# Sequence Modelling with Features: Linear-Chain Conditional Random Fields

COMP-599

Oct 6, 2015

# Announcement

A2 is out. Due Oct 20 at 1pm.

# Outline

Hidden Markov models: shortcomings

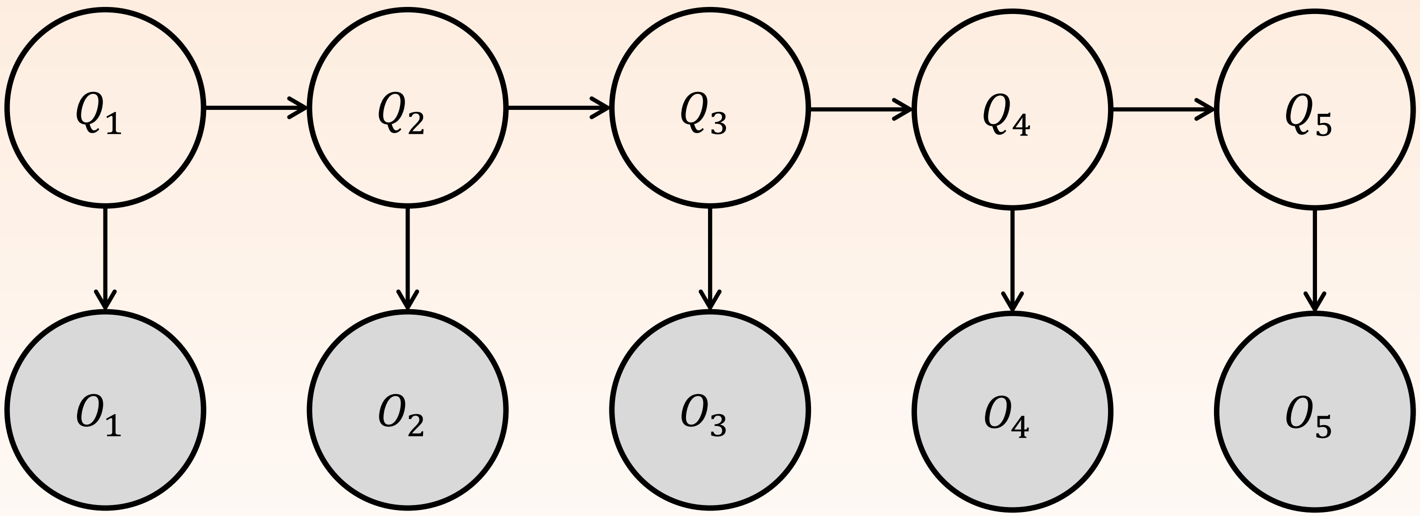
Generative vs. discriminative models

Linear-chain CRFs

- Inference and learning algorithms with linear-chain CRFs

# Hidden Markov Model

Graph specifies how join probability decomposes




$$P(\boldsymbol{O},\boldsymbol{Q}) = P(Q_1)\prod_{t=1}^{T-1} P(Q_{t+1}|Q_t)\prod_{t=1}^{T} P(O_t|Q_t)$$

Initial state probability

State transition probabilities

Emission probabilities

# Shortcomings of Standard HMMs

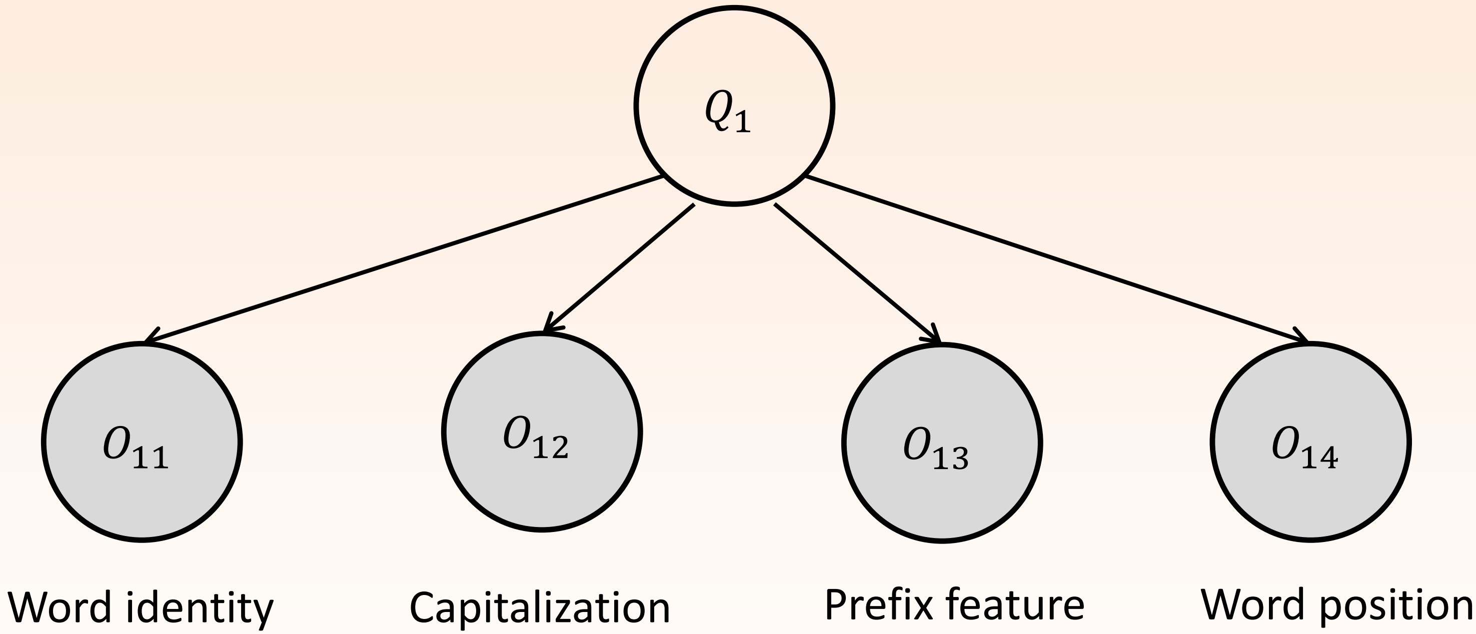
How do we add more features to HMMs?

Might be useful for POS tagging:

- Word position within sentence ($1^{st}$, $2^{nd}$, last…)
- Capitalization
- Word prefix and suffixes (-*tion*, -*ed*, -*ly*, -*er*, *re*-, *de*-)
- Features that depend on more than the current word or the previous words.

# Possible to Do with HMMs

Add more emissions at each timestep




Clunky

Is there a better way to do this?

# Discriminative Models

HMMs are **generative models**

- Build a model of the joint probability distribution $P(\boldsymbol{O}, \boldsymbol{Q})$,
- Let's rename the variables
- Generative models specify $P(X, Y; \theta^{\mathrm{gen}})$

If we are only interested in POS tagging, we can instead train a **discriminative model**

- Model specifies $P(Y|X; \theta^{\mathrm{disc}})$
- Now a task-specific model for sequence labelling; cannot use it for generating new samples of word and POS sequences

# Generative or Discriminative?

Naive Bayes

$$P(y|\vec{x}) = P(y) \prod_i P(x_i|y) \,/\, P(\vec{x})$$

Logistic regression

$$P(y|\vec{x}) \;= \frac{1}{Z} e^{a_1 x_1 \,+\, a_2 x_2 \,+\, \ldots \,+\, a_n x_n \,+\, b}$$

# Discriminative Sequence Model

The parallel to an HMM in the discriminative case: **linear-chain conditional random fields (linear-chain CRFs)** (Lafferty et al., 2001)

$$P(Y|X) = \frac{1}{Z(X)} \exp \sum_t \sum_k \theta_k f_k(y_t, y_{t-1}, x_t)$$

sum over all features

sum over all time-steps

Z(X) is a normalization constant:

$$Z(X) = \sum_y \exp \sum_t \sum_k \theta_k f_k(y_t, y_{t-1}, x_t)$$

sum over all possible sequences of hidden states

# Intuition

**Standard HMM**: product of probabilities; these probabilities are defined over the identity of the states and words

- Transition from state DT to NN: $P(y_{t+1} = NN | y_t = DT)$
- Emit word the from state DT: $P(x_t = the | y_t = DT)$

**Linear-chain CRF**: replace the products by numbers that are NOT probabilities, but linear combinations of weights and feature values.

# Features in CRFs

Standard HMM probabilities as CRF features:

- Transition from state DT to state NN
  $f_{DT\rightarrow NN}(y_t, y_{t-1}, x_t) = \mathbf{1}(y_{t-1} = DT)\,\mathbf{1}(y_t = NN)$
- Emit *the* from state DT
  $f_{DT\rightarrow the}(y_t, y_{t-1}, x_t) = \mathbf{1}(y_t = DT)\,\mathbf{1}(x_t = the)$
- Initial state is DT
  $f_{DT}(y_1, x_1) = \mathbf{1}(y_1 = DT)$

**Indicator function**:

Let $\mathbf{1}(condition) = \begin{cases} 1 & \text{if } condition \text{ is true} \\ 0 & \text{otherwise} \end{cases}$

# Features in CRFs

Additional features that may be useful

- Word is capitalized

  $f_{cap}(y_t, y_{t-1}, x_t) = \mathbf{1}(y_t = ?\,)\mathbf{1}(x_t \text{ is capitalized})$
- Word ends in –*ed*

  $f_{-ed}(y_t, y_{t-1}, x_t) = \mathbf{1}(y_t = ?\,)\mathbf{1}(x_t \text{ ends with } ed)$
- **Exercise**: propose more features

# Inference with LC-CRFs

Dynamic programming still works – modify the forward and the Viterbi algorithms to work with the weight-feature products.

| | HMM | LC-CRF |
|---|---|---|
| Forward algorithm | $P(X\|\theta)$ | $Z(X)$ |
| Viterbi algorithm | $\underset{Y}{\operatorname{argmax}} P(X, Y\|\theta)$ | $\underset{Y}{\operatorname{argmax}} P(Y\|X, \theta)$ |

# Forward Algorithm for HMMs

Create trellis $\alpha_i(t)$ for $i = 1 \ldots N, t = 1 \ldots T$

$\alpha_j(1) = \pi_j b_j(O_1)$ for j = 1 … N

for t = 2 … T:

for j = 1 … N:

$$\alpha_j(t) = \sum_{i=1}^{N} \alpha_i(t-1) a_{ij} b_j(O_t)$$

$$P(\boldsymbol{O}|\theta) = \sum_{j=1}^{N} \alpha_j(T)$$

Runtime: O($N^2 T$)

# Forward Algorithm for LC-CRFs

Create trellis $\alpha_i(t)$ for $i = 1 \ldots N, t = 1 \ldots T$

$\alpha_j(1) = \exp \sum_k \theta_k^{init} f_k^{init}(y_1 = j, x_1)$ for j = 1 … N

for t = 2 … T:

for j = 1 … N:

$$\alpha_j(t) = \sum_{i=1}^{N} \alpha_i(t-1) \exp \sum_k \theta_k f_k(y_t = j, y_{t-1}, x_t)$$

$$Z(X) = \sum_{j=1}^{N} \alpha_j(T)$$

Transition and emission probabilities replaced by exponent of weighted sums of features.

Runtime: O($N^2 T$)

Having $Z(X)$ allows us to compute $P(Y|X)$

# Viterbi Algorithm for HMMs

Create trellis $\delta_i(t)$ for $i = 1 \dots N, t = 1 \dots T$

$\delta_j(1) = \pi_j b_j(O_1)$ for j = 1 … N

for t = 2 … T:

  for j = 1 … N:

$$\delta_j(t) = \max_i \delta_i(t-1) a_{ij} b_j(O_t)$$

Take $\max_i \delta_i(T)$

Runtime: O($N^2T$)

# Viterbi Algorithm for LC-CRFs

Create trellis $\delta_i(t)$ for $i = 1 \ldots N, t = 1 \ldots T$

$\delta_j(1) = \exp \sum_k \theta_k^{init} f_k^{init}(y_1 = j, x_1)$ for j = 1 … N

for t = 2 … T:

for j = 1 … N:

$$\delta_j(t) = \max_i \delta_i(t-1) \exp \sum_k \theta_k f_k(y_t = j, y_{t-1}, x_t)$$

Take $\max_i \delta_i(T)$

Runtime: O($N^2 T$)

Remember that we need backpointers.

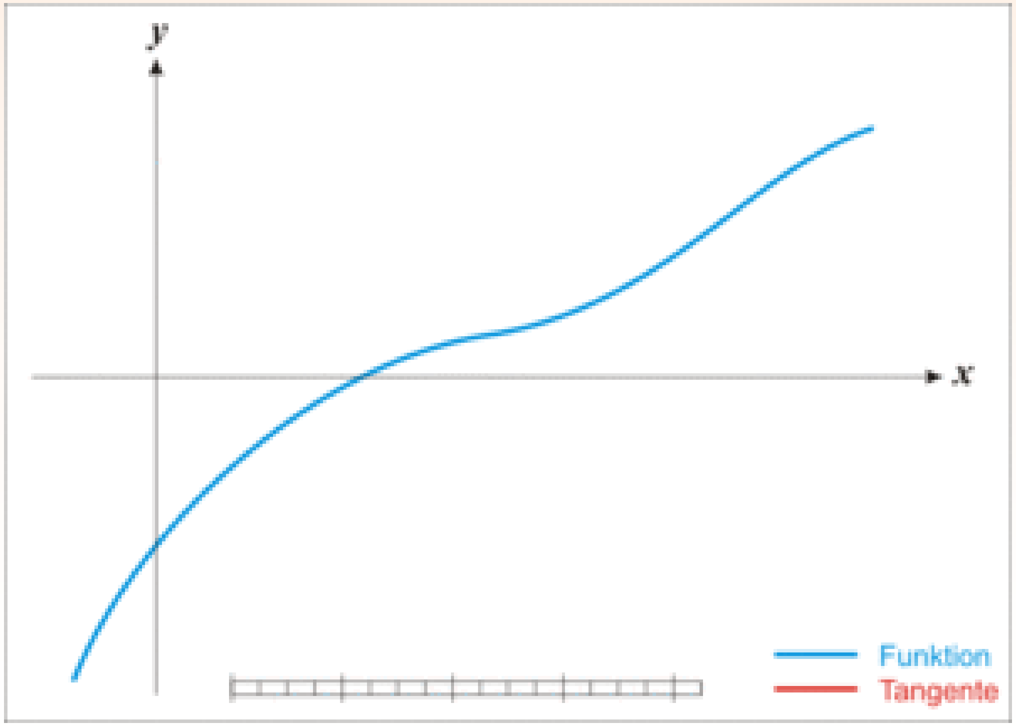
# Training LC-CRFs

Unlike for HMMs, no analytic MLE solution

Use iterative method to improve data likelihood

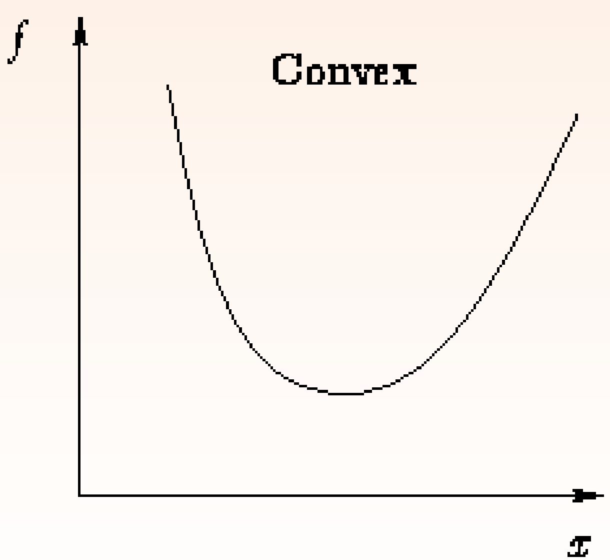
**Gradient descent**

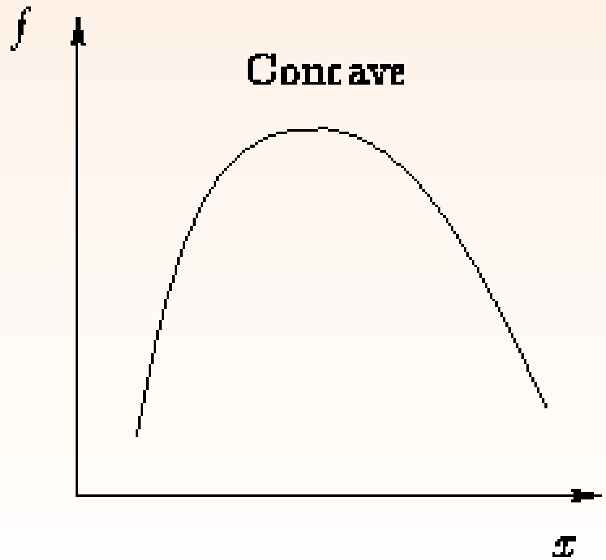
A version of Newton's method to find where the gradient is 0



# Convexity

Fortunately, $l(\theta)$ is a **concave** function (equivalently, its negation is a **convex** function). That means that we will find the global maximum of $l(\theta)$ with gradient descent.



# Gradient of Log-Likelihood

Find the gradient of the log likelihood of the training corpus:

$$l(\theta) = \log \prod_i P(Y^{(i)} | X^{(i)})$$

# Regularization

To avoid overfitting, we can encourage the weights to be close to zero.

Add term to corpus log likelihood:

$$l^*(\theta) = \log \prod_i P(Y^{(i)} | X^{(i)}) \; - \exp \sum_k \frac{\theta_k^2}{2\sigma^2}$$

$\sigma$ controls the amount of **regularization**

Results in extra term in gradient:

$$-\frac{\theta_k}{\sigma^2}$$

# Gradient Ascent

Walk in the direction of the gradient to maximize $l(\theta)$

- a.k.a., gradient descent on a loss function

$$\theta^{\text{new}} = \theta^{\text{old}} + \gamma \nabla l(\theta)$$

$\gamma$ is a learning rate that specifies how large a step to take.

There are more sophisticated ways to do this update:

- Conjugate gradient
- L-BFGS (approximates using second derivative)

# Gradient Descent Summary

Initialize $\theta = \{\theta_1, \theta_2, \dots, \theta_k\}$ randomly

Do for a while:

Compute $\nabla l(\theta)$, which will require dynamic programming (i.e., forward algorithm)

$\theta \leftarrow \theta + \gamma \nabla l(\theta)$

# Interpretation of Gradient

Overall gradient is the difference between:

$$\sum_i \sum_t f_k(y_t^{(i)}, y_{t-1}^{(i)}, x_t^{(i)})$$

the empirical distribution of feature $f_k$ in the training corpus

and:

$$\sum_i \sum_t \sum_{y,y'} f_k\left(y, y', x_t^{(i)}\right) P(y, y' | X^{(i)})$$

the expected distribution of $f_k$ as predicted by the current model

# Interpretation of Gradient

When the corpus likelihood is maximized, the gradient is zero, so the difference is zero.

Intuitively, this means that finding MLE by gradient descent is equivalent to telling our model to predict the features in such a way that they are found in the same distribution as in the gold standard.

# Stochastic Gradient Descent

In the standard version of the algorithm, the gradient is computed over the entire training corpus.

- Weight update only once per iteration through training corpus.

**Alternative**: calculate gradient over a small mini-batch of the training corpus and update weights then

- Many weight updates per iteration through training corpus
- Usually results in much faster convergence to final solution, without loss in performance

# Stochastic Gradient Descent

Initialize $\theta = \{\theta_1, \theta_2, \dots, \theta_k\}$ randomly

Do for a while:

Randomize order of samples in training corpus

For each mini-batch in the training corpus:

Compute $\nabla^* l(\theta)$ over this mini-batch

$\theta \leftarrow \theta + \gamma \nabla^* l(\theta)$