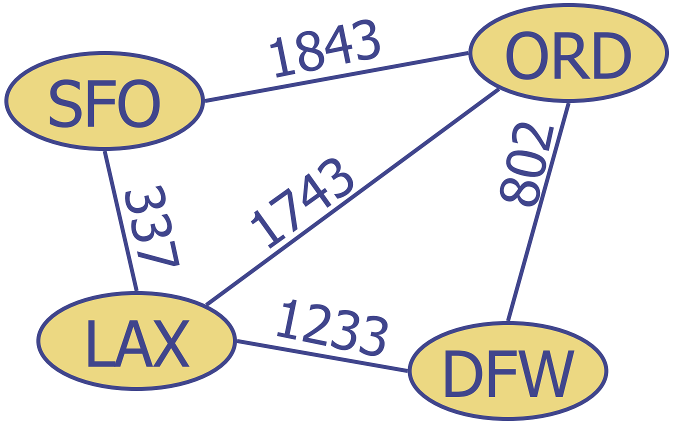
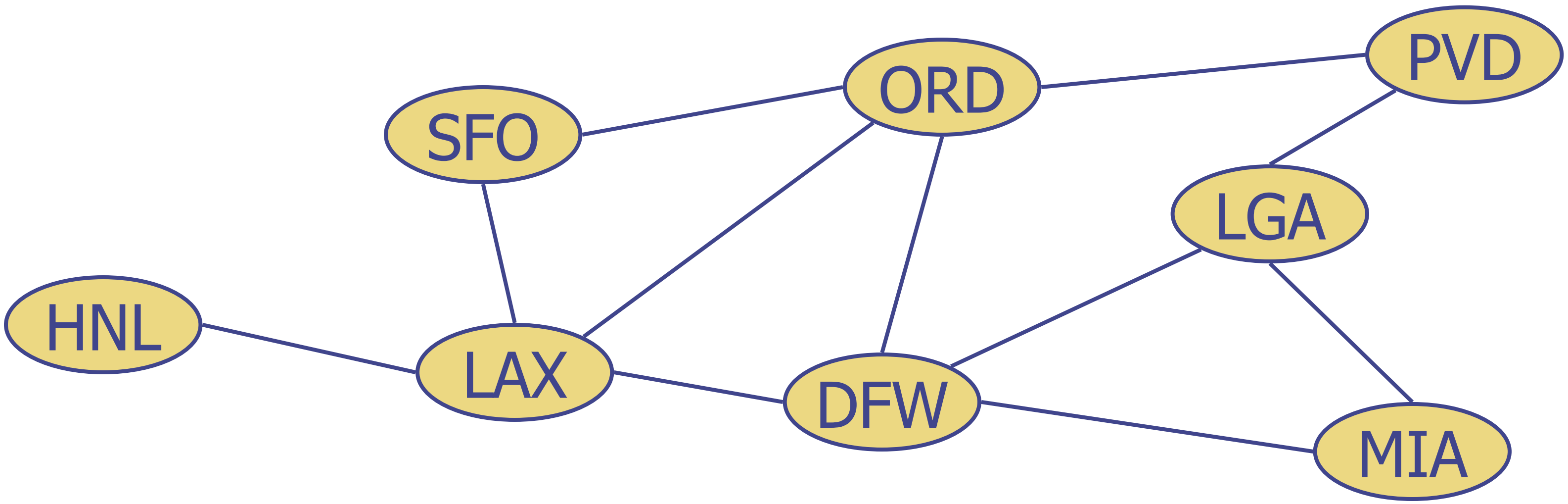
Graphs



Lecture notes adapted from Goodrich and Tomassia

Graph

- A graph is a pair (V, E), where
  - V is a set of nodes, called vertices
  - E is a collection of pairs of vertices, called edges
- Example:
  - A vertex represents an airport and stores the airport code
  - An edge represents a flight route between two airports



Edge Types

- Directed edge
  - ordered pair of vertices (u,v)
  - first vertex u is the origin
  - second vertex v is the destination
  - e.g., a flight

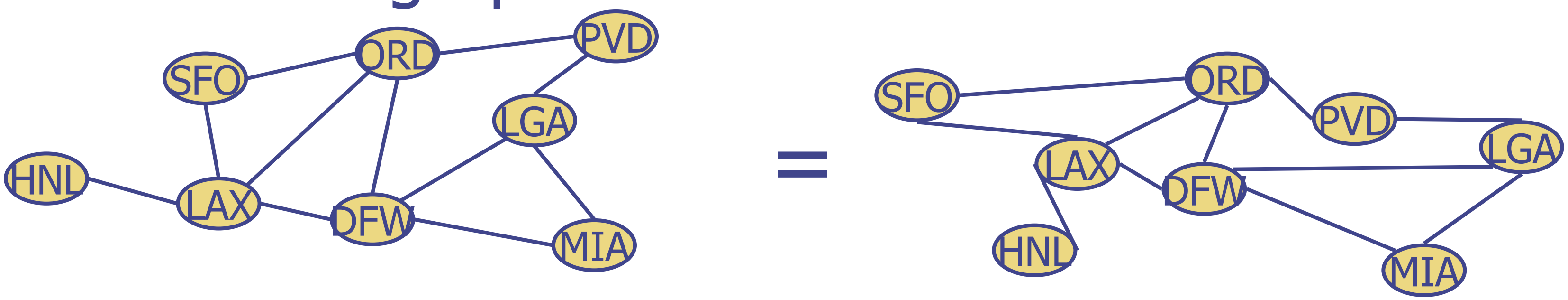
- Undirected edge
  - unordered pair of vertices (u,v)
  - e.g., a street

- Directed graph: all edges are directed
- Weighted edge: has a real number associated to it
  - e.g. distance between cities
  - e.g. bandwidth between internet routers

- Weighted graph: all edges have weights

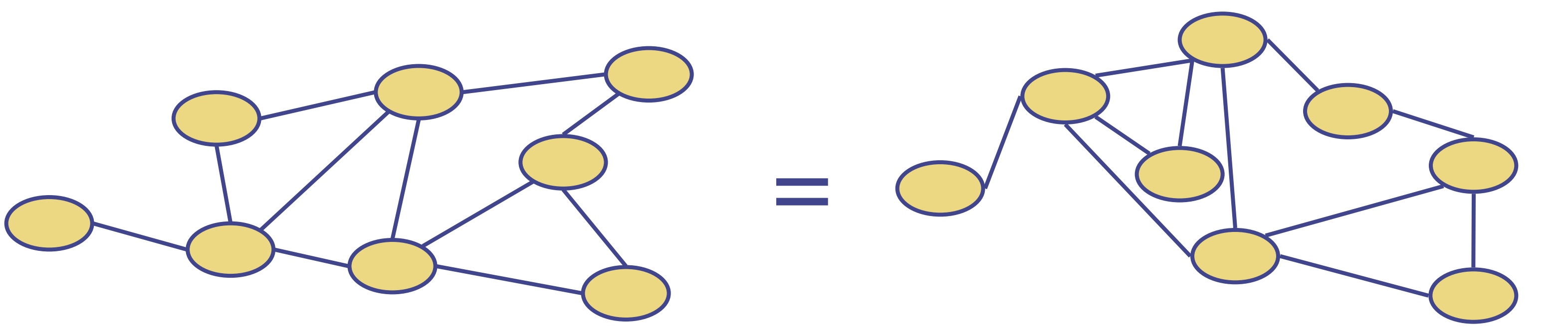
Labeled graphs

- Labeled graphs: vertices have identifiers



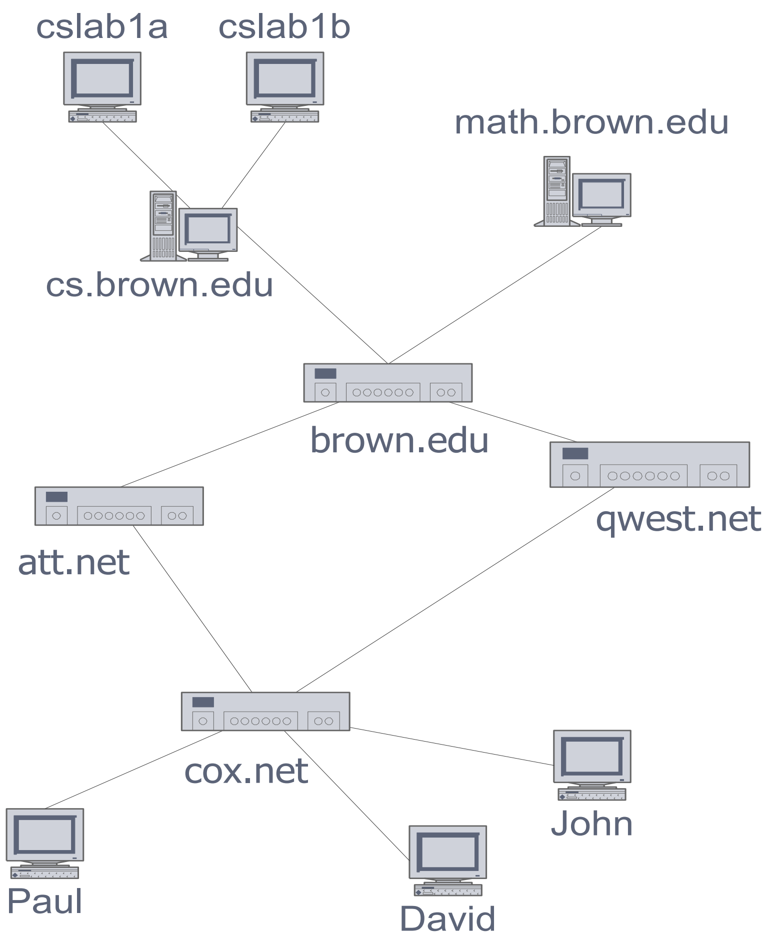
  - Note: Geometric layout doesn't matter - only connections matter

- Unlabeled graph: vertices have no identifiers



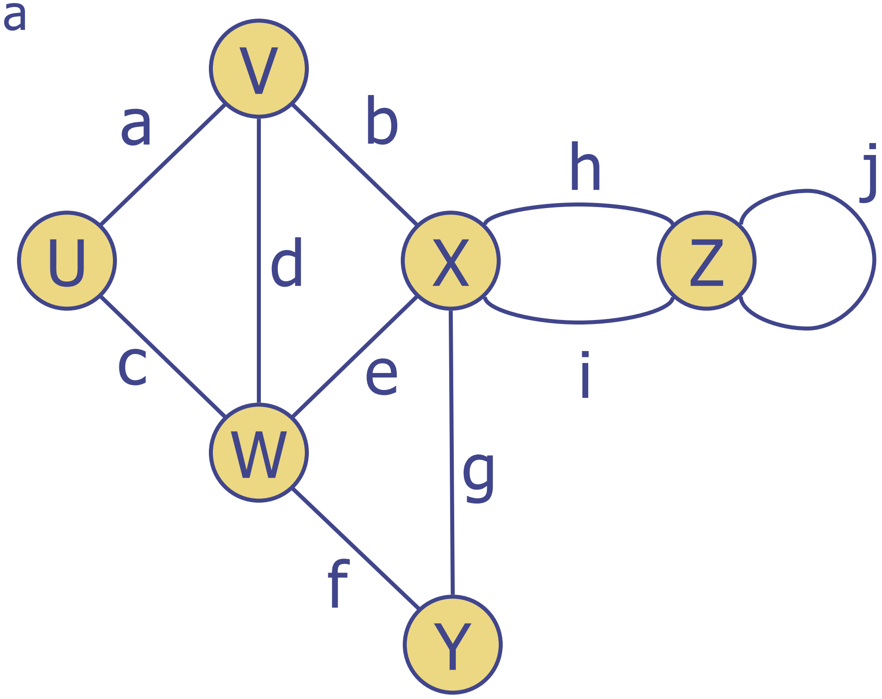
Applications

- Electronic circuits
  - Printed circuit board
  - Integrated circuit
- Transportation networks
  - Highway network
  - Flight network
- Computer networks
  - Local area network
  - Internet
  - Web
- Databases
  - Entity-relationship diagram



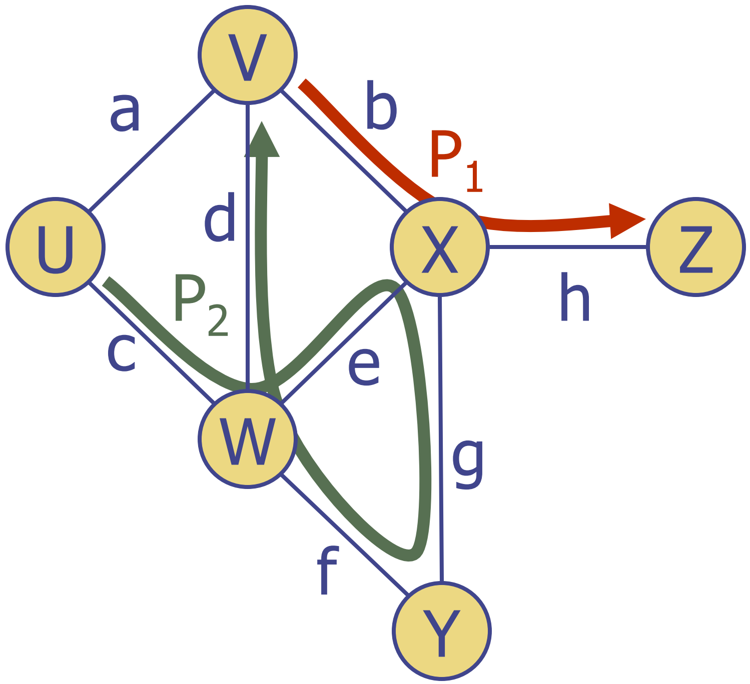
Terminology

- Endpoints of an edge
  - U and V are the endpoints of a
- Edges incident on a vertex
  - a, b, and d are incident on V
- Adjacent vertices
  - Connected by an edge
  - U and V are adjacent
- Degree of a vertex
  - Number of incident edges
  - X has degree 5
- Parallel edges
  - h and i are parallel edges
- Self-loop
  - j is a self-loop



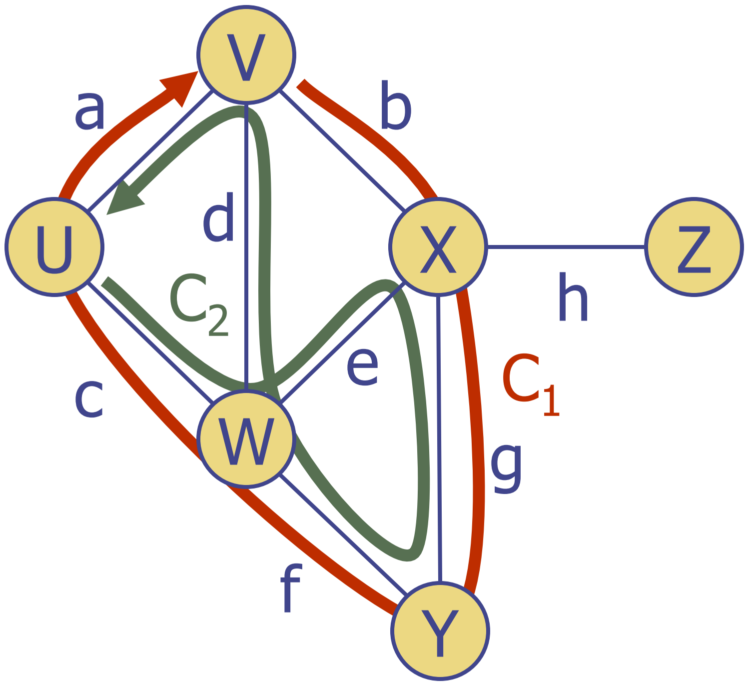
Terminology (cont.)

- Path
  - sequence of adjacent vertices
- Simple path
  - path such that all its vertices are distinct
- Examples
  - P_1=(V, X, Z) is a simple path
  - P_2=(U, W, X, Y, W, V) is a path that is not simple
- Graph is connected iff
  - For all pair of vertices u and v, there is a path between u and v



Terminology (cont.)

- Cycle
  - path that starts and ends at the same vertex
- Simple cycle
  - cycle where each vertex is distinct
- Examples
  - C_1=(V, X, Y, W, U, ↵) is a simple cycle
  - C_2=(U, W, X, Y, W, V, ↵) is a cycle that is not simple
- A tree is a connected acyclic graph



Properties

Property 1

$\Sigma_{v \in V} \deg(v) = 2|E|$

Why?

Notation

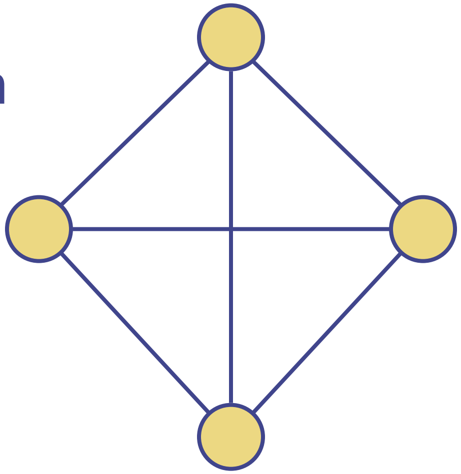
$|\mathbf{V}|$	number of vertices
$|\mathbf{E}|$	number of edges
$\deg(v)$	degree of vertex v

Property 2

In an undirected graph with no self-loops and no multiple edges

$|E| \leq |V| (|V| - 1)/2$

Why?

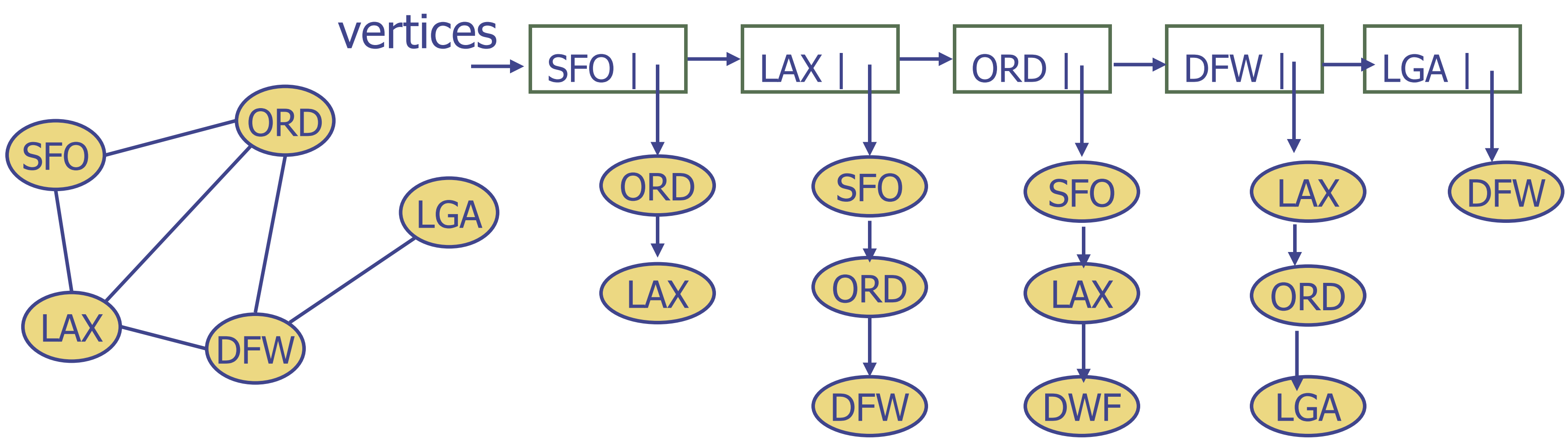


Example

- $|\mathbf{V}| = 4$
- $|\mathbf{E}| = 6$
- $\deg(v) = 3$

Data structure for graphs - Adjacency lists

- Graph can be stored as
  - A dictionary of pairs (key, info) where
  - key = vertex identifier
  - info contains a list (called adj) of adjacent vertices
- Example: if the dictionary is implemented as a linked-list

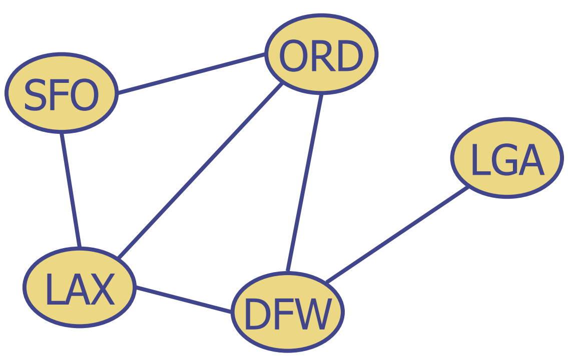


Adjacency lists - Operations

- addVertex(key k):
  vertices.insert(k, emptyList)
- addEdge(key k, key l):
  vertices.find(k).adj.insert(l)
  vertices.find(l).adj.insert(k)
- areAdjacent(key k, key l):
  return vertices.find(k).adj.find(l)

Data structure for graphs - Adjacency matrix

- Define some order on the vertices, for example:
  DFW, LAX, LGA, ORD, SFO
- Graph with n vertices is stored as
  - n x n array M of boolean, where
  - $M[i][j] = \begin{cases} 1 & \text{if there is an edge between i-th and j-th vertices} \\ 0 & \text{otherwise} \end{cases}$



	DFW	LAX	LGA	ORD	SFO
DFW	0	1	1	1	0
LAX	1	0	0	1	1
LGA	1	0	0	0	0
ORD	1	1	0	0	1
SFO	0	1	0	1	0

Adjacency matrix - Operations

- addEdge(i,j): matrix[i][j] = 1
- removeEdge(i,j): matrix[i][j] = 0
- Not very good for inserting/removing vertices: requires shifting elements of matrix.
- Requires space $O(n^2)$

Lists vs Matrices

- Adjacency lists are better if:
  - You frequently need to add/remove vertices
  - The graph has few edges
  - Need to traverse the graph
- Adjacency matrices are better if
  - you frequently need to
    - add/remove edges, but NOT vertices
    - Check for the presence/absence of an edge between i,j
  - matrix is small enough to fit in memory