AN OBSERVER-BASED APPROACH TO MODELING ERROR IDENTIFICATION

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Abstract

Information about the location of modeling errors is crucial for the efficient improvement of an invalid model. This article discusses how to pinpoint modeling errors through comparison of experimental data with data obtained through simulation of the invalid model. An observer-based approach is developed. By designing a dedicated observer for the system using the invalid model, a signal vector is generated, on which each modeling error imposes an easily identifiable feature. An algorithm to analyze the featured signal is then developed. With this algorithm, the features of each of the modeling errors are extracted. To illustrate and verify the developed approach, a simulation study of a biological model is performed and promising results are obtained.
1 INTRODUCTION

The use of modeling and simulation has proven to be invaluable in the design and analysis of complex systems. As depicted in Figure 1 (see also [1]), a typical modeling process starts by identifying an Experimental Frame. The frame, as introduced by Zeigler [2][3], represents the experimental conditions under which the modeller wants to investigate the system. As such, it reflects the modeller’s goals and questions. Based on the identified frame, a class of matching models can be identified. Through structure characterization, the appropriate model structure is selected based on the \textit{a priori} knowledge and/or the measurement data. Subsequently, parameter estimation yields optimal parameter values for reproducing the measurement data. Using the identified model and parameters, simulation allows one to mimic the system behavior (virtual experimentation).

The question remains however whether the model has predictive validity: is it capable not only of reproducing data which was used to choose the model and parameters but also of predicting new behavior? The predictive validity of a model is usually substantiated by comparing experimental data sets to those produced by simulation, an activity known as model validation. Due to its special importance in the communication between model builders and users, model validation has received considerable attention in the past few decades (for surveys, see for example [4]-[7]). Problems from general validation methodologies [8][9] to concrete testing technologies [10]-[14] have been extensively studied. The comparison of the experimental and simulation data are accomplished either subjectively, such as through graphical comparison [8], Turing test [15], or statistically, such as through analysis of the mean and variance of the residual signal employing the standard $F$ statistics [8], Hotelling’s $T^2$ tests [16], multivariate analysis of variance [17], regression analysis [18], spectral analysis [19], autoregressive analysis [20], autocorrelation function testing [21], error analysis [22][14] and some non-parametric methods. An excellent presentation of the different issues as well as a classification of verification, validation, and testing techniques is given by Balci in [6].

As indicated by the feedback arrows in Figure 1, a model has to be corrected once proven invalid. The above mentioned methods are designed to determine, through comparison of measured and simulated data, the validity of a model. The aim of the current paper is to add to the validation process, a step to systematically provide modeling error information. As such, the presented technique is not part of the validation process, but rather complements it. As one might intuitively expect, different modeling errors usually cause the behavior of the model to deviate in different ways from that of the real system. Or, in other words, different modeling errors correspond to different “patterns” in the error signal, the difference between experimental data and simulated data. These “patterns”, if extract-able, can obviously be used to identify the modeling errors. In the context of fault diagnosis, a technique of detecting and identifying faults by means of designing fault diagnosis observers has been developed (for surveys, see for instance [23][24]). In this paper, the concept is generalized to the identification of modeling errors. By specially designing an observer of the system with the invalid model, a signal vector is generated, on which each modeling error imposes an easily identifiable feature. An algorithm to analyze the featured signal is then developed, with which the features of the modeling errors are extracted. As a result of this specific approach, once a model is found to contain significant errors, the type as well as the location of the errors can be determined.

The paper is organized as follows. In order to better present the ideas, a biological process and its modeling are discussed in Section 2, which serves as an example throughout the paper for both theoretical discussions and simulation studies. In Section 3, the different types of modeling errors which may occur and their unified representation are discussed. It is shown, by analyzing the biological process, that each modeling error can be characterized by a feature matrix. The latter forms a basis for the identification algorithm. In Section 4, the identification of the modeling errors for the case where all the state variables of the system are measurable is addressed. Firstly, a feature equation is derived which characterizes the relations between the modeling errors and an error vector produced by an observer that is specially designed for the system.
based on the invalid model. Then, an algorithm to extract the features that are contained in the error vector is designed. In Section 5, identification of modeling errors in the case of incomplete state measurement is studied. In Section 6, simulation studies are reported to illustrate and verify the ideas presented.

2 MODELING A BIOLOGICAL PROCESS

Figure 2 shows a biological denitrification plant, which aims to remove the nitrate as well as the carbon organics contained in the influent water by means of biological reactions. It consists of two functional units, a bio-reactor and a settler. In the reactor, which is often completely mixed, heterotrophic biomass is present. It biodegrades the carbon organics with nitrate as the electron acceptor. The carbon organics and the nitrate are thus both removed. The ‘overflow’ of the reactor, containing the substrate residuals and the sludge flocks (where the biomass resides), flows into the settler. There, the sludge settles and thus separates itself from the treated water, and is subsequently recycled to the reactor through the recycling line. In order to prevent the sludge concentration in the reactor from becoming too high due to its continuous growth in the reactor, surplus sludge is removed from the waste flow (see Figure 2). Models of the denitrification process usually aim to predict the effluent quality (the amount of carbon organics and nitrate in the effluent) and the sludge production. This implies that the following three variables are crucial to the model: the carbon organics concentration, the nitrate concentration, and the biomass concentration. The main biological reaction occurring in the reactor is known to be,

\[ S + NO_3^- + H^+ \xrightarrow{r} X + N_2 + CO_2 + H_2O \]

where \( S, NO_3^-, H^+, X, N_2, CO_2 \) and \( H_2O \) denote, respectively, the carbon organics, nitrate, proton, biomass, nitrogen gas, carbon dioxide gas and water. \( r \) denotes the reaction rate. The “feedback” arrow in the scheme expresses the auto-catalytic action of the biomass \( X \). As clearly shown in the scheme, the reaction results in the removal of the nitrate and carbon organics and in the growth of the biomass. Another reactor process is the decay of the biomass which causes the decrease of the biomass on the one hand and the consumption of the nitrate on the other hand. In the context of modeling the effluent quality, the \textit{a priori} knowledge allows one to model the process by making mass balances for the three materials,

\[
\begin{align*}
\dot{X}(t) &= \mu(t)X(t) - bX(t) - \frac{Q_\text{in}(t)}{V}X(t) \\
\dot{S}_S(t) &= -\frac{1}{Y_S} \mu(t)X(t) - \frac{Q_\text{in}(t)}{V}S_S(t) + \frac{Q_\text{in}(t)}{V}S_{S,\text{in}}(t) \\
\dot{S}_\text{NO}(t) &= -\frac{1}{2.86Y_S} \mu(t)X(t) - \frac{1 - f_P}{2.86}bX(t) - \frac{Q_\text{in}(t)}{V}S_{\text{NO}}(t) + \frac{Q_\text{in}(t)}{V}S_{\text{NO},\text{in}}(t)
\end{align*}
\]

where \( X, S_S, S_{\text{NO}} \) denote the biomass, the carbon organics and the nitrate concentrations in the bioreactor, respectively; \( S_{S,\text{in}} \) and \( S_{\text{NO},\text{in}} \) denote the carbon organics and the nitrate concentrations in the influent, respectively; \( Q_\text{in} \) is the influent flow rate; \( Q_\text{w} \) is the waste flow rate; \( V \) is the volume of the bioreactor; \( Y_S \) is the yield coefficient; \( b \) is the biomass decay coefficient; \( f_P \) is the fraction of the inert materials in biomass; \( \mu(t) = r(t)/X(t) \) is the specific biomass growth rate, which is still to be modeled.

Experiments show that \( \mu \) is a nonlinear function of \( S_S \) and \( S_{\text{NO}} \). It has been revealed that \( \mu \) increases almost linearly with \( S_S \) and \( S_{\text{NO}} \) when they are low, but becomes independent of them when they are high. Several empirical laws have been proposed to model this relationship. The following double Monod law is commonly used [25],

\[
\mu(t) = \mu_{\text{max}} \frac{S_S(t)}{K_S + S_S(t)} \frac{S_{\text{NO}}(t)}{K_{\text{NO}} + S_{\text{NO}}(t)}
\]
where $\mu_{\text{max}}$ is the maximum specific growth rate, $K_S$ and $K_{NO}$ are the so-called half saturation coefficients for the carbon organics and the nitrate, respectively.

Equation (1), together with equation (2), gives a parametric model of the denitrification process. All the parameters involved are plant dependent and hence have to be specifically estimated for each individual case based on the data obtained either from on-site measurements or from laboratory analyses (of on-site samples).

### 3 DIFFERENT TYPES OF MODELING ERRORS AND THEIR UNIFIED REPRESENTATION

Assume that the paramerized model to be validated takes the form,

$$\dot{x}_m(t) = f_m(x_m(t), \Theta_m, u(t), t)$$  \hspace{1cm} (3)

where $x_m(t) \in \mathbb{R}^n$ is the state variable vector of the model, $u(t) \in \mathbb{R}^p$ is the input vector, and $\Theta_m$ is the model parameter vector, which is known. On the basis of this model, the real behavior of the system can generally be represented as,

$$\dot{x}_r(t) = f_m(x_r(t), \Theta_m, u(t), t) + e_m(t)$$  \hspace{1cm} (4)

where $x_r(t) \in \mathbb{R}^n$ is the state vector of the system, $e_m(t) \in \mathbb{R}^p$ is the modeling error vector. It is assumed in equation (4) that the real system has the same number of state variables as the model. This representation does not limit the generality of the representation since the errors introduced by erroneous state aggregations in deriving model (3) can also be represented by the error term $e_m(t)$.

In order to make the modeling error identification possible, an appropriate representation of the error term $e_m(t)$ in equation (4) is required. This representation should be obtained by making use of the a priori knowledge about the possible modeling errors. Basically, modeling errors may be introduced in each stage of the modeling process as depicted in Figure 1. In this section, it will be shown, taking the biological model developed in the previous section as an example, how the a priori knowledge concerning the modeling errors can be obtained through the analysis of the modeling process and the model itself. The mathematical representation of the modeling errors will also be discussed. As will be shown in the next section, such a representation allows the identification of the modeling errors based on the comparison of the observed data with data produced by simulation of the erroneous model.

**Modeling Errors due to an improperly defined Experimental Frame**

In defining the boundaries of the process or system to be modeled, some important components may be missed, some significant disturbances to the system may be improperly neglected and so on. All of these introduce errors into the model. The Experimental Frame is the formalisation of the experimental conditions (inputs applied to the system, outputs observed, criteria of acceptance, ...) (see [3]) and as such the above mentioned modeling errors can be formally expressed as Experimental Frame errors. For a more rigorous treatment, see [13].

For instance, an assumption underlying model (1) is that no other reactions occur in the process which affect the mass balance of the concerned materials. One knows, however, that this assumption is not valid when dissolved oxygen is present in the influent. In fact, when dissolved oxygen is fed to the bioreactor, the following reaction, which is called the aerobic oxidation, will also occur, accompanying the denitrification
reaction described in the previous section,

\[ S + O_2 \xrightleftharpoons{r_o} X + CO_2 + H_2O \]

where \( r_o \) denotes the oxidation reaction rate. The reaction scheme clearly shows how the aerobic oxidation affects the mass balance of the carbon organics and the biomass. This will inevitably introduce errors in the prediction of these two variables. Since, as shown in model (1), both \( S \) and \( X \) appear in the equation concerning the dynamics of \( S_{NO} \), the prediction of the nitrate concentration will be affected indirectly.

A characteristic of the modeling error described above is that it does not directly affect the third equation in model (1). The above aerobic oxidation introduces an \( r_o \) term into the first equation of (1) and an \( \frac{1}{Y_S} r_o \) term into the second equation. The modeling error term in equation (4) takes the following form,

\[ e_{m,o}(t) = \left[ 1 - \frac{1}{Y_S} \right]^T r_o(t) \quad (5) \]

While \( \left[ 1 - \frac{1}{Y_S} \right]^T \) is apparently a known vector, \( r_o(t) \) is an unknown, time-variant scalar.

**Modeling Errors due to an improperly characterized Model Structure**

Due to for instance lack of knowledge of the mechanism of the process to be modeled or due to an over-simplification of the model, one may assume a wrong model structure. Typical errors include choosing an incorrect number of state variables or incorrectly assuming non-linear behavior. Structural errors may accidentally be produced through incorrect choice of parameters (usually, 0), whereby some part of the model structure vanishes, thereby altering the model structure.

For instance, in model (1), there does not exist a fundamental law that precisely characterizes the dependence of the denitrification reaction rate on the concentrations of the materials. The “laws” which have hitherto been proposed are all quite empirical. A problem of this type of laws is that they have a limited applicability range. An inappropriate choice of the “laws” may introduce errors. For example, when the model of the denitrification rate given in equation (2) is not a good description of the real reaction rate: \( \mu_r(t) = \mu(t) + \delta \mu_r(t) \), where \( \mu_r(t) \) is the real specific reaction rate and \( \delta \mu_r(t) \) is the modeling error, the following error term is found by substitution in equation (4),

\[ e_{m,\mu}(t) = \left[ 1 - \frac{1}{Y_S} - \frac{1 - Y_S \delta \mu_r(t)}{2.86Y_S^2} \right]^T \delta \mu_r(t) X(t) \quad (6) \]

**Modeling Errors due to inaccurate estimates of the Model Parameters**

Either by improper or inadequate data used for parameter estimation or by ill designed estimation algorithms, one may use incorrect parameter values. The error terms in equation (4) due to the estimate errors of the parameters in model (1) are as follows,

**modeling error of \( b \)**

Assuming \( b_r = b + \delta b \), where \( b_r \) is the real decay coefficient and \( \delta b \) is the modeling error, one obtains,

\[ e_{m,b}(t) = \left[ -1 \ 0 \right]^T \delta b X(t) \quad (7) \]
modeling error of \( f_P \)
Assuming \( f_{Pr} = f_P + \delta f_P \), where \( f_{Pr} \) is the real inert fraction in a biomass cell and \( \delta f_P \) is the modeling error, one obtains,
\[
e_{m,f_P}(t) = [0 \ 0 \ 1]^T \frac{\delta f_P}{2.86} X(t)
\] (8)

modeling error of \( Y_S \)
Assuming \( \frac{1}{Y_{Sr}} = \frac{1}{Y_S} + \delta(\frac{1}{Y_S}) \), where \( Y_{Sr} \) is the real yield coefficient and \( \delta(\frac{1}{Y_S}) \) is the modeling error, one obtains,
\[
e_{m,Y_S}(t) = [0 \ 1 - \frac{1}{2.86}]^T \delta(\frac{1}{Y_S}) \mu(t) X(t)
\] (9)

modeling errors of \( \mu_{\text{max}}, K_{NO} \) and \( K_S \)
Assuming \( \mu_r(t) = \mu(t) + \delta \mu_p(t) \), where \( \mu_r(t) \) is the real specific reaction rate and \( \delta \mu_p(t) \) is the error caused by the modeling error of \( \mu_{\text{max}}, K_{NO} \) or \( K_S \), one obtains,
\[
e_{m,\mu_{\text{max}},K_{NO},K_S}(t) = [1 - \frac{1}{Y_S} \ rac{1 - Y_S}{2.86Y_S}]^T \delta \mu_p(t) X(t)
\] (10)

One finds that every single modeling error shown above takes the form of a product of a known constant vector and an unknown time-varying variable. This is not an artifact of this particular example, but is in fact a general property. Usually, each modeling error affects only a subspace of the \( n \)-dimensional state space, and can hence be represented in equation (4) with a term \( F_i d_i(t) \), where \( F_i \in \mathbb{R}^{n \times n} \), \( d_i(t) \in \mathbb{R}^n \). The vectors of \( F_i \) span the subspace affected by the concerned modeling error. \( F_i \) is called the feature vector or feature matrix of the modeling error. \( d_i(t) \) represents the magnitude of the modeling error, and is generally unknown and time-varying. Thus, equation (4) can be rewritten as,
\[
\dot{x}_r(t) = f_m(x_r(t), \theta_m(t), u(t), t) + \sum_{i=0}^l F_i d_i(t)
\] (11)

Since it is usually not possible to predict all possible modeling errors, it is necessary to include a special feature matrix, say \( F_0 \), in equation (11) to represent modeling errors which were not explicitly modeled. Obviously, the \( n \)-dimensional identity matrix is suitable for that purpose.

To allow for meaningful error identification, some assumptions are made with respect to equation (11):

- The individual errors are written in “additive” form:
  \[
  v_r = v + \delta v
  \]
  Such a “choice” of individual error terms is always possible without loss of generality. One may be required to “lump” non-linear errors as in \( \delta(Y_S) \) or \( \delta \mu_p \) above.
- Simultaneously occurring errors are assumed to be either additive, or sufficiently small to allow for an linear approximation:
  \[
  f(A + \delta A, B + \delta B) \approx f(A, B) + \frac{\partial f}{\partial A}(A, B) \delta A + \frac{\partial f}{\partial B}(A, B) \delta B
  \]
  Though such an assumption is not necessary per se, as non-linear effects can always be lumped into an extra error term (using the above mentioned \( F_0 \)), this would defeat our purpose of isolating individual error contributions.
4 MODELING ERROR IDENTIFICATION WITH COMPLETE STATE MEASUREMENT

The above has lead us to an intuitively understandable representation of the error term. Since each modeling error affects a specific subspace of the state space, it is reasonable to believe that the modeling errors can be identified by investigating the difference between the behaviors. Intuitively, the quality of modeling error identification techniques will depend on their ability to separate (classify) error subspaces. In this section, an observer-based approach is developed in which a signal, called the featured signal, is generated based on equations (3) and (11), on which each modeling error imposes an easily distinguishable feature. The featured signal is then further analyzed so that the modeling errors are identified. It is assumed in this section that all the states of the system are measurable.

4.1 A Feature Equation

A straightforward way to compare the behaviors of system (11) and model (3) is to directly compare the data observed from the system to those produced by simulation. The error signal,

\[ e(t) = x_r(t) - x_m(t) \]

is characterized by,

\[ \dot{e}(t) = f_m(x_r(t), \theta_m, u(t), t) - f_m(x_m(t), \theta_m, u(t), t) + \sum_{i=0}^{l} F_i d_i(t) \]  

which is obtained by subtracting equation (3) from equation (11). Integration of equation (12) results in,

\[ \sum_{i=0}^{l} F_i \int_{t_0}^{t} d_i(\tau) d\tau = e(t) - e(t_0) - \int_{t_0}^{t} (f_m(x_r(\tau), \theta_m, u(\tau), \tau) - f_m(x_m(\tau), \theta_m, u(\tau), \tau)) d\tau \]  

Since \( x_r(t) \), the states of the system, are completely measurable, the right-hand side of equation (13), which can be defined as a new error signal \( \bar{e}(t) \), is a known vector. Equation (13) clearly shows that each modeling error imposes a subspace, which is spanned by its feature matrix, on the \( n \)-dimensional \( \bar{e} \) space. This indicates that one may identify the modeling errors associated with the model by investigating the properties of the \( \bar{e} \) space.

Equation (13) is theoretically meaningful. However, the numerical errors accumulated in the integration may easily destroy the ‘subspace’ property of \( \bar{e}(t) \) since equation (12) is not asymptotically stable. It is therefore necessary to improve the design such that the error equation is stabilized. As will be shown in the following, this can be accomplished by building appropriate observers for system (11).

When model (3) is linear and time-invariant, an approach to designing observers with the desired properties have been developed [26] in the context of fault diagnosis. In the sequel, we will focus on the observer design for general models (linear or nonlinear, time-invariant or time varying). It can be shown that the observer designed in [26] is a special case of the observers designed below.

A standard design leads to the following observer of system (11),

\[ \dot{x_o}(t) = f_m(x_o(t), \theta_m, u(t), t) + G \cdot (x_r(t) - x_o(t)) \]  

6
where $G \in \mathbb{R}^{n \times n}$ is a gain matrix which should be designed such that observer (14) is asymptotically stable. Defining $e_o(t) = x_r(t) - x_o(t)$, the following error equation is obtained by subtracting equation (14) from equation (11),

$$
\dot{e}_o(t) = -Ge_o(t) + f_m(x_r(t), \Theta_m, u(t), t) - f_m(x_o(t), \Theta_m, u(t), t) + \sum_{i=0}^l F_i d_i(t)
$$

Solving equation (15), one obtains,

$$
\sum_{i=0}^l \int_{t_0}^t e^{-G(t-\tau)} F_i d_i(\tau) d\tau = \int_{t_0}^t e^{-G(t-\tau)} (f_m(x_r(\tau), \Theta_m, u(\tau), \tau) - f_m(x_o(\tau), \Theta_m, u(\tau), \tau)) d\tau
$$

(16)

$e_o(t_0) = 0$ is assumed in equation (16) since $x_r(t)$ are completely measurable. Assuming that,

$$
G = gI
$$

(17)

where $I$ is an unitary matrix, equation (16) can be rewritten as,

$$
\sum_{i=0}^l F_i \int_{t_0}^t e^{-g(t-\tau)} d_i(\tau) d\tau = \int_{t_0}^t e^{-g(t-\tau)} (f_m(x_r(\tau), \Theta_m, u(\tau), \tau) - f_m(x_o(\tau), \Theta_m, u(\tau), \tau)) d\tau
$$

(18)

Similar to equation (13), equation (18) characterizes a relationship between the modeling errors and a generated error signal $\tilde{e}_o(t)$, the right-hand side of equation (18). Again, each modeling error imposes a subspace on the $n$-dimensional $\tilde{e}_o(t)$ space. As opposed to equation (13), however, $\tilde{e}_o(t)$ in equation (18) can easily be solved numerically since equation (15) is asymptotically stable.

Equation (18) is called the feature equation. Correspondingly, $\tilde{e}_o(t)$ is called the featured error signal.

A question to be answered is, whether a matrix $G$ constrained by equation (17) is able to asymptotically stabilize observer (14). It can be generally proven that such a $G$ exists as long as function $f_m(x_m(t), \Theta_m, u(t), t)$ is Lipschitz in $x_m$. In fact, assuming that,

$$
\| f_m(x_{m,2}(t), \Theta_m, u(t), t) - f_m(x_{m,1}(t), \Theta_m, u(t), t) \| \leq k \| x_{m,2} - x_{m,1} \| \quad \forall x_{m,1}, x_{m,2}
$$

where $k > 0$ is a Lipschitz constant, it is easy to prove, by means of a Lyapunov function $V = (x_o(t) - x_{o,0}(t))^T (x_o(t) - x_{o,0}(t))$ where $x_{o,0}(t)$ satisfies $f_m(x_{o,0}(t), \Theta_m, u(t), t) + G \cdot (x_r(t) - x_{o,0}(t)) = 0$, that observer (14) is asymptotically stable when $g > k$ as then,

$$
V \leq 2(k-g)(x_o(t) - x_{o,0}(t))^T (x_o(t) - x_{o,0}(t)) < 0 \quad \forall x_o(t) \neq x_{o,0}(t)
$$

The calculation of $\tilde{e}_o(t)$ is schematically shown in Figure 3. Obviously, model (14) has to be implemented instead of model (3). Model (3) can easily be obtained by letting $g = 0$ in model (14). This nice property allows an elegant unified implementation of model (3) and observer (14). In addition to the ‘model’ block, two extra blocks are required to calculate $e_1(t)$ and $e_2(t)$.

A problem underlying observer (14) is that, for some models, a large $g$ has to be chosen in order to obtain an asymptotically stable observer. As shown on the left-hand side of equation (18), a large $g$ will reduce the manifestation ($\int_{t_0}^t e^{-g(t-\tau)} d_i(\tau) d\tau$) of the modeling errors in the error signal $\tilde{e}_o(t)$ such that it may be
difficult to distinguish the effects of modeling errors from noise. In such cases, the design of the observer is improved as follows,

\[ \dot{x}_O(t) = f_m(x_r(t), \Theta_m, u(t), t) + g \cdot (x_r(t) - x_O(t)) \tag{19} \]

which is asymptotically stable as long as \( g > 0 \). The error signal \( e_O(t) \), which is defined as,

\[ e_O(t) = x_r(t) - x_O(t) \]

is characterized by,

\[ \dot{e}_O(t) = -ge_O(t) + \sum_{i=0}^{l} F_i d_i(t) \tag{20} \]

Thus,

\[ \sum_{i=0}^{l} F_i \int_{0}^{t} e^{-g(t-\tau)} d_i(\tau) d\tau = e_O(t) \tag{21} \]

Obviously, equation (21) is also a feature equation and \( e_O(t) \) another featured error signal.

The advantage of this design over the previous one is that the choice of \( g \) is independent of the dynamic characteristics of the model. Hence it can be chosen relatively small to enhance the manifestation of the modeling errors. In addition, as shown in Figure 4, the calculation is also simpler than that of \( e_o(t) \). However, since observer (19) is far different from model (3), a unified implementation of both will lead to an implementation which is hard to accept as a final simulation model. If observer (19) is implemented separately, what is validated is the conceptual model, provided that the computerized implementation of observer (19) is correct, rather than the computerized model which will be finally delivered.

For use in section 4.2, equations (18) and (21) are unified into the following feature equation,

\[ \sum_{i=0}^{l} F_i d_i(t) = e(t) \tag{22} \]

where \( e(t) \) is the featured signal, \( d_i(t) \) is the manifestation of the \( i \)-th modeling error in the featured signal.

### 4.2 Modeling Error Identification

As has been mentioned in the previous subsection, feature equation (22) provides a relationship between modeling errors and subspaces of the \( n \)-dimensional space of the featured signal. This implies that information about modeling errors can be obtained when the featured signal \( e(t) \) is localized to a certain subspace. Obviously, the identification process involves two essential steps, localizing \( e(t) \) to a subspace and interpreting the localization based on the feature equation.

Given a signal \( e(t) \in \mathbb{R}^n \) and a subspace \( S \subseteq \mathbb{R}^n \), to verify if \( e(t) \) lies in \( S \) one may investigate the projection of \( e(t) \) on \( S^\perp \), the orthogonal complementary subspace of \( S \), making use of the fact that any signal lying in \( S \) has zero projection on \( S^\perp \). Assume \( V = [V_1 \ V_2] \in \mathbb{R}^{n \times n} \), where the columns of \( V_1 \) and \( V_2 \) constitute a unitary, orthogonal basis of the subspaces \( S \) and \( S^\perp \), respectively. Obviously, \( V \) is a unitary orthogonal matrix, \( i.e. \), \( V^T V = VV^T = I \). The projection of signal vector \( e(t) \) on \( S^\perp \), represented under the basis constituted by the columns of \( V_2 \), is \( V_2^T e(t) \). If \( e(t) \in S \), we have,

\[ V_2^T e(t) = 0 \tag{23} \]

Equation (23) gives a theoretical criterion to judge whether or not \( e(t) \) lies in \( S \). Practically, however, equation (23) will never strictly hold due to, for instance, noise contained in the measurement of \( x_r(t) \), the
existence of insignificant modeling errors, etc. Noting that the objective is to identify the most significant modeling errors, criteria (23) is changed to,

\[
\frac{\|V_2^T e(t)\|_2}{\|V_1^T e(t)\|_2} \leq \varepsilon_1
\]

(24)

where \(V_1^T e(t)\) is the projection of vector \(e(t)\) on subspace \(S\), represented under the basis constituted by the columns of \(V_1\); \(\varepsilon_1\) is a threshold, which should be chosen according to the magnitude of the system noise and of the measurement noise.

Let \(M_{\text{ex}}\) be a subset of the pre-defined modeling error set \(M_{\text{p}} = \{F_1, F_2, \cdots, F_l\}\) where \(F_i\) denotes the \(i\)-th modeling error as well as its feature matrix. Assume \(M_{\text{ex}}\) satisfies,

\[
\sum_{F_i \in M_{\text{ex}}} \mathcal{R}(F_i) = S
\]

(25)

where \(\mathcal{R}(F_i)\) denotes the space spanned by the columns of \(F_i\). By introducing feature equation (22) in inequality (24), one obtains,

\[
\frac{\|\sum_{F_i \in M_{\text{ex}}} V_2^T F_i \bar{d}_i(t) + \sum_{F_i \in M_{\text{p}} - M_{\text{ex}}} V_2^T F_j \bar{d}_j(t)\|_2}{\|\sum_{F_i \in M_{\text{ex}}} V_1^T F_i \bar{d}_i(t) + \sum_{F_i \in M_{\text{p}} - M_{\text{ex}}} V_1^T F_j \bar{d}_j(t)\|_2} \leq \varepsilon_1
\]

(26)

Inequality (26) obviously holds when,

\[
\bar{d}_j(t) = 0 \quad \forall F_j \in M_{\text{p}} - M_{\text{ex}}
\]

(27)

\(i.e.,\) when no modeling error exists in \(M_{\text{p}} - M_{\text{ex}}\), or

\[
\max(\bar{d}_j)|_{F_j \in M_{\text{p}} - M_{\text{ex}}} \ll \max(\bar{d}_j)|_{F_i \in M_{\text{ex}}}
\]

(28)

\(i.e.\) the magnitudes of the modeling errors in \(M_{\text{p}} - M_{\text{ex}}\) are much smaller than those of the modeling errors in \(M_{\text{ex}}\). However, inequality (26) may still hold, even when equation (27) and inequality (28) do not hold. A typical situation is,

\[
\frac{\|V_2^T F_j\|_2}{\|V_1^T F_j\|_2} = \frac{\sigma(V_2^T F_j)}{\sigma(V_1^T F_j)} \leq \varepsilon_1 \quad \forall F_j \in M_{\text{p}} - M_{\text{ex}} \quad \text{with non-small } \bar{d}_j(t)
\]

(29)

where \(\sigma(V_2^T F_j)\) and \(\sigma(V_1^T F_j)\) are the largest singular values of matrices \(V_2^T F_j\) and \(V_1^T F_j\), respectively. The physical interpretation of this condition is that \(\mathcal{R}(F_j)\) is very ‘close’ to \(S\) so that its projection on \(S^\perp\) is much smaller than that on \(S\). Note condition (29) is necessary only when \(\sum_{F_i \in M_{\text{ex}}} V_1^T F_i \bar{d}_i(t)\) is zero or small. When this is not the case, inequality (26) will hold even when \(\sigma(V_2^T F_j)/\sigma(V_1^T F_j) > \varepsilon_1\). In order to take this situation into consideration, condition (29) is modified to,

\[
\frac{\sigma(V_2^T F_j)}{\sigma(V_1^T F_j)} \leq \varepsilon_2 \quad \text{where } \varepsilon_1 < \varepsilon_2 \quad \forall F_j \in M_{\text{p}} - M_{\text{ex}} \quad \text{with non-small } \bar{d}_j(t)
\]

(30)

Generally, \(\varepsilon_2\) should be chosen sufficiently large in order to reduce the risk of excluding significant modeling errors contained in \(M_{\text{p}} - M_{\text{ex}}\). However, a too high \(\varepsilon_2\) decreases the effectiveness of the algorithm. A compromise is thus needed.

Equation (27) and inequalities (28) and (30) indicate that, when inequality (24) holds, the most significant
modeling errors are among those contained in $M_{ex}$ and those contained in $M_p - M_{ex}$ satisfying condition (30). The rest can thus be excluded.

When inequality (24) does not hold, which implies that $e(t)$ does not lie in subspace $S$, no information about which modeling errors are the most significant ones can be drawn.

The above discussions lead to the following algorithm for identifying the pre-defined modeling errors.

**Step 1**: Construct set $S_c$ as,

$$S_c = \{S_i \mid S_i = \sum_{i_j \in M_p} \mathbb{R}(F_{i,j}) \text{ where } M_i = \{F_{i,1}, F_{i,2}, \cdots, F_{i,k}\} \subseteq M_p, \text{ dim}(S_i) < n\}$$  \hspace{1cm} (31)

Apparantly, $S_c$ is a set constituted by all the true subspaces of $R^n$ that are spanned by the feature matrices of one or more pre-defined modeling errors.

**Step 2**: For each $S_i \in S_c$, construct matrices $V_1$ and $V_2$ such that their columns constitute unitary bases of $S_i$ and $S_i^\perp$, respectively. Define a modeling error set $M_{S_i}$ as,

$$M_{S_i} = \{F_j \mid F_j \in M_p, \text{ } V_2F_j = 0 \text{ or } \frac{\sigma(V_2^TF_j)}{\sigma(V_1^TF_j)} \leq \varepsilon_2\}$$  \hspace{1cm} (32)

According to the above analysis, $M_{S_i}$ is such a set that all the modeling errors in $M_p - M_{S_i}$ can be excluded when inequality (24) holds.

**Step 3**: For each $S_i \in S_c$, define $f_i$ as,

$$f_i = \begin{cases} 0 & \text{if inequality (24) holds;} \\ 1 & \text{otherwise} \end{cases}$$

The most significant modeling errors are thus localized to,

$$M_e = \bigcap_i[M_{S_i} \cup (M_p - M_{S_i}) f_i]$$  \hspace{1cm} (33)

When inequality (24) does not hold for all $S_i \in S_c$, which means that $e(t)$ does not lie in any true subspaces of $R^n$ that are spanned by the feature matrices of one or more pre-defined modeling errors, the algorithm gives $M_e = M_p$. In this case, the algorithm is not able to provide any useful information for model improvement.

**5 MODELING ERROR IDENTIFICATION WITH INCOMPLETE MEASUREMENT**

It was assumed in the previous section that $x_r(t)$, the states of the system, are completely measurable. When this is not the case, modeling error identification is still possible, though more complicated. How to identify modeling errors in such situations is studied in this section.

Assume that the measurement of the system is,

$$y_r(t) = h(x_r(t), u(t), t)$$  \hspace{1cm} (34)

which motivates the following decomposition of model (3),

$$\dot{x}_m(t) = A(\theta_m)x_m(t) + f_{mn,h}(x_m(t), \theta_m, u(t), t) + F_{u}f_{mn,u}(x_m(t), \theta_m, u(t), t)$$  \hspace{1cm} (35)
where \( A(\theta_m) \in R^{m \times n} \) is the system matrix of the linear time-invariant part of the model, 
\( f_{mn,k}(x_m(t), \theta_m, u(t), t) + F_u f_{mn,u}(x_m(t), \theta_m, u(t), t) \in R^n \) is the nonlinear and/or time varying part of the model, in which \( f_{mn,k}(x_m(t), \theta_m, u(t), t) \in R^{n \times 1} \) is a function vector satisfying that all elements in \( f_{mn,k}(x_r(t), \theta_m, u(t), t) \) are solvable from equation (34), \( F_u \in R^{n \times k} \) is a constant matrix, \( f_{mn,u}(x_m(t), \theta_m, u(t), t) \in R^{k \times 1} \) contains those independent functions satisfying that \( f_{mn,u}(x_r(t), \theta_m, u(t), t) \) are not solvable from equation (34). As an illustration, a decomposition of model (1) is given. When \( Q_{in} \) and \( Q_w \) are constant and \( S_S(t) \) and \( S_{NO}(t) \) are measurable, the terms in equation (35) are as follows,

\[
A(\theta_m) = \begin{bmatrix} -Q_w/V - b & 0 & 0 \\ 0 & -Q_w/V & 0 \\ (1 - f_p)b/2.86 & 0 & -Q_w/V \end{bmatrix}
\]

\( f_{mn,k}(x_m(t), \theta_m, u(t), t) = 0 \)

\( F_u f_{mn,u}(x_m(t), \theta_m, u(t), t) = \begin{bmatrix} 1 \\ -1/Y_S \\ -(1 - Y_S)/(2.86 Y_S) \end{bmatrix} \mu(t) X(t) \)

Corresponding to model representation (35), system (11) is rewritten as,

\[
\dot{x}_r(t) = A(\theta_m)x_r(t) + f_{mn,k}(x_r(t), \theta_m, u(t), t) + F_u f_{mn,u}(x_r(t), \theta_m, u(t), t) + \sum_{i=0}^{l} F_i d_i(t) \tag{36}
\]

Assume furthermore that the following linear combination of the states can be obtained from measurement equation (34),

\[
y_{r,lin} = C x_r(t) \tag{37}
\]

where \( C \in R^{m \times n} \) is a constant matrix, satisfying that \((C, A(\theta_m)) \) is observable. Based on the above measurement equation, the following reduced-order observer can be designed for system (36),

\[
\begin{align*}
\dot{z}(t) &= Fz(t) + G y_{r,lin}(t) + T f_{mn,k}(x_r(t), \theta_m, u(t), t) \\
\epsilon(t) &= K z(t) + P y_{r,lin}(t)
\end{align*} \tag{38}
\]

where the parametric matrices \( F, G, T, K, P \) are subject to the structure conditions:

\[
\begin{align*}
TA - FT &= GC \\
KT + PC &= 0 \\
F &= \text{stable}
\end{align*} \tag{39-41}
\]

With the definition,

\( \epsilon(t) = z(t) - T x_r(t) \)

the following error equation can easily be obtained from equations (36)–(41),

\[
\begin{align*}
\dot{\epsilon}(t) &= Fe(t) - TF_u f_{mn,u}(x_r(t), \theta_m, u(t), t) - T \sum_{i=0}^{l} F_i d_i(t) \\
\epsilon(t) &= Ke(t)
\end{align*} \tag{42}
\]

If matrices \( F, G, T, K, P \) are designed such that,

\[
\begin{align*}
TF_u &= 0 \\
TF &\not= 0, \, \forall \, i \in 1 - w \\
K(F | TF_i) &\not= 0, \, \forall \, i \in w
\end{align*} \tag{43-45}
\]
where \( I = \{0, 1, 2, \ldots, k\} \), \( \mathbf{w} \subseteq \mathbf{1} \), \( (F \mid TF_i) \) denotes the controllability matrix of the pair \( (F, TL_i) \). It is obvious that \( \varepsilon(t) \) will remain 'zero' as long as no modeling error in \( \mathbf{w} \) exists. In other words, if the error signal vector \( \varepsilon(t) \) is detected as 'zero’, all the modeling errors in subset \( \mathbf{w} \) are excluded. Similar decision logic to that used in the previous section should be used here.

With different choices of \( \mathbf{w} \subseteq \mathbf{k} \), a bank of such observers, which are called robust observers, can be designed. The modeling errors that can not be excluded by the residual of these observers are concluded as the possible most significant ones.

Algorithms for designing robust observers have been extensively studied. The readers are referred to [27][28], for the algorithms and proofs.

### 6 Simulation Study

To illustrate and verify the techniques presented in previous sections, the identification of the modeling errors that are associated with model (1) are studied in this section.

#### 6.1 Design

Assume the real behavior of the denitrification process shown in Figure 2 is characterized by,

\[
\begin{align*}
\dot{X}_r(t) &= \mu_r(t)X_r(t) + \mu_o(t)X_r(t) - b_rX_r(t) - \frac{Q_w(t)}{V}X_r(t) \\
\dot{S}_{S,r}(t) &= -\frac{1}{Y_{S,r}}\mu_r(t)X_r(t) - \frac{1}{Y_{S,r}}\mu_o(t)X_r(t) - \frac{Q_w(t)}{V}S_{S,r}(t) + \frac{Q_w(t)}{V}S_{S,in}(t) \\
\dot{S}_{NO,r}(t) &= -\frac{1}{2.86Y_{S,r}}\mu_r(t)X_r(t) - \frac{1}{2.86Y_{S,r}}\mu_o(t)X_r(t) - \frac{Q_w(t)}{V}S_{NO,r}(t) + \frac{Q_w(t)}{V}S_{NO,in}(t) \\
\dot{S}_O(t) &= -\frac{1}{Y_{S,r}}\mu_o(t)X_r(t) - \frac{Q_w(t)}{V}S_O(t) + \frac{Q_w(t)}{V}S_{O,in}(t)
\end{align*}
\]

(46)

where \( S_O \) and \( S_{O,in} \) are the oxygen concentrations in the reactor and in the influent, respectively; \( \mu_r(t) \) and \( \mu_o(t) \) are the specific denitrification and oxidation rates, respectively,

\[
\begin{align*}
\mu_r(t) &= \frac{\mu_{\max,r}}{K_{S,r} + S_{S,r}(t)} \frac{S_{NO,r}(t)}{K_{NO,r} + S_{NO,r}(t)} \frac{K_{OI}}{K_{OI} + S_O(t)} \\
\mu_o(t) &= \frac{\mu_{\max,o}}{K_{S,r} + S_{S,r}(t)} \frac{S_O(t)}{K_{O,I} + S_O(t)}
\end{align*}
\]

(47)

(48)

Note how \( \mu_r(t) \) has a different structure from its model shown in equation (2). The extra term \( K_{OI}/(K_{OI} + S_O(t)) \) in \( \mu_r(t) \) reflects the effect of the inhibition of oxygen on the denitrification rate.

While states \( S_{S,r}(t) \) and \( S_{NO,r}(t) \) in system (46) can easily be measured through chemical analysis, state \( X_r(t) \), the biomass concentration, is not directly measurable. However, with a respiratory meter [29], \( \frac{\mu_{\max,r}X_r(t)}{Y_{S,r}}, \) denoted as \( \tilde{X}_r(t) \), can be measured [30]. In order to make this measurement usable in identifying the modeling errors, model (1) is rewritten as,

\[
\dot{\tilde{X}}(t) = \mu_{\max}(\tilde{t})\tilde{b}(t)\tilde{X}(t) - b\tilde{X}(t) - \frac{Q_w(t)}{V}\tilde{X}(t)
\]
\[
\begin{align*}
\dot{S}_S(t) &= -\ddot{\mu}(t)X(t) - \frac{Q_{in}(t)}{V}S_S(t) + \frac{Q_{in}(t)}{V}S_{S,in}(t) \\
\dot{S}_{NO}(t) &= -\frac{1-Y_S}{2.86}\ddot{\mu}(t)X(t) - \frac{Y_S}{\mu_{max}} \frac{1-f_p}{2.86}b\dot{X}(t) - \frac{Q_{in}(t)}{V}S_{NO}(t) + \frac{Q_{in}(t)}{V}S_{NO,in}(t)
\end{align*}
\]

where,
\[
\begin{align*}
\dot{X}(t) &= \frac{\mu_{max}}{Y_S}X(t) \\
\ddot{\mu}(t) &= \frac{S_S(t)}{K_S + S_S(t)} \frac{S_{NO}(t)}{K_{NO} + S_{NO}(t)}
\end{align*}
\]

Based on model (49), an observer is designed with the second approach presented in Section 4.1 (equation (19)) for system (46), where \( g = 3 \) is chosen as the observer gain. The error signal,
\[
e_O(t) = [\ddot{X}_r(t) \, S_{S,r}(t) \, S_{NO,r}]^T - [\ddot{X}_O(t) \, S_{S,0}(t) \, S_{NO,0}]^T
\]
is a featured signal.

Corresponding to the reformulation of the model, the modeling errors modeled in Section 3 are reformulated as,
\[
\begin{align*}
e_{m,0}(t) &= [\mu_{max} - 1 \, 0]^T d_1(t) = F_1 d_1(t) \\
e_{m,d1}(t) &= [\mu_{max} - 1 - \frac{1-Y_S}{2.86}]^T d_2(t) = F_2 d_2(t) \\
e_{m,b}(t) &= [-1 \, 0 - \frac{Y_S}{\mu_{max}} \frac{1-f_p}{2.86}]^T d_3(t) = F_3 d_3(t) \\
e_{m,fr}(t) &= [0 \, 0 \, 1]^T d_4(t) = F_4 d_4(t) \\
e_{m,Y}(t) &= [0 \, 0 \, 1]^T d_5(t) = F_5 d_5(t) \\
e_{m,K_{NO},K_S}(t) &= [\mu_{max} - 1 - \frac{1-Y_S}{2.86}]^T d_6(t) = F_6 d_6(t) \\
e_{\mu_{max}}(t) &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} d_{7,1}(t) \\ d_{7,2}(t) \end{bmatrix} = F_7 d_7(t)
\end{align*}
\]

With the model parameters shown in Table 1, the feature matrices \( F_1 \)–\( F_7 \) are calculated and shown in Table 2. In order to improve the identifiability of the modeling errors, a row wise transformation is applied to all the feature matrices, and the improved feature matrices are also shown in Table 2 where \( W \), the transformation matrix, is chosen as \( diag(1 \, 1 \, 8) \). Correspondingly, the feature equation is changed to,
\[
\sum_{i=0}^{7} F_{iw} \int_{t_0}^{t} e^{-\varepsilon(t-\tau)} d_i(\tau) d\tau = \ddot{e}_O(t) = We_O(t)
\]

With feature matrices \( F_{iw} = -F_{7w} \), \( S_c \) is constructed as,
\[
S_c = \{S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8\} = \{\Re(F_{iw}), \Re(F_{2w}), \Re(F_{3w}), \Re(F_{4w}), \Re(F_{7w}), \Re(F_{7w}), \Re(F_{7w}) + \Re(F_{2w}), \\
\Re(F_{7w}) + \Re(F_{3w}), \Re(F_{2w}) + \Re(F_{3w})\}
\]

With \( \varepsilon_2 = 0.2 \), sets \( M_{S_i}, i = 1, 2, \ldots, 7 \), are calculated as follows,
\[
M_{S_i} = \{F_{iw}\}
\]
\[M_{52} = \{F_{2w}, F_{6w}\}\]
\[M_{53} = \{F_{3w}\}\]
\[M_{54} = \{F_{4w}, F_{5w}\}\]
\[M_{55} = \{F_{7w}, F_{3w}, F_{4w}, F_{5w}\}\]
\[M_{56} = \{F_{1w}, F_{2w}, F_{4w}, F_{5w}, F_{6w}\}\]
\[M_{57} = \{F_{1w}, F_{3w}\}\]
\[M_{58} = \{F_{2w}, F_{3w}, F_{6w}\}\]

6.2 Simulation

A simulation program for system (46) is designed. Random noise is added to all the three measurements: \(\bar{X}_r, S_{Sr}\) and \(S_{NO,r}\). According to the level of the noise, \(\varepsilon_1\) and \(\varepsilon_2\) are designed as 0.1 and 0.2, respectively. The designed observer of system (46) (equation (19) with \(g = 3\)) is also implemented. The system and its observer are simulated with different modeling errors, and the identification results are shown in below. The conditions and parameters that are used in the simulations, when not specifically mentioned, are: \(S_{O,in} = 0, K_O = 0.5 \text{ mg.l}^{-1}, K_{OI} = 1.5 \text{ mg.l}^{-1}, b_r, f_{P_r}, Y_{S,r}, K_{NO,r}\) and \(K_{S,r}\) take the same values as those of the model (see Table 1).

The cases studied are,

- **Case 1**: \(S_{O,in} = 2 \text{ mg.l}^{-1}\);
- **Case 2**: \(S_{O,in} = 10 \text{ mg.l}^{-1}\);
- **Case 3**: \(b_r = 0.3 \text{ day}^{-1}\);
- **Case 4**: \(f_{P_r} = 0.3\);
- **Case 5**: \(Y_{S,r} = 0.5\);
- **Case 6**: \(K_{NO,r} = 1 \text{ mg.l}^{-1}\);
- **Case 7**: \(\mu_{max,r} = 1.2 \text{ day}^{-1}\);
- **Case 8**: \(b_r = 0.3 \text{ day}^{-1}, K_{NO,r} = 1 \text{ mg.l}^{-1}\);
- **Case 9**: \(b_r = 0.28 \text{ day}^{-1}, K_{NO,r} = 0.2 \text{ mg.l}^{-1}\);
- **Case 10**: \(\mu_{max,r} = 1.2 \text{ day}^{-1}, K_{NO,r} = 0.3 \text{ mg.l}^{-1}\);

The identification results are shown in Table 3, where \(r_i\) denotes \(\|V_{ij}^T\text{We}(t)\|_2/\|V_{ij}^T\text{We}(t)\|_2\), for \(i = 1, 2, \cdots, 7\).

The following observations are made on Table 3.

- For Case 2 to Case 8, the algorithm successfully localizes the modeling errors to a subset of \(M_p\) in which the real modeling errors are contained;
- For Case 1 and Case 9, the modeling errors identified are only a subset of those that are really associated with the model. In Case 1, both \(F_{1w}\) and \(F_{2w}\) are associated with the model due to the existence of oxygen in the influent. \(F_{2w}\) is not identified because the influence of the oxygen inhibition on denitrification, when the oxygen concentration in the reactor is low, is much smaller than the influence of
oxygen on carbon organics consumption and biomass growth. A similar reasoning explains why $F_{3w}$ is not identified in Case 9. This is considered to be a nice property since it means that the algorithm will only identify the most significant modeling errors that have caused the invalidity of the model.

As a general conclusion, the simulation studies demonstrate how the approach developed in the previous sections can be used to identify modeling errors.

7 CONCLUDING REMARKS

An approach to modeling error identification has been presented in this paper. It consists of three major steps: analyzing the process and its model to predict the possible modeling errors and extract the feature matrices of these errors, designing observers for the process based on the invalid model to generate the featured signal, and analyzing the featured signal to identify the modeling errors. Since the approach requires enumeration of the modeling errors in advance, it is more suitable for mechanistic models than for black box ones. For gray box models, one may consider each small black box as a unit and localize the modeling errors to this level.

The approach has been verified by simulation studies. However, more practical verification is still necessary. This will be one subject for future study. One disadvantage of the approach is that it makes use of only the structural information of the modeling errors. By considering more types of knowledge, the modeling errors may be better identified. How to unite different types of knowledge in making the decision is also worth of further investigation.

References


Information Sources  

- a priori knowledge
- modeller's and experimenter's goals
- experiment observation (measurement) data

Activities

- Experimental Frame Definition
  - class of parametric model candidates
- Structure Characterisation
  - parametric model
- Parameter Estimation
  - model with meaningful parameter values
- Simulation
  - simulated measurements
- Validation
  - validated model

Figure 1: Model Development Process

![Diagram of model development process]

Figure 2: Biological Denitrification Process

![Diagram of biological denitrification process]

Table 1: Model Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{\text{max}}$</td>
<td>$1 \text{ day}^{-1}$</td>
</tr>
<tr>
<td>$b$</td>
<td>0.24 $\text{ day}^{-1}$</td>
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<tr>
<td>$f_p$</td>
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</tr>
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<td>$Y_S$</td>
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</tr>
<tr>
<td>$K_{NO}$</td>
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</tr>
<tr>
<td>$K_S$</td>
<td>2 $\text{ mg.l}^{-1}$</td>
</tr>
</tbody>
</table>

18
\[
\dot{e}_1(t) = -ge_1(t) + f_m(x_r(t), \theta_m, u(t), t)
\]

**Figure 3:** Generation of a Featured Error Signal

\[
\dot{x}_o(t) = f_m(x_o(t), \theta_m, u(t), t) + g \cdot (x_r(t) - x_o(t))
\]

**Figure 4:** Another Scheme for Generating the Featured Error Signal

<table>
<thead>
<tr>
<th>Table 2: Feature Matrices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_1$</td>
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</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>-1</td>
</tr>
<tr>
<td>0</td>
</tr>
</tbody>
</table>

$F_{1w} = WF_1$  $F_{2w} = WF_2$  $F_{3w} = WF_3$  $F_{4w} = WF_4$  $F_{5w} = WF_5$  $F_{6w} = WF_6$  $F_{7w} = WF_7$

<table>
<thead>
<tr>
<th></th>
<th>$F_1$</th>
<th>$F_2$</th>
<th>$F_3$</th>
<th>$F_4$</th>
<th>$F_5$</th>
<th>$F_6$</th>
<th>$F_7$</th>
</tr>
</thead>
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<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$F_{2w}$</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$F_{3w}$</td>
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<td>1</td>
<td>8</td>
<td>8</td>
<td>-0.923</td>
<td>0</td>
</tr>
</tbody>
</table>
## Table 3: Identification Results

<table>
<thead>
<tr>
<th>Case</th>
<th>$r_1$</th>
<th>$r_2$</th>
<th>$r_3$</th>
<th>$r_4$</th>
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