Population Dynamics

- Deductive modelling: based on physical laws
- Inductive modelling: based on observation + intuition
- Single species:
  Birth (in migration) Rate, Death (out migration) Rate

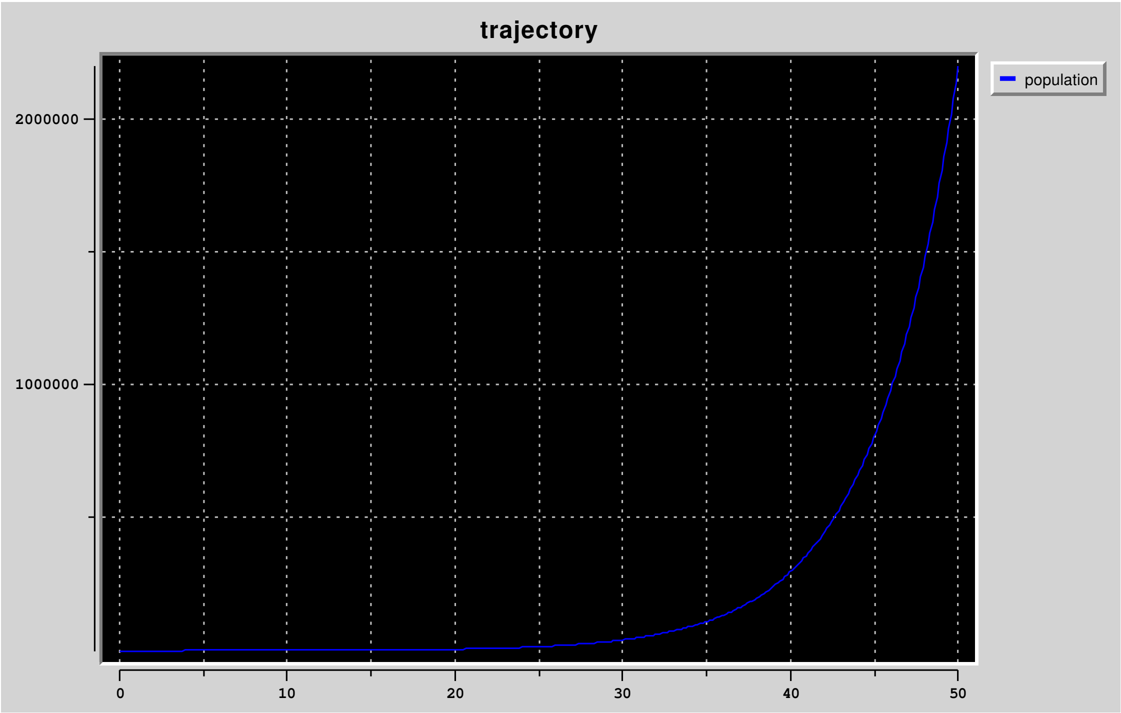
$$\frac{dP}{dt} = BR - DR$$

- Rates proportional to population

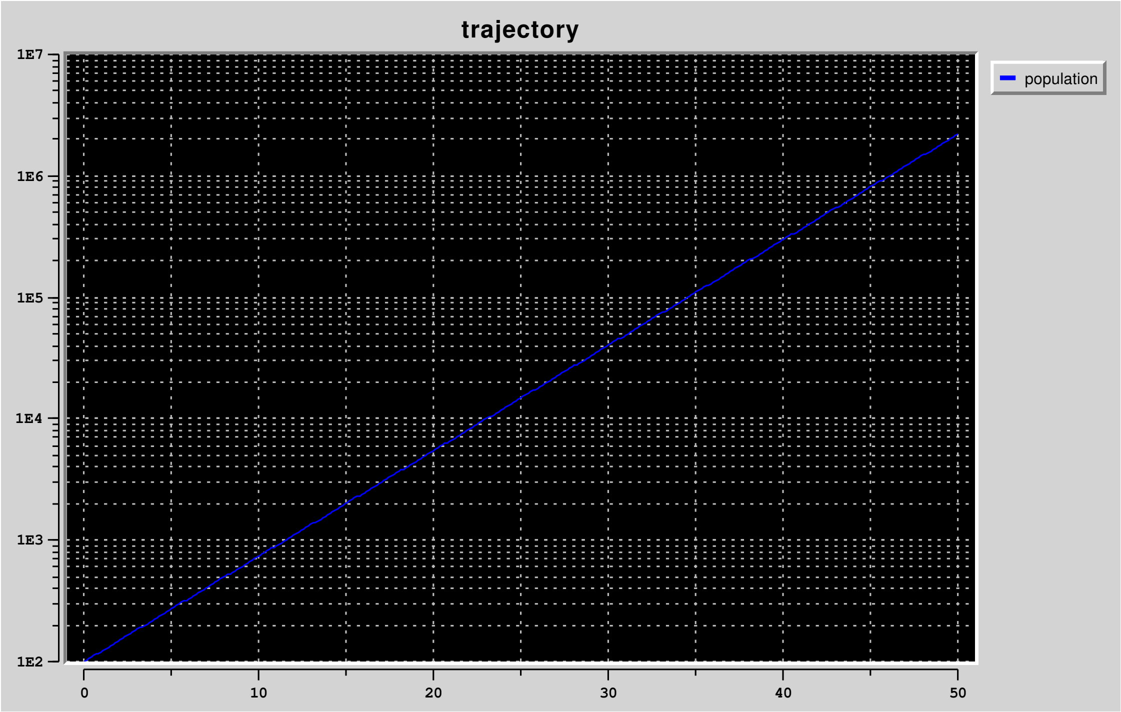
$$BR = k_{BR} \times P; DR = k_{DR} \times P$$

$$\frac{dP}{dt} = (k_{BR} - k_{DR})P$$

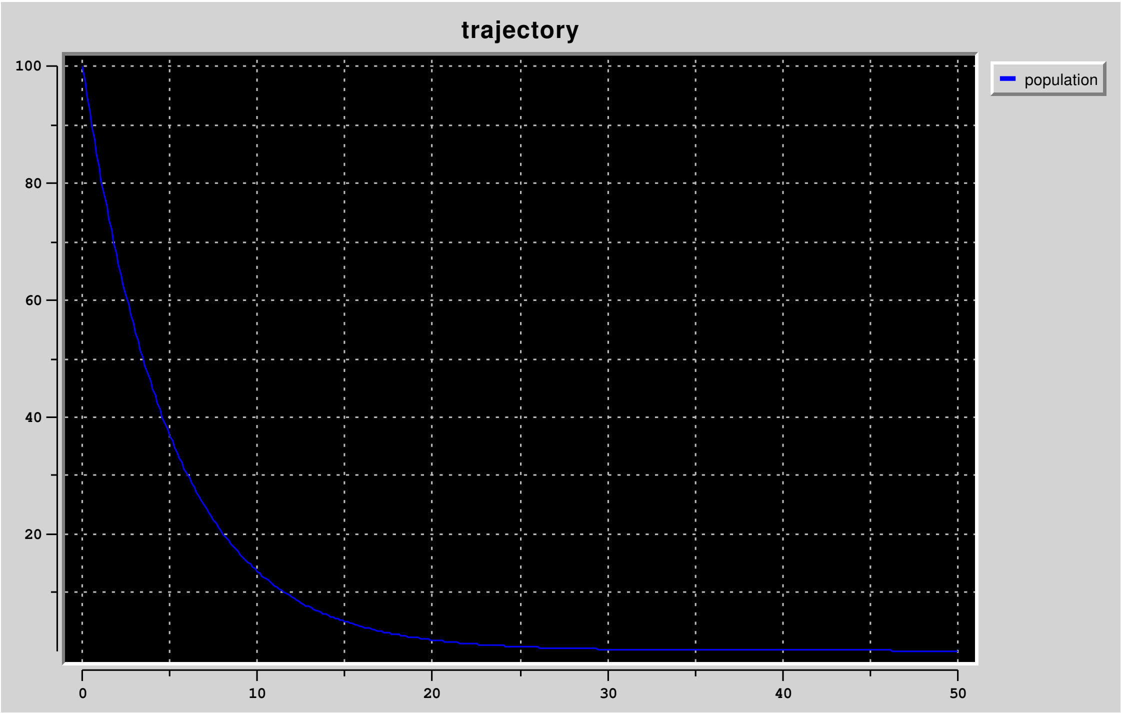
$k_{BR} = 1.4, k_{DR} = 1.2$: Exponential Growth



$k_{BR} = 1.4, k_{DR} = 1.2$: log(Exponential Growth)



$k_{BR} = 1.2, k_{DR} = 1.4$: Exponential Decay



Logistic Model

- Are k_{BR} and k_{DR} really constant ?
- Energy consumption in a *closed* system $\rightarrow$ limits growth

$$E_{pc} = \frac{E_{tot}}{P}$$

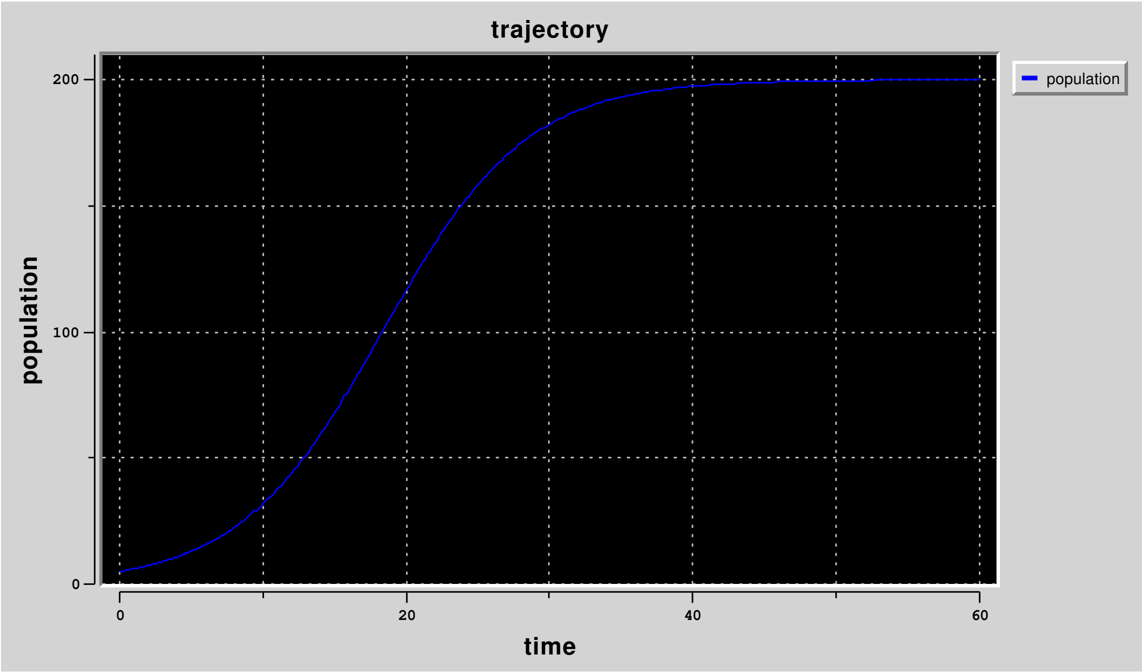
$P \uparrow \rightarrow E_{pc} \downarrow \rightarrow k_{BR} \downarrow$ and $k_{DR} \uparrow$ until equilibrium

- "crowding" effect:
  ecosystem can support maximum population P_{max}

$$\frac{dP}{dt} = k \times (1 - \frac{P}{P_{max}}) \times P$$

- crowding is a quadratic effect

$$k_{BR} = 1.2, k_{DR} = 1.4, crowding = 0.001$$



Disadvantages

- *NO* physical evidence for model structure !
- But, many phenomena can be well *fitted* by logistic model.
- P_{max} can only be *estimated* once steady-state has been reached. Not suitable for control, optimisation, ...
- Many-species system: P_{max}, steady-state ?

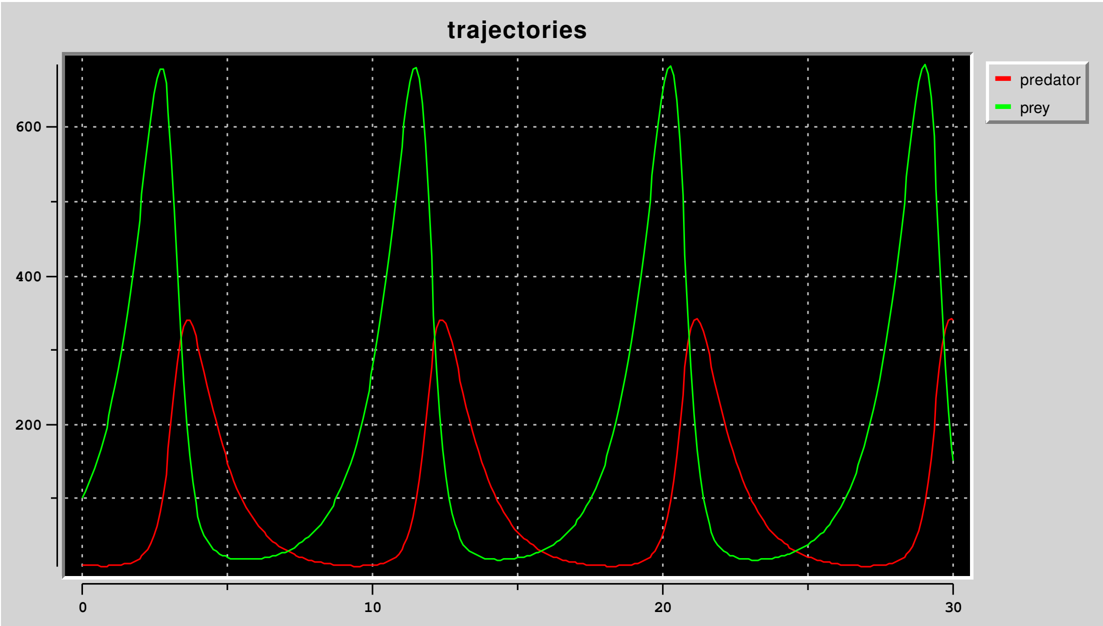
Multi-species: Predator-Prey

- Individual species behaviour + *interactions*
- Proportional to species, no interaction when one is extinct: *product* interaction $P_{pred} \times P_{prey}$

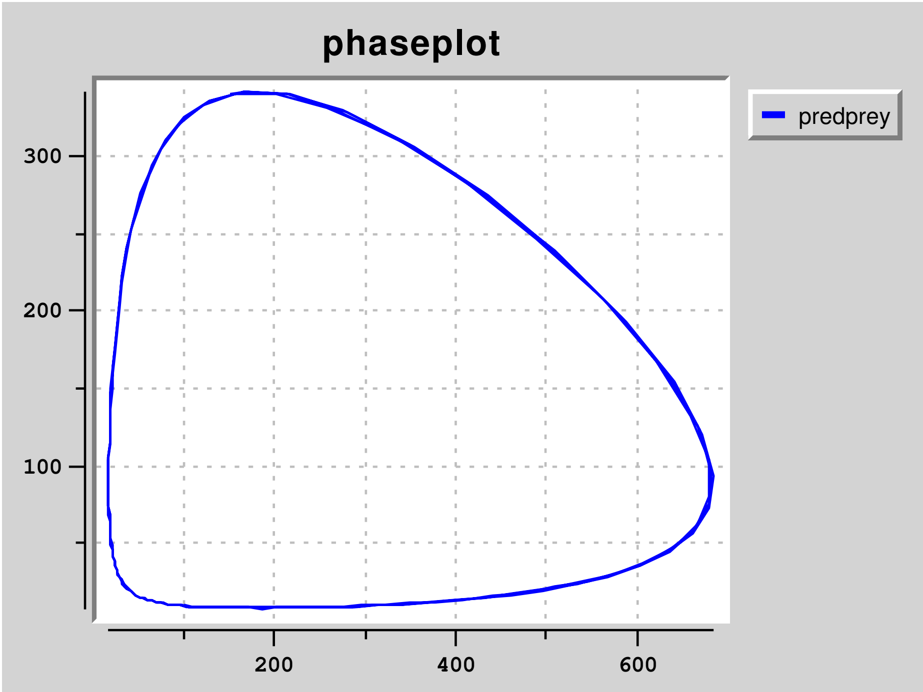
$$\begin{aligned}
\frac{dP_{pred}}{dt} &= -a \times P_{pred} + k \times b \times P_{pred} \times P_{prey} \\
\frac{dP_{prey}}{dt} &= c \times P_{prey} - b \times P_{pred} \times P_{prey}
\end{aligned}$$

- Excess death rate $a > 0$, excess birth rate $c > 0$, grazing factor $b > 0$, efficiency factor $0 < k \leq 1$
- *Lotka-Volterra* equations (1956): periodic steady-state

Predator Prey (population)



Predator Prey (phase)



Competition and Cooperation

- Several species competing for the *same* food source

$$\begin{aligned} \frac{dP_1}{dt} &= a \times P_1 - b \times P_1 \times P_2 \\ \frac{dP_2}{dt} &= c \times P_2 - d \times P_1 \times P_2 \end{aligned}$$

- Cooperation of different species (symbiosis)

$$\begin{aligned} \frac{dP_1}{dt} &= -a \times P_1 + b \times P_1 \times P_2 \\ \frac{dP_2}{dt} &= -c \times P_2 + d \times P_1 \times P_2 \end{aligned}$$

Grouping and general n-species Interaction

- Grouping (opposite of crowding)

$$\frac{dP}{dt} = -a \times P + b \times P^2$$

- n-species interaction

$$\frac{dP_i}{dt} = (a_i + \sum_{j=1}^{n} b_{ij} \times P_j) \times P_i, \forall i \in \{1, \ldots, n\}$$

- Only *binary* interactions,
  no $P_1 \times P_2 \times P_3$ interactions

Forrester System Dynamics

- based on observation + physical insight
- semi-physical, semi-inductive *methodology*

Methodology

1. levels/stocks and rates/flows

Level	Inflow	Outflow
population	birth rate	death rate
inventory	shipments	sales
money	income	expenses

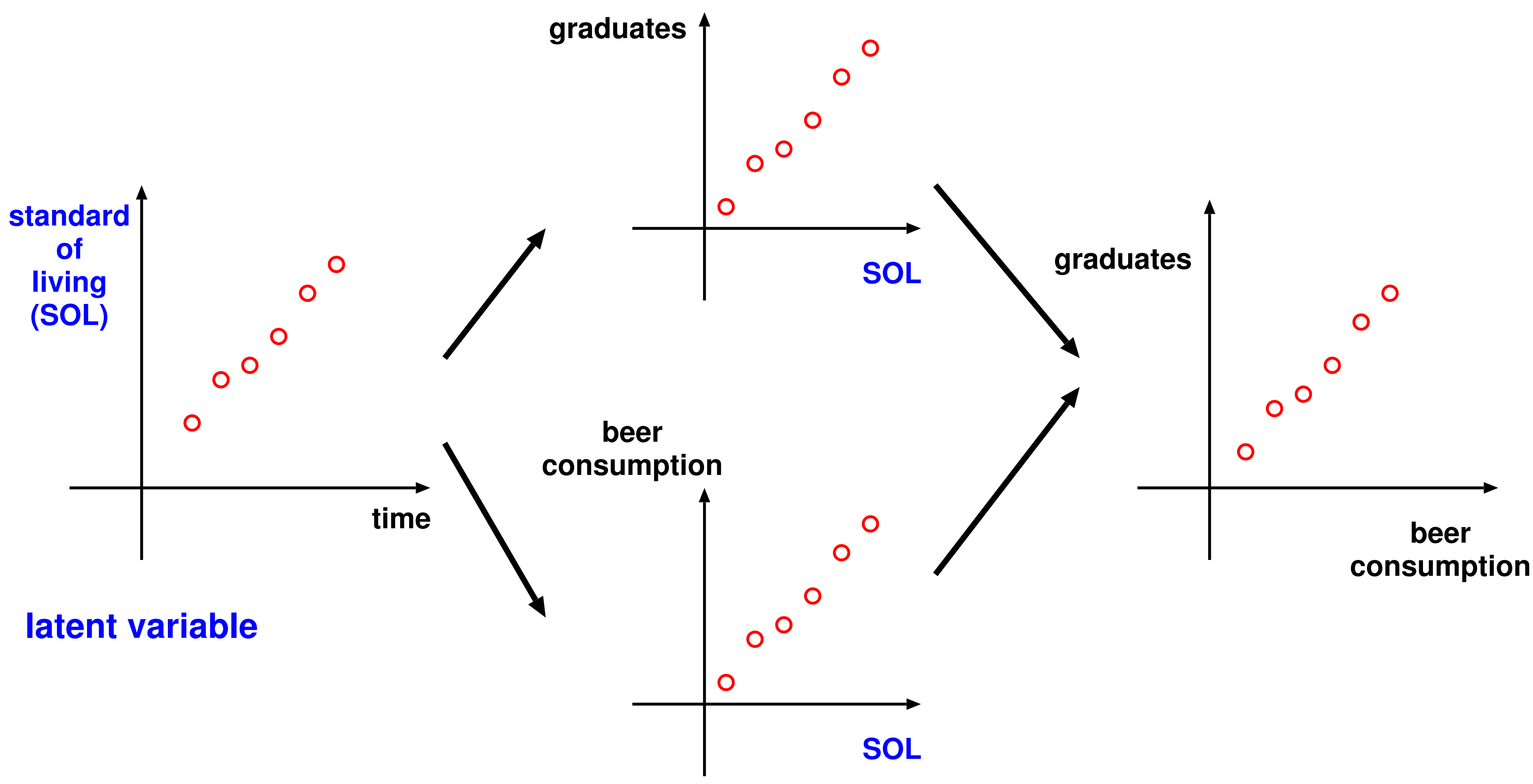
2. laundry list: levels, rates, and causal relationships
   birth rate $\rightarrow$ birth $\rightarrow$ population

3. Influence Diagram (+ and -)

4. Structure Diagram

$$\frac{dP}{dt} = BR - DR$$

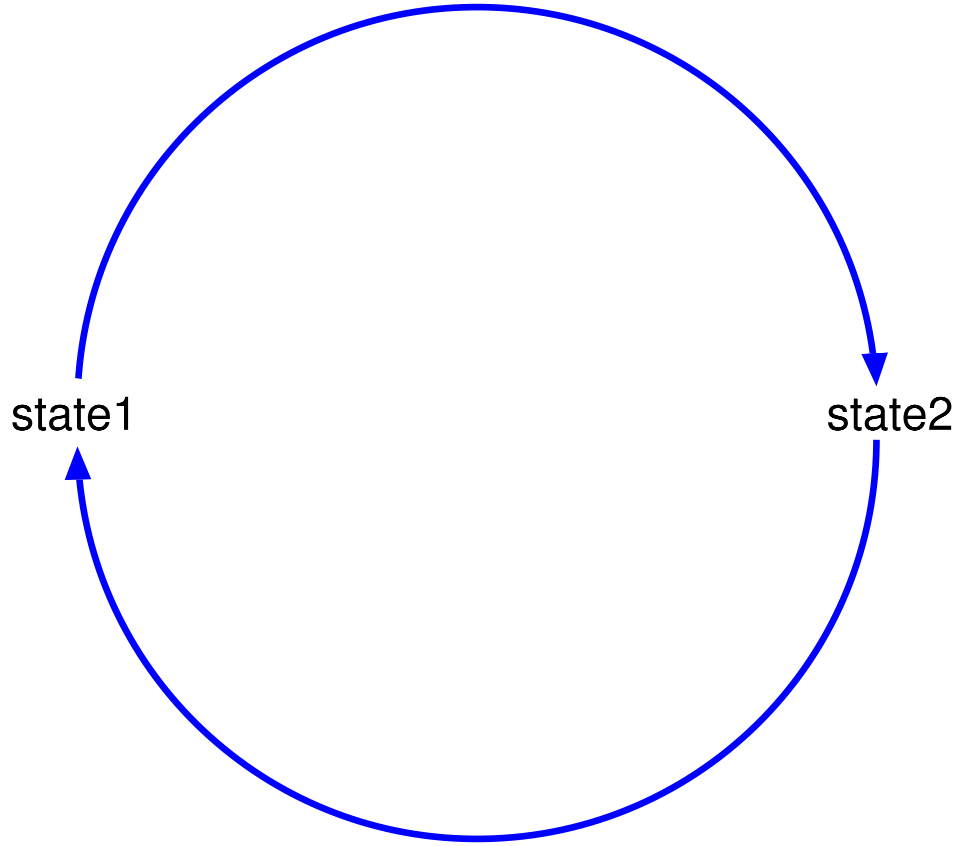
Causal Relationships



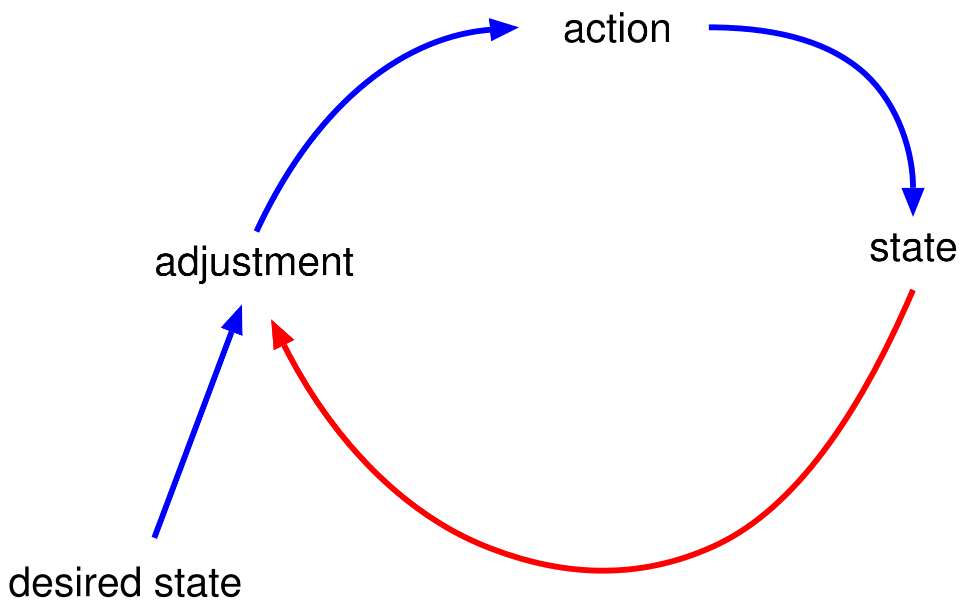
Archetypes

- Bellinger `http://www.outsights.com/systems/`
- influence diagrams
- Common combinations of reinforcing and balancing structures

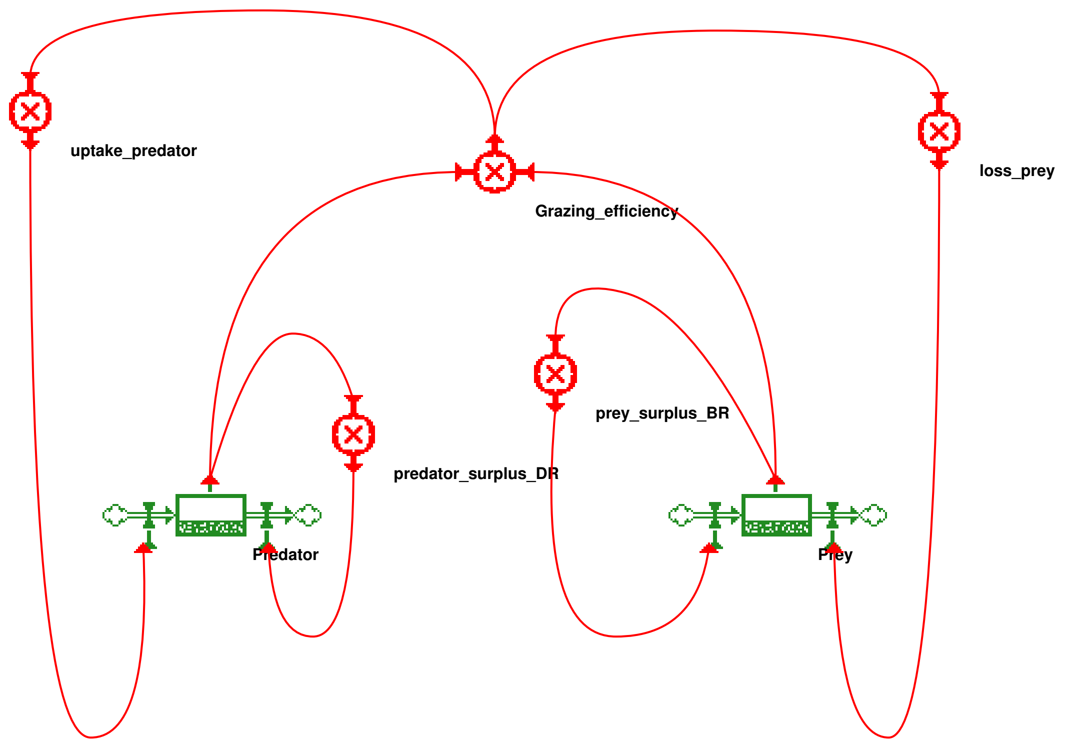
Archetypes: Reinforcing Loop



Archetypes: Balancing Loop



Forrester System Dynamics



2–species predator–prey system

Inductive Modelling: World Dynamics

- BR: BirthRate
- P: Population
- POL: Pollution
- MSL: Mean Standard of Living
- $\ldots$

Inductive Modelling: Structure Characterization

$$BR = f(P, POL, MSL, \ldots)$$

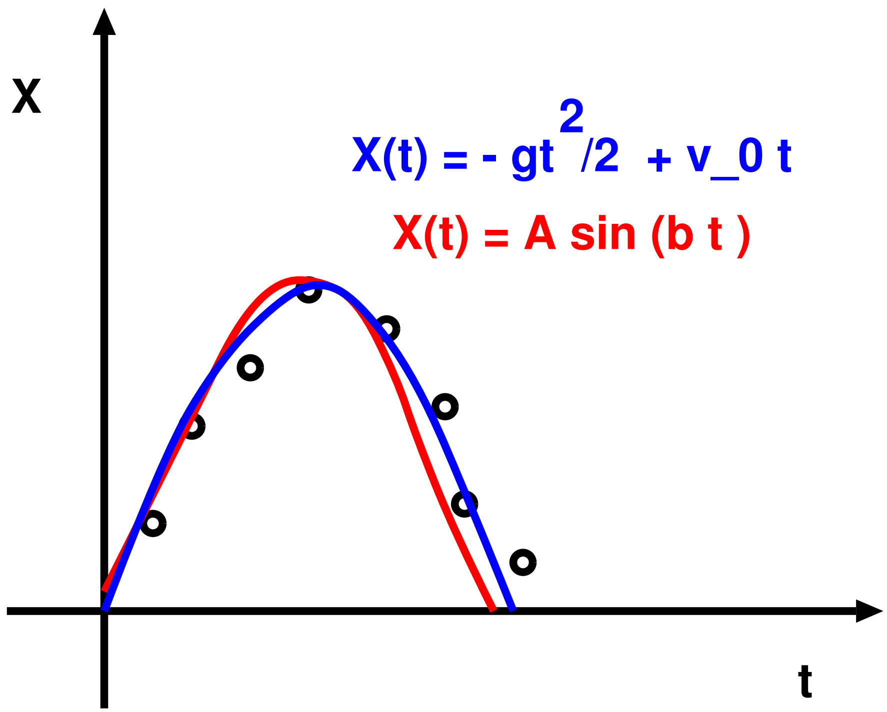
$$BR = BRN \times f^{(1)}(P, POL, MSL, \ldots)$$

$$BR = BRN \times P \times f^{(2)}(POL, MSL, \ldots)$$

$$BR = BRN \times P \times f^{(3)}(POL) \times f^{(4)}(MSL) \ldots$$

- $f^{(3)}(POL)$ inversely proportional
- $f^{(4)}(MSL)$ proportional
- compartmentalize to find correllations
- $\ldots$ Structure Characterization !

Structure Characterisation: LSQ fit

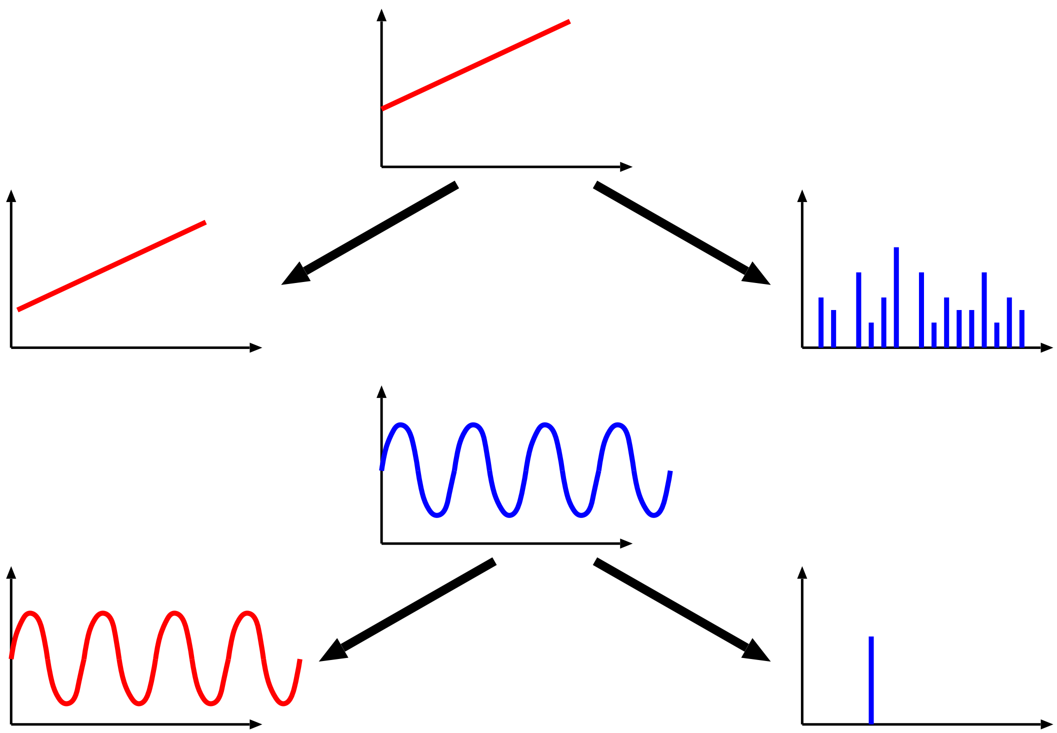


$$\text{LSQ (sin)} < \text{LSQ}(t^2)$$

Feature Extraction

1. Measurement data *and* model candidates
2. Structure selection and validation
3. Parameter estimation
4. Model use

Feature Rationale



Minimum Sensitivity to Noise

Maximum Discriminating Power

Throwing Stones

Candidate Models

1. $x = -\frac{1}{2}gt^2 + v_0 t$
2. $x = A sin(bt)$

Feature 1 (quadratic model)

$$g_i = \frac{2x_i}{t_i^2} - \frac{2\dot{x}_i}{t_i}, i = A, B$$

$$F1 = g_A / g_B$$

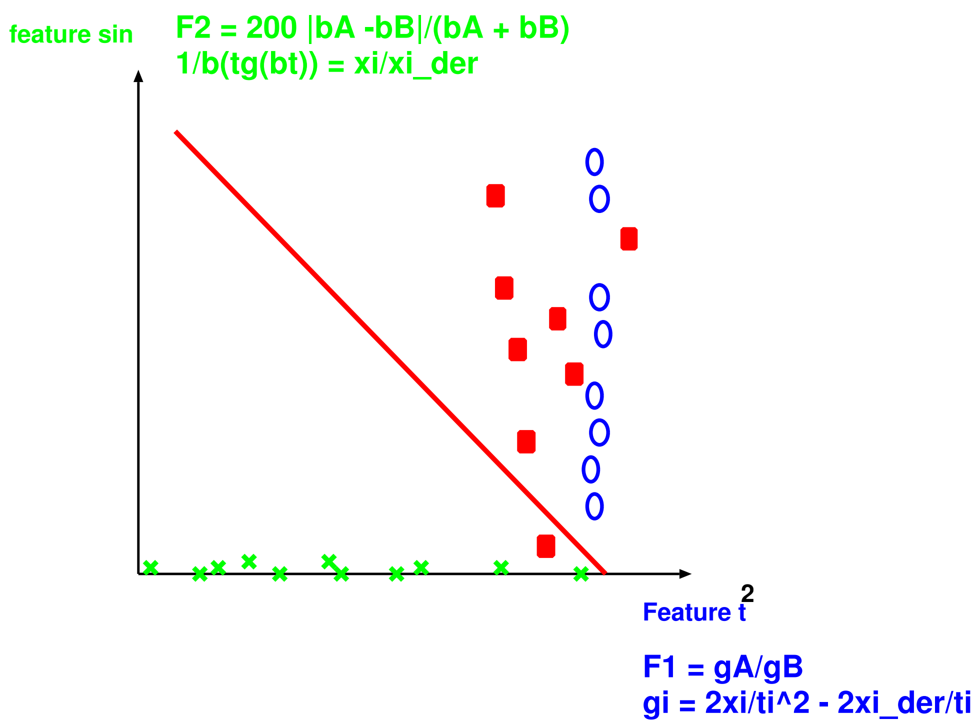
Feature 2 (sin model)

$$\frac{1}{b} tg(bt) = \frac{x_i}{\dot{x}_i}$$

solve numerically for b

$$F2 = 200 \frac{|b_A - b_B|}{b_A + b_B}$$

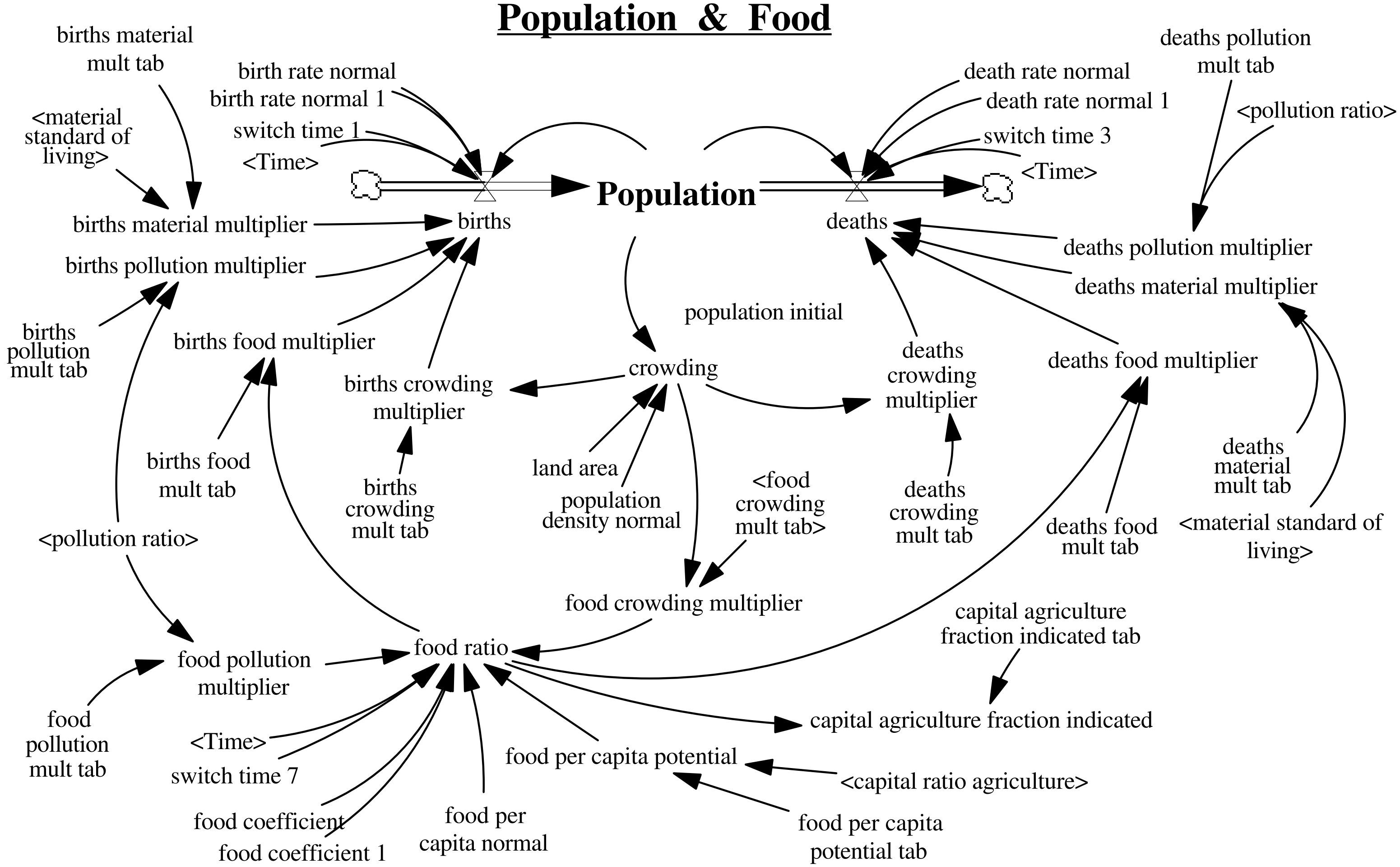
Feature Space Classification



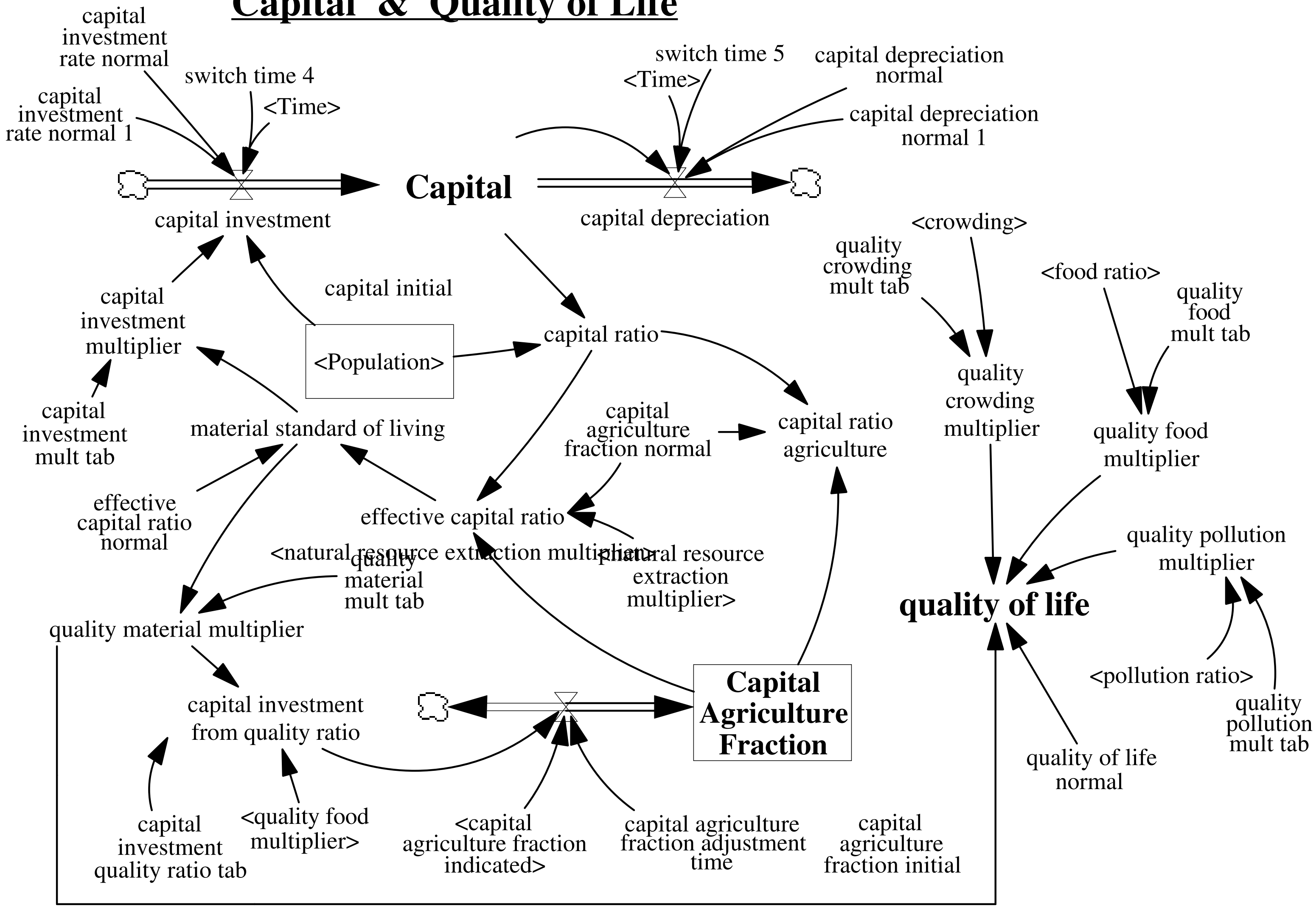
Forrester's World Dynamics model

- "Club of Rome" World Dynamics model
- Few "levels", note the depletion of natural resources
- implemented in `Vensim PLE` (www.vensim.com)

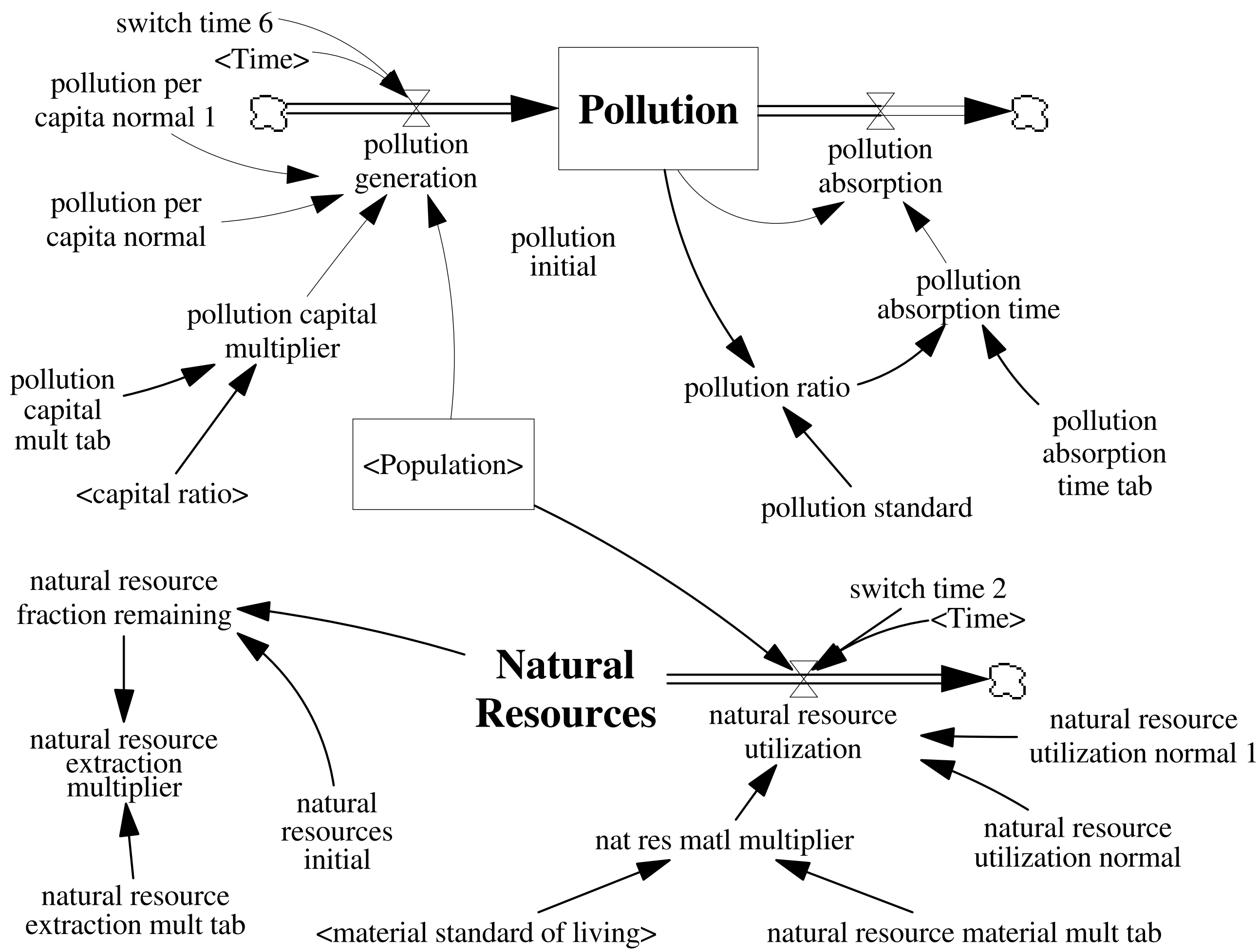
Population & Food



Capital & Quality of Life



Pollution & Natural Resources



World Model Results