into 1st position.
- Repeat for the 2nd position, the 3rd, and so on until array is sorted.

Original list: *Partially sorted list:*



What is the "cost" of Selection sort?

- Number of pairwise comparisons
  - Let's say *n* is the number of items in the array.
  - Need *n-1* comparisons on the 1st pass through the array.
  - Need *n-2* comparisons on the 2nd pass through the array.
  - And so on until we reach the last two elements.
  - So in total: *(n-1) + (n-2) + (n-3) + … + 1 =* ***n * (n-1) / 2* pairwise comparisons.**
  - This is better than Bubble sort. (But only by a factor of 2.)
- Amount of memory we need (in addition to the original array)
  - Everything happens within the original array.
  - Need to keep track of the index of the current item being compared.
  - Need to keep track, during each pass, of the index of the best value found so far.
  - So **only 2 integers** in memory. Roughly the same as Bubble sort.

Why do we care about the "cost"?

- Need to know whether we can use our program or not!
- Can we use Selection sort to alphabetically sort the words in the English Oxford dictionary?
  - About 615,000 entries in the 2nd edition (1989).
  - So we would need 189 trillion pairwise comparisons!
- What if we try to sort websites according to hostnames:
  - About 127.4 million active domain names (as of January 2011).
  - So we would need 8.06×10^{15} pairwise comparisons!
- Fortunately, not much "extra" memory is needed :-))

Let's find a better way: Merge sort

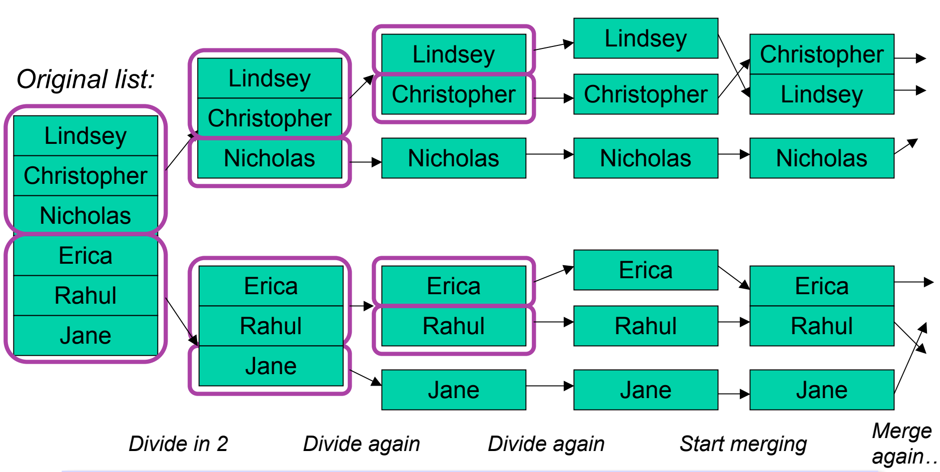
- Divide-and-Conquer! (This is our old friend "Recursion".)
- Main idea:
  1. Divide the problem into subproblems.
  2. Conquer the sub-problems by solving them recursively.
  3. Merge the solution of each subproblem into the solution of the original problem.
- What does this have to do with sorting?

Merge sort

- Example:
  – Sort an array of names to be in alphabetical order.

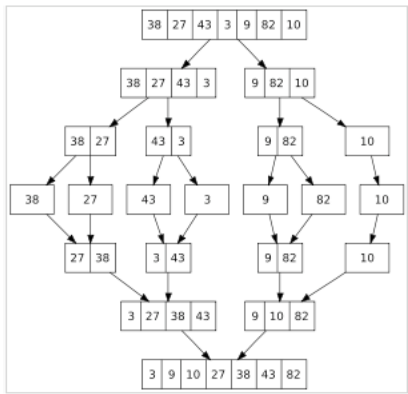
- Algorithm:
  1. Divide the array into left and right halves.
  2. Conquer each half by sorting them (recursively).
  3. Merge the sorted left and right halves into a fully sorted array.

Merge sort: An example



Another example of Merge sort

- Consider sorting an array of numbers:



Let's think about Merge sort

- Possibly harder to implement than Bubble sort or Selection sort.
- Number of pairwise comparisons:
  - How many times we divide into left/right sets? At most $log_2(n)$
  - How many items to sort once everything is fully split? None!
  - How many comparisons during merge, if subsets are sorted?
    - Need about *n* comparisons if sorted subsets have *n/2* items each.
  - So in total: *n* comparisons per level * $log_2(n)$ levels = $n * log_2(n)$
  - This is better than Bubble sort and Selection sort (by a lot).
- Amount of memory we need (in addition to the original array):
  - Every time we merge 2 lists, we need extra memory.
  - For the last merge, we need a full *n*-item array of extra memory.
  - This is worse than Bubble sort and Selection sort, but not a big deal.
  - We also need 2 integers (1 for each list) to keep track of where we are during merging.

Merge sort is a bargain!

- Using Merge sort to alphabetically sort the words in the English Oxford dictionary.
  - Recall: about 615,000 entries in the 2nd edition (1989).
  - So we would need 11.8 million pairwise comparisons.
  - Versus 1.89 trillion if using Selection sort!
- Using Merge sort to organize websites according to hostnames:
  - Recall: about 127.4 million active domain names (as of January 2011).
  - So we would need 3.4 billion pairwise comparisons.
  - Versus $8.06*10^{15}$ if using Selection sort!

Number of comparions

- Between Dec. 2007 and Jan. 2011 number of domains names grew from 62 millions to 127 millions.
  - Number of comparisons with Bubblesort grows from $3.4*10^{15}$ to $1.6*10^{16}$.
  - Number of comparisons with Mergesort grows from 1.6 to to 3.4 billion comparisons.

Quick recap on the number of operations

- Number of operations (y) as a function of the problem size (n)
  - Constant: $y = c$ *Best*
  - Linear: $y = n$
  - Log-linear: $y = n*log_2(n)$
  - Quadratic: $y = n^2$
  - Exponential: $y = 2^n$ *Worse*
- Bubble sort and Selection sort take a **quadratic** number of comparisons.
  - This is as bad as it gets, for sorting algorithms.
- Merge sort takes a **linear*log** number of comparisons.
  - This is as good as it gets, for sorting algorithms.
- This is a worst-case analysis (i.e. maximum number of operations.)

A word about memory

- Merge sort uses twice as much memory as Selection sort.
  - This is not a big deal. If you can store the array once, you can probably store it twice.
- But computers have 2 types of memory:
  - RAM (rapid-access memory) and hard-disk memory.
  - RAM is much faster, but usually there is less of it.
  - As long as everything fits into RAM, no problem!
- If array is too large for RAM, then you need to worry about:
  - Number of times sections of the array are copied / swapped to and from disk.

Take-home message

- Sorting is one of the most useful algorithms.
  - Applications are everywhere.
- There are many ways to solve a problem.
  - For sorting: Bubble sort, Selection sort, Merge sort, and many more.
  - Some methods use $n*log_2(n)$ comparisons and (almost) no extra memory!
- When choosing an algorithm to solve a problem, it's important to think about the **cost** (= time and memory) of this algorithm.
- It's also useful to think about how "easy" the algorithm is to program (more complicated = more possible mistakes), but this is harder to quantify.