COMP 102: Computers and Computing
Lecture 4: Finite State Machines and Memory

Instructor: Joelle Pineau (jpineau@cs.mcgill.ca)

Class web page: www.cs.mcgill.ca/~jpineau/comp102

What have we seen so far?

- Representing many types of data (text, numbers, images, sound) using binary representations.
- Solving problems using logical variables, logical expressions, truth tables, and logic gates.

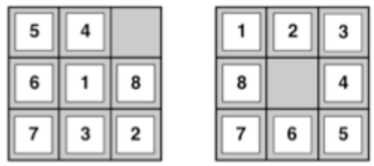
Any limitations to this?

- So far we have assumed that the state of each logical variable stays constant over time.

  E.g. Reset the computer between each round of Rock-Paper-Scissors.

Today: Representing many configurations with memory.

The eight-puzzle



Start state Goal state

- Can define a sequence of moves to go from start configuration to goal configuration.
- Need to be able to represent any configuration in memory.
- BUT! Need a way to describe how the configuration (a.k.a. the state) changes over time. What state-to-state transition is allowed in 1 move?

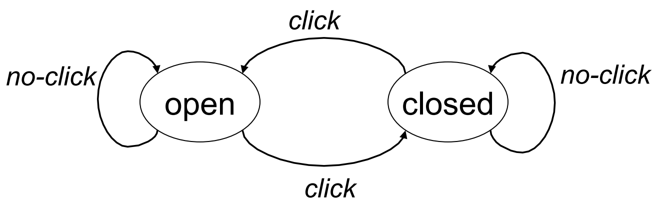
Other examples

- Retractable ballpoint pen
- Automatic door
- Traffic-light control
- Combination lock
- Elevator control system
- Language generation

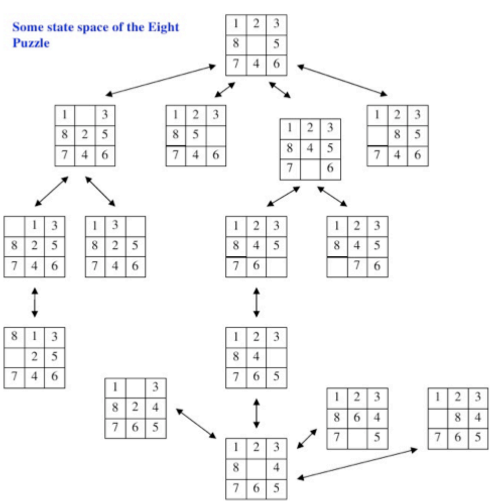
What do these things have in common?

- The state changes over time.
- Need memory to capture the state of the system at any point in time.

Ballpoint pen



Eight-puzzle



Finite State Machine

- The Finite State Machine combines a look-up table (constructed with binary logic) with a memory device (to store the state).

- Components of the finite state machine:
  - Set of states: $S = \{s_1, s_2, \ldots, s_n\}$
  - Set of observations: $O = \{o_1, o_2, \ldots, o_n\}$
  - A transition function: describing how the state changes in response to the observation seen.

Transition function

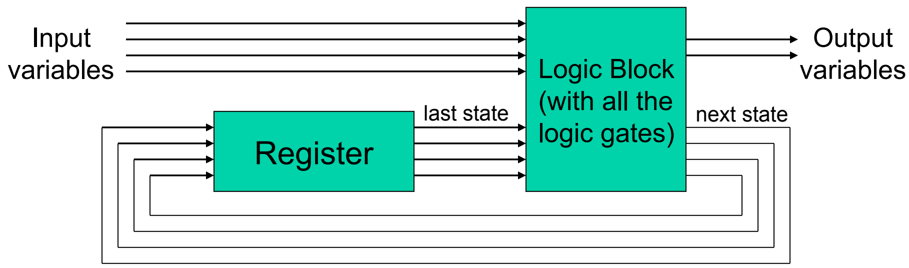
- For each state and observation, need to know what is the next state.
- Denote this as: $T(s,o) = s'$

  where s is the current state of the device

  o is the most recent observation

  s' is the next state of the device

Storing the state of a finite-state machine



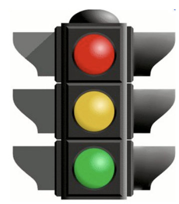
- Observations are set through inputs.
- Register is used to store bits. It has an additional timing input that tells it when to change state (*think of a clock that 'ticks' every second*).
- The logic block implements the transition function.

To implement a finite-state machine (FSM)

1. Select the set of states and the set of observations.
2. Choose a different pattern of bits for each state
   - In traffic light example: need 3 bits for state, 1 bit for observation.
3. Generate transition table or transition graph.
4. Implement transition table using logic gates.
   - Input = bits representing the observation and the last state.
   - Output = bits representing the next state.

Traffic-light controller

- **State** = 2-way car traffic light
  - S = {red, yellow, green}$_{\text{direction 1}}$ x {red, yellow, green}$_{\text{direction 2}}$
  - S = {red-red, red-yellow, yellow-red, red-green, green-red}
  - Not all configurations of lights are included (e.g. green-green) at least one light must be red.

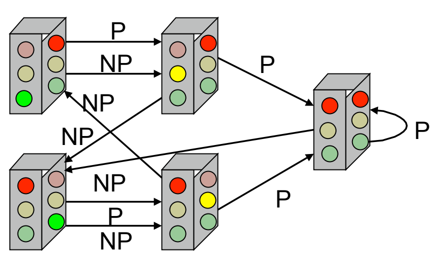


- **Observation** = pedestrian light request
  - O = {pressed, not pressed}
- **Memory** = Current state of the light in both directions.

Traffic-light controller: Transition function

- **State** = 2-way car traffic light
  - S = {red-red, red-yellow, yellow-red, red-green, green-red}
- **Observation** = pedestrian light request
  - O = {pressed, not pressed}
- **Transition function**: T(current state, observation) = next state

Here is one way to show the transition function as a graph.



Traffic-light controller: Transition table

State	Observation	Next state
Green-Red	Pressed	Yellow-Red
Green-Red	Not-Pressed	Yellow-Red
Yellow-Red	Pressed	Red-Red
Yellow-Red	Not-Pressed	Red-Green
Red-Green	Pressed	Red-Yellow
Red-Green	Not-Pressed	Red-Yellow
Red-Yellow	Pressed	Red-Red
Red-Yellow	Not-Pressed	Green-Red
Red-Red	Pressed	Red-Red
Red-Red	Not-Pressed	Red-Green

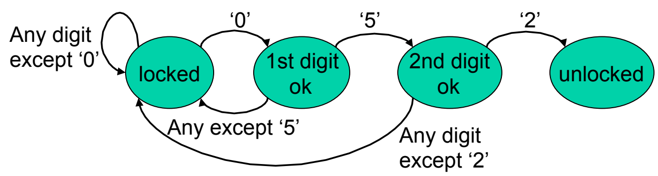
- This can be seen as just a standard truth table.
- Simply need to pick logical variables for the states and observations.
- Only subtlety: Need to use same set of variables for the "State" and "Next state" variables.
- Also need to store those variables between time steps.

Combination Lock

- **State** = Summary of the sequence of numbers.
- **Observation** = Number entered.
- **Memory** = Not <u>all</u> numbers ever dialed need to be stored, but need to remember enough about recent numbers to know if the sequence opens the lock.

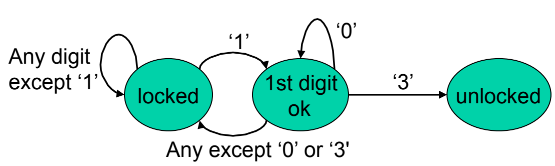
Recognizing sequences with a FSM

- Recall: Combination Lock
  - Assume the lock opens only when it sees sequence **0 - 5 - 2**.
- States: Number of digits in the sequence that have been recognized already = {0, 1, 2, 3, done}
- Observations: Digits that can be selected = {0, 1, …, 9}
- Transition function:



Recognizing sequences with patterns

- Consider a lock which recognizes sequences that start with "1", have any number of "0"'s, and end with "3".
- Transition function:



Which sequences can be recognized?

- Can recognize any pre-specified sequences of numbers or letters of a finite length.
  - E.g. Misspelled word within a stream of text.
- Cannot recognize all types of patterns.
  - E.g. Cannot build a finite-state machine that unlocks a lock whenever you enter any palindrome: 3-2-1-1-2-3
  - Why? Palindromes can be <u>of any length</u>, and to recognize the 2nd half, you need to <u>remember every character in the first half.</u> Because there are <u>infinitely many possible first halves</u>, this would <u>require a machine with an infinite number of states</u>.

Other tasks we cannot do with an FSM

- Problems with non-deterministic transitions, e.g. backgammon.
- Problems where we don't know the set of state/observations in advances.
- Problems where the transitions change over time.

Example: Behavior-based robotics

- Consider building a Light Seeking robot
  - States = {Seek-light, Follow-light, Avoid-obstacle}
  - Observations = {Light, NoLight, Obstacle, NoObstacle}
- Draw the transition graph.
- What other states/observations could we include?

Take-home message

- Finite-state machines let us reason about functions that change over time.
- Understand the components of finite state machines **(state, observation, transition**) and the implementation steps **(transition table, logic encoding through gates, use of registers**)
- Understand what tasks <u>can</u> and <u>cannot</u> be done with a FSM.