Overview

- Untimed models of discrete event systems
- Languages
- Regular Expressions
- Automata
  - Finite State Automata
  - Nondeterministic Finite State Automata
  - State Aggregation
  - Discrete Event Systems as State Automata

Untimed models

- Level of specification: I/O System (state based, deterministic)
- Time Base = $\mathbb{N}$ (time $=$ progression index)
- Dynamic but
  - only *sequence* (order) of states traversed matters
  - not *when* in state or *how long* in state
- Discrete Event: event set E

Languages – Regular Expressions – Automata

- *language* L, defined over alphabet E (events) $\equiv$
  set of *strings* formed from E

- Example: all possible input behaviours:

$$L = \{\varepsilon, ARR, DEP, ARR\ ARR\ DEP, \ldots\}$$

- Regular expression: shorthand notation for a regular language

$$ARR\ DEP, ARR * DEP*, (DEP + ARR)*$$

  Concatenation ($+$), Kleene closure ($*$).

- Finite State Automaton (model): *generate*/*accept* a language

Finite State Automaton

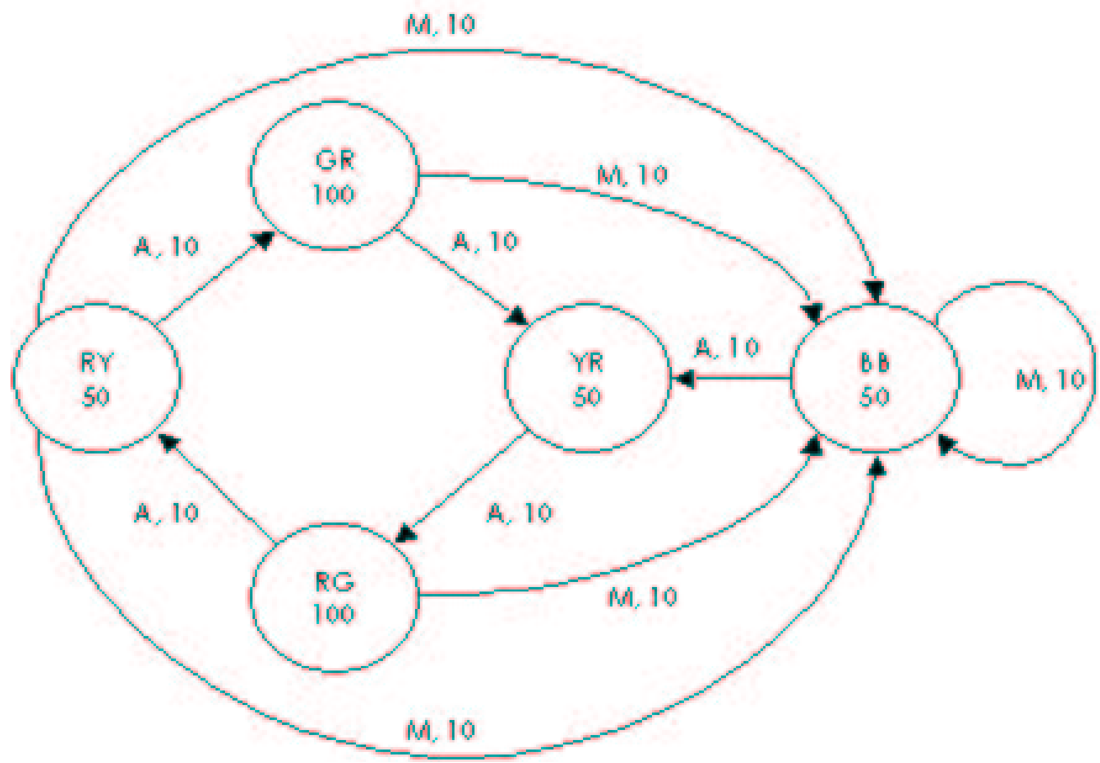
$$(E, X, f, x_0, F)$$

- E is a finite alphabet
- X is a finite state set
- f is a state transition function,
  $f : X \times E \rightarrow X$
- x_0 is an initial state, $x_0 \in X$
- F is the set of final states

Dynamics (x' is next state):

$$x' = f(x, e)$$

FSA graphical notation: State Transition Diagram



FSA recognizes Language

- *extended* transition function:

$$f : X \times E* \rightarrow X$$

$$f(x, ue) = f(f(x, u), e)$$

- A string u over the alphabet E is recognized by a FSA (E, X, f, x_0, F) if $f(x_0, u) = x$ where $x \in F$.

- The language recognized by a FSA $A = (E, X, f, x_0, F)$ is the set of strings $\{u : f(x_0, u) \in F\}$.
  The language recognized by A is $L(A)$.

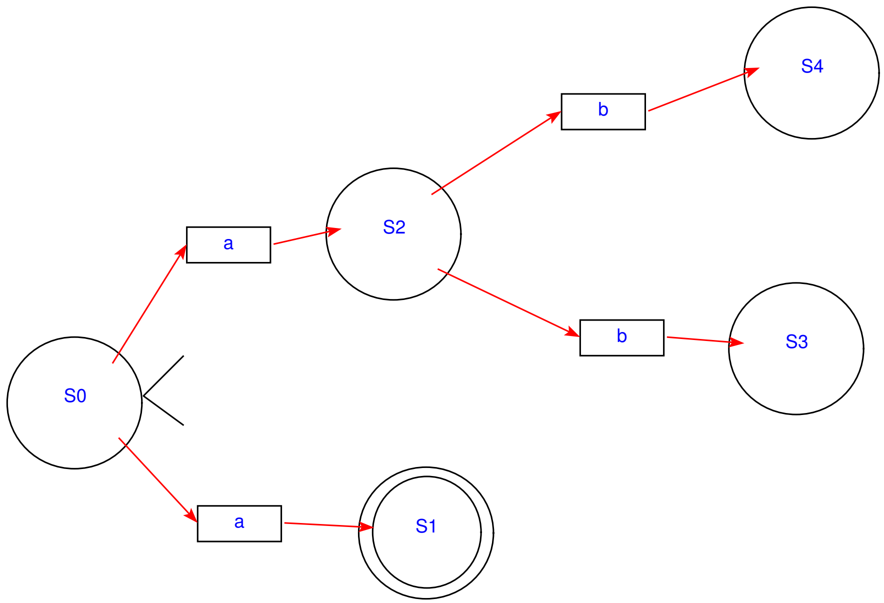
Nondeterministic Finite State Automaton

$$NFA = (E, X, f, x_0, F)$$

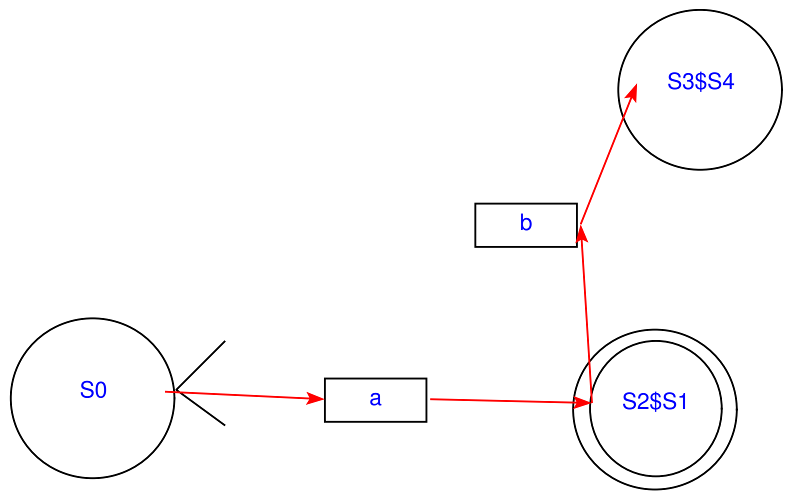
$$f : X \times E \rightarrow 2^X$$

- Monte Carlo simulation (if probabilities added)
- Transform to equivalent FSA (*aka* DFA)

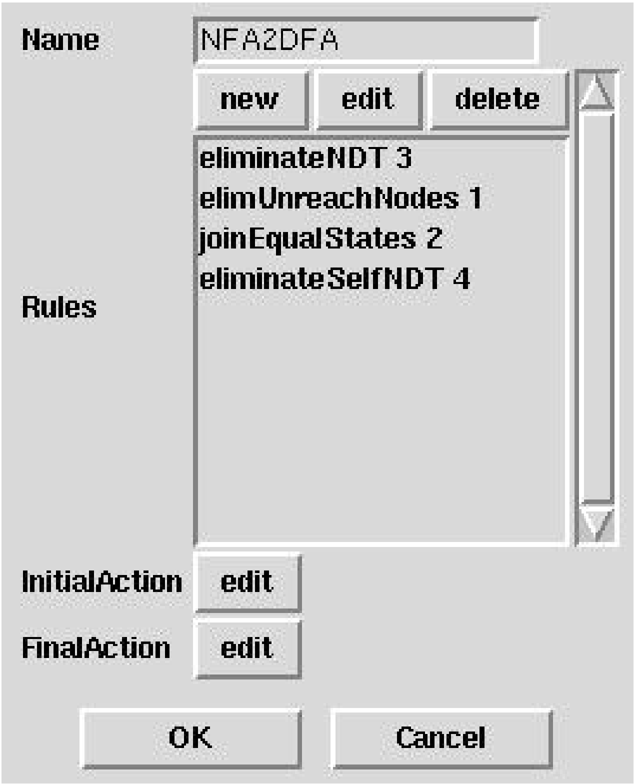
Nondeterministic Finite State Automaton



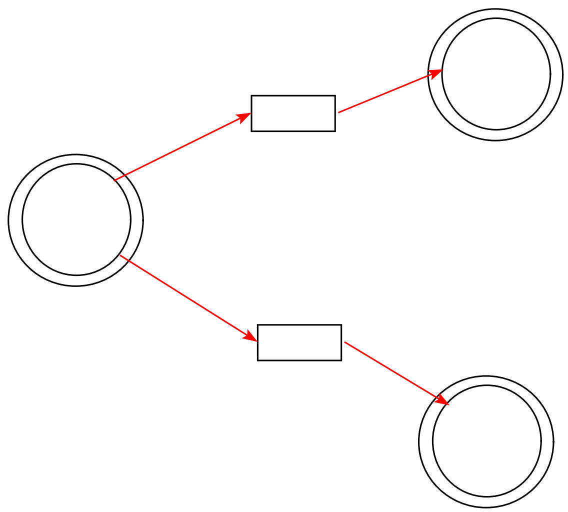
Constructed Deterministic Finite State Automaton



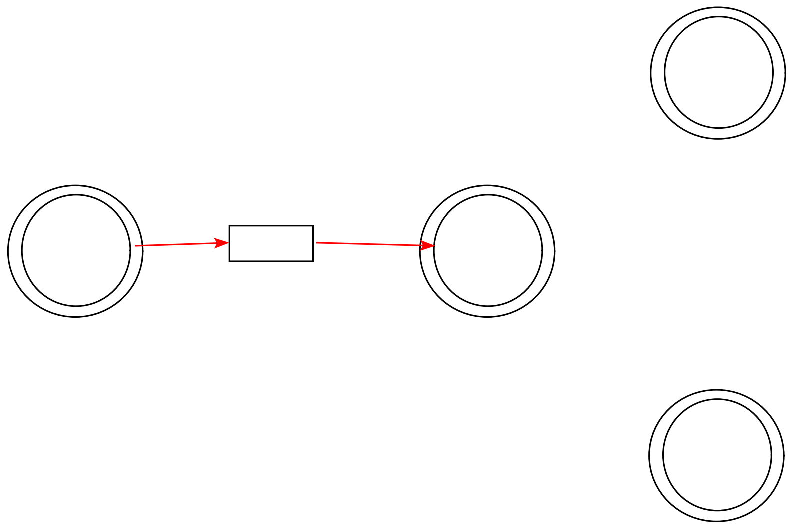
Transformation Rules



Rule LHS



Rule RHS



Managing Complexity: State Aggregation

$$(E, X, f, x_0, F)$$

$$R \subseteq X$$

R consists of *equivalent states with respect to F*
if for any $x, y \in R, x \neq y$ and any string u,

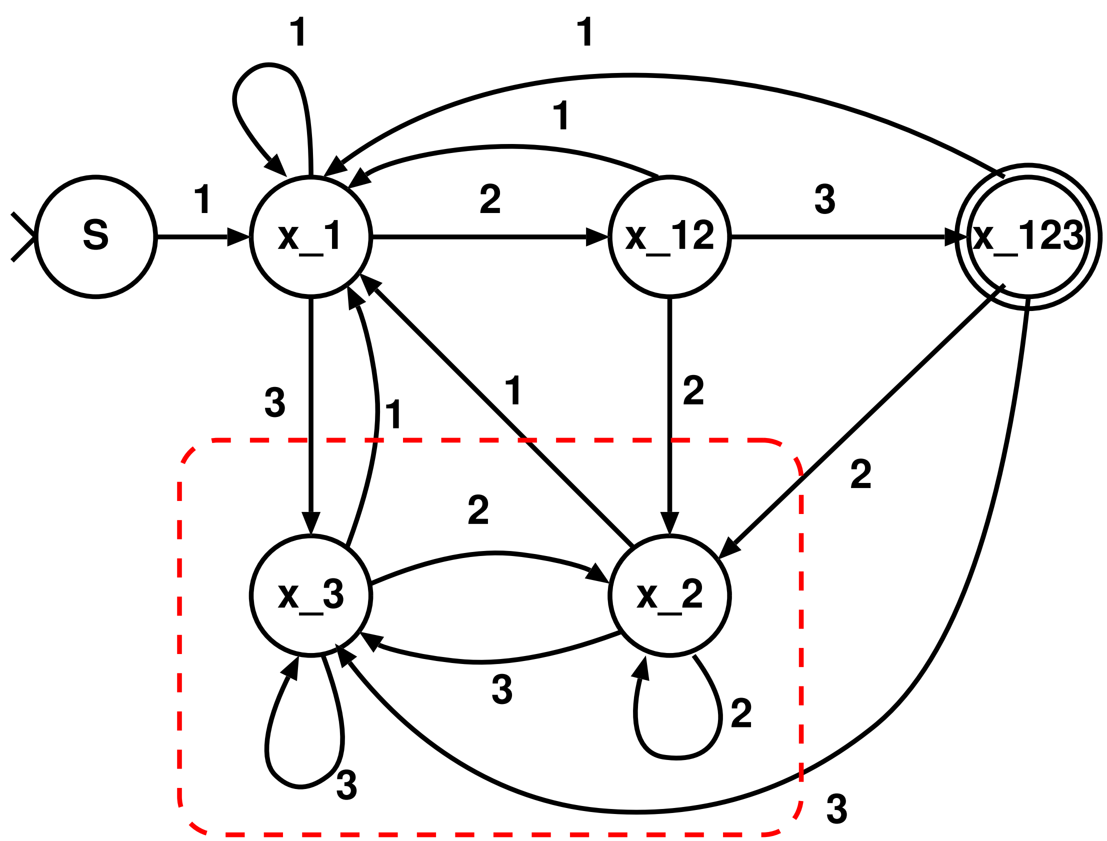
$$f(x, u) \in F \Leftrightarrow f(y, u) \in F$$

State Aggregation Algorithm

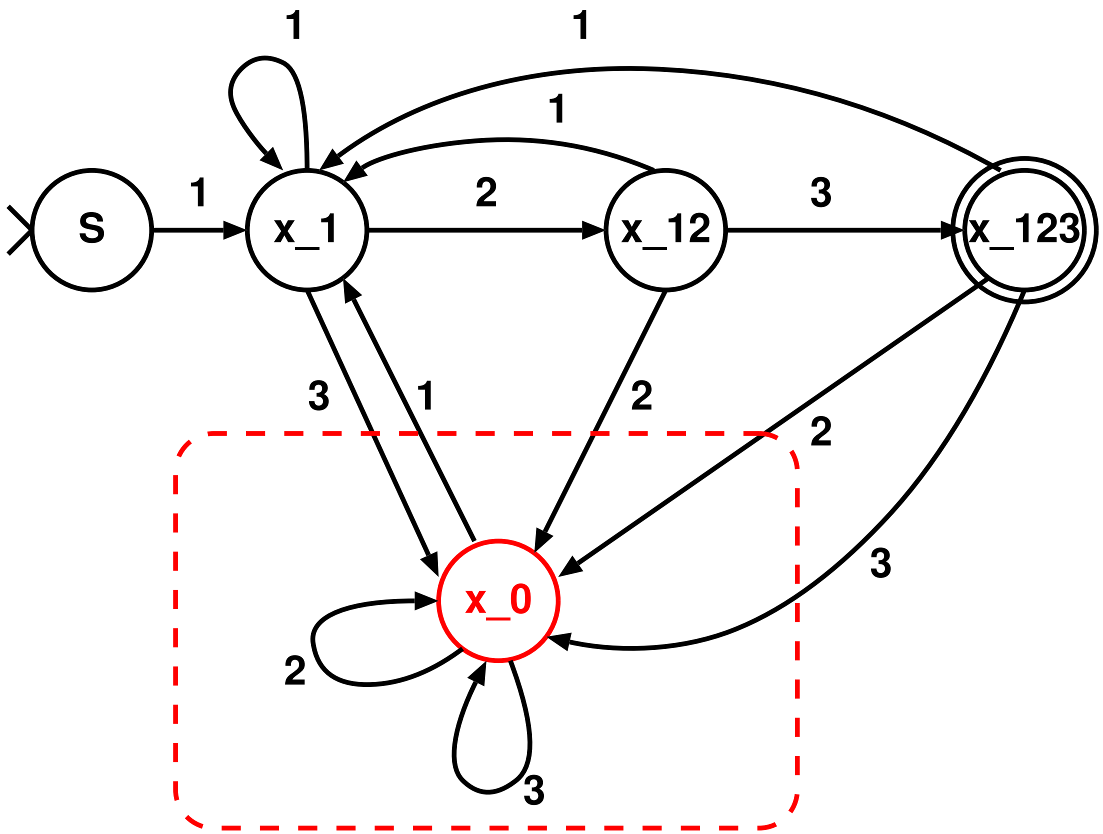
1. Mark (x, y) for all $x \in F, y \notin F$
2. For every pair (x, y) not marked in previous step:
   (a) If $(f(x, e), f(y, e))$ is marked for some $e \in E$, then:
      i. Mark (x, y)
      ii. Mark all unmarked pairs (w, z) in the list of (x, y). Repeat this step for each (w, z) until no more markings possible.
   (b) If no $(f(x, e), f(y, e))$ is marked, the for every $e \in E$:
      i. If $f(x, e) \neq f(y, e)$ then add (x, y) to the list of $f(x, e) \neq f(y, e)$

Pair which remain unmarked are in *equivalence set*

digit sequence (123) detector FSA



State Reduced FSA



State Automata to model Discrete Event Systems

- X is state space
- All inputs are strings from an alphabet E (the events)
- State transition function $x' = f(x, e)$
- Allow X and E to be *countable* rather than finite
- Introduce *feasible events*

State Automaton

$$(E, X, \Gamma, f, x_0)$$

- E is a countable event set
- X is a countable state space
- $\Gamma(x)$ is the set of feasible or enabled events
  $x \in X, \Gamma(x) \in E$
- f is a state transition function,
  $f : X \times E \to X$, only defined for $e \in \Gamma(x)$
- x_0 is an initial state, $x_0 \in X$

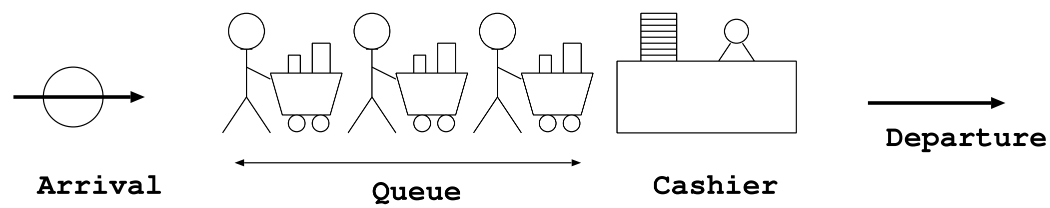
$$(E, X, \Gamma, f)$$

omits x_0 and describes a class of State Automata.

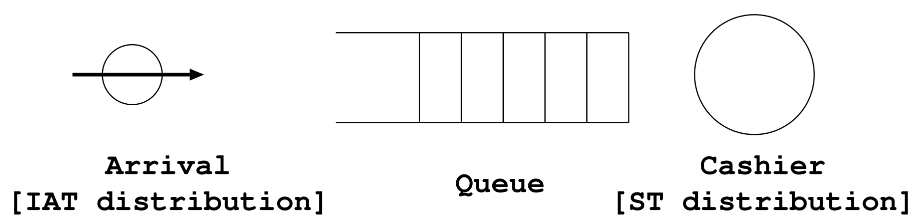
Feasible/Enabled Events

- On transition diagram: not feasibe $\Rightarrow$ not marked
- Meaning: *ignore* non-feasible events
- Why not $f(x,e) = x$ for non-feasible events ?

State Automata for Queueing Systems

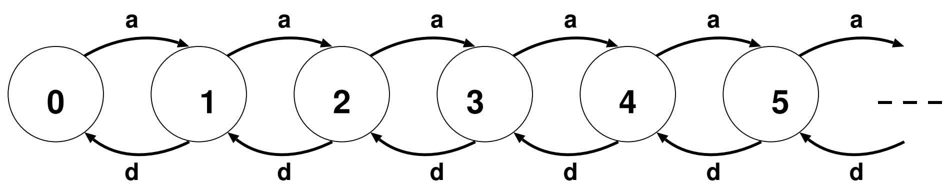


Physical View



Abstract View

State Automata for Queueing Systems: customer centered



$$E = \{a, d\}$$

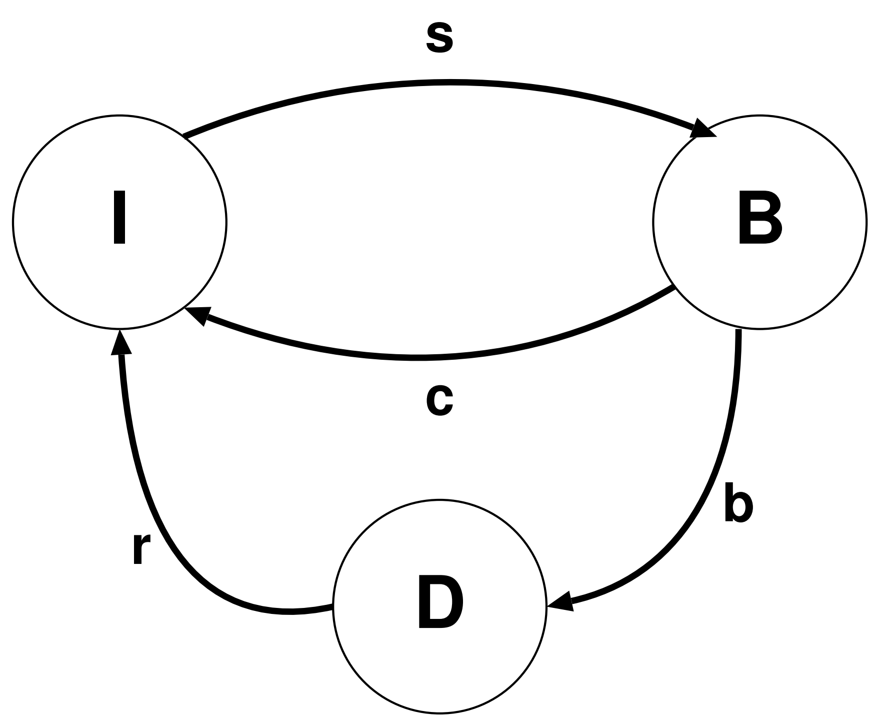
$$X = \{0, 1, 2, \ldots\}$$

$$\Gamma(x) = \{a, d\}, \forall x > 0, \Gamma(0) = \{a\}$$

$$f(x, a) = x + 1, \forall x \geq 0$$

$$f(x, d) = x - 1, \forall x > 0$$

State Automata for Queueing Systems: server centered (with breakdown)



State Automata for Queueing Systems: server centered (with breakdown)

$$E = \{s, c, b, r\}$$

Events: s denotes service starts, c denotes service completes, b denotes breakdown, r denotes repair.

$$X = \{I, B, D\}$$

State: I denotes idle, B denotes busy, D denotes broken down.

$$\Gamma(I) = \{s\}, \Gamma(B) = \{c, b\}, \Gamma(D) = \{r\}$$

$$f(I, s) = B, f(B, c) = I, f(B, b) = D, f(D, r) = I$$

Interpretations/Uses

- *Generate* all possible behaviours.
- *Accept* all allowed input sequences $\Rightarrow$ code generation.
- *Verification* of properties.

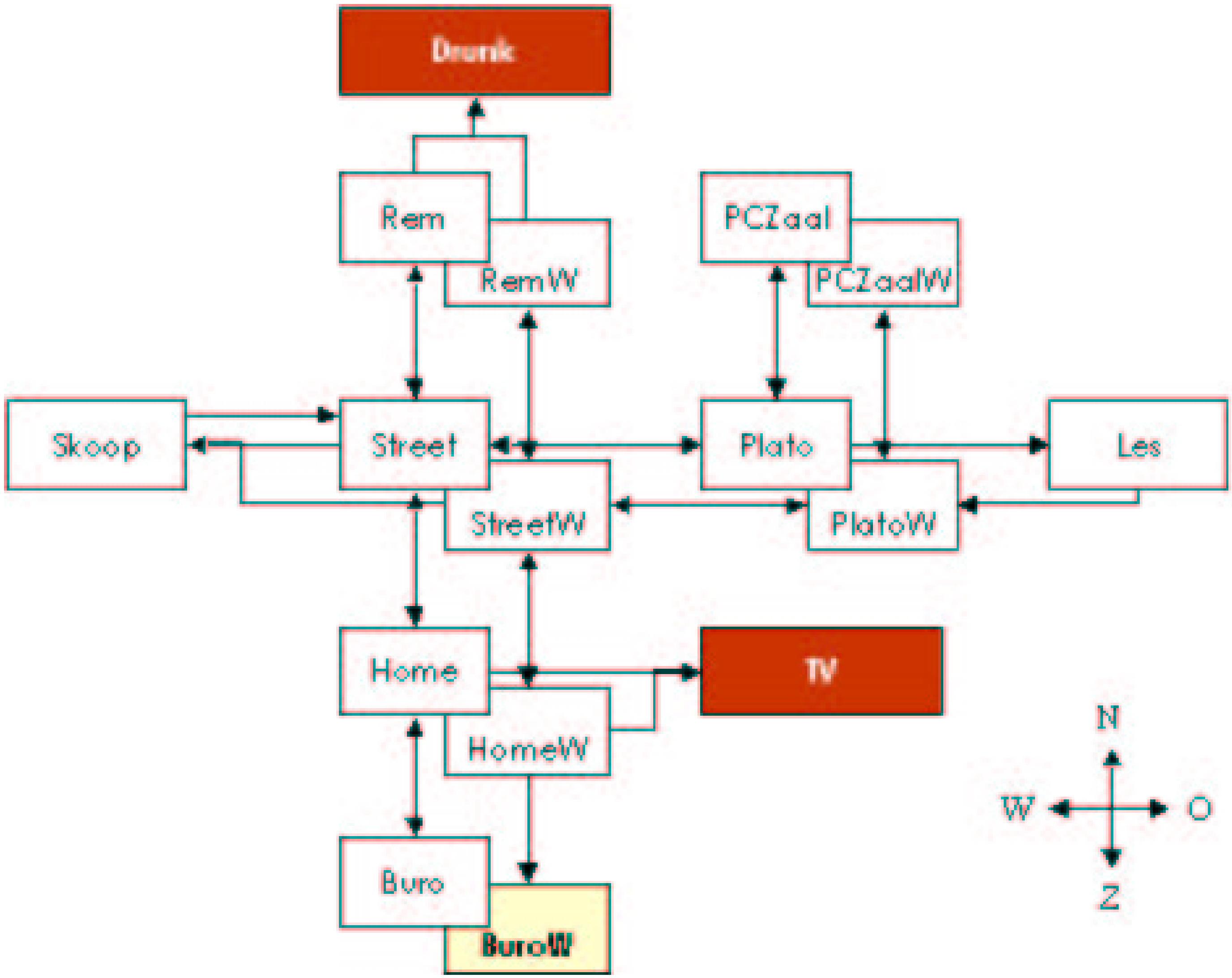
State Automata with Output

$$(E, X, \Gamma, f, x_0, Y, g)$$

- Y is a countable output set,
- g is an output function

$$g : X \times E \to Y, e \in \Gamma(x)$$

State Automata for Adventure Games



State Automata (later: StateCharts) for Graphical User Interface Specification

Limitiations/extensions of State Automata

- Adding time ?
- Hierarchical modelling ?
- Concurrency by means of $\times$
- States are represented explicitly
- Specifying control logic, synchronisation ?