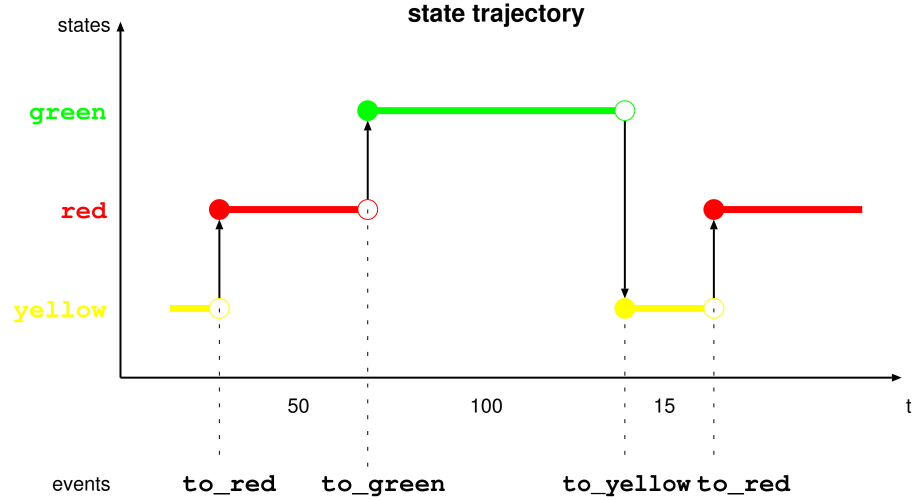
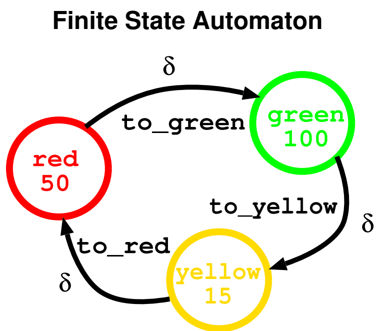
Timed Discrete Event Modelling and Simulation

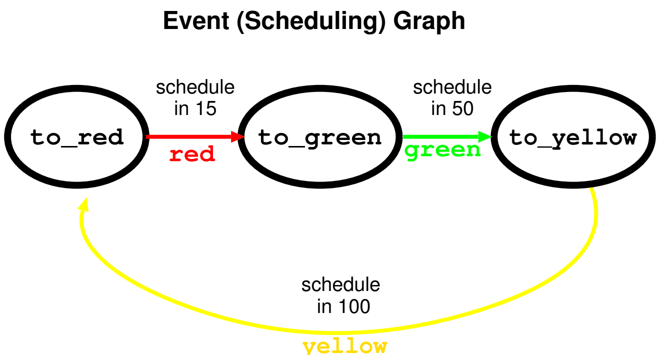
- extend State Automata with “time in state”
- equivalent to Event Graphs “time to transition”

$\Rightarrow$ *schedule events*

(timed) Discrete Event Models







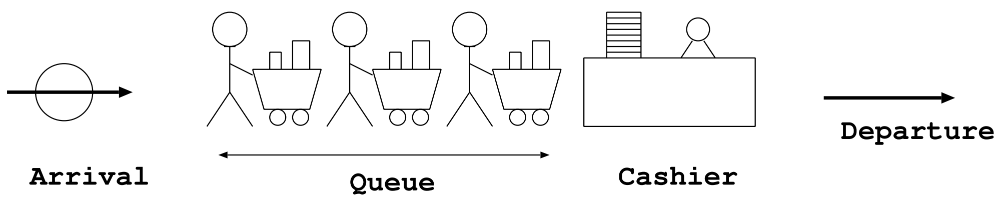
Discrete Event Modelling and Simulation

- Model : objects and relationships among objects
- Object : characterized by attributes to which values can be assigned
- Attributes:
  - indicative
  - relational
- Values: of a type

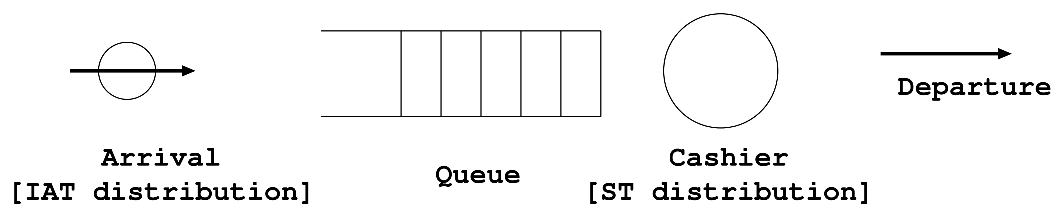
Time and State Relationships

- Indexing Attribute: enables state transitions
  Time is most common.
- Instant: value of System Time at which the value of at least one attribute of an object can be assigned.
- Interval: duration between two successive instants.
- Span: contiguous succession of one or more intervals.
- State of an object: enumeration of all attribute values at a particular instant.
- State of the system: all object states at a particular instant.

Single Server Queueing System

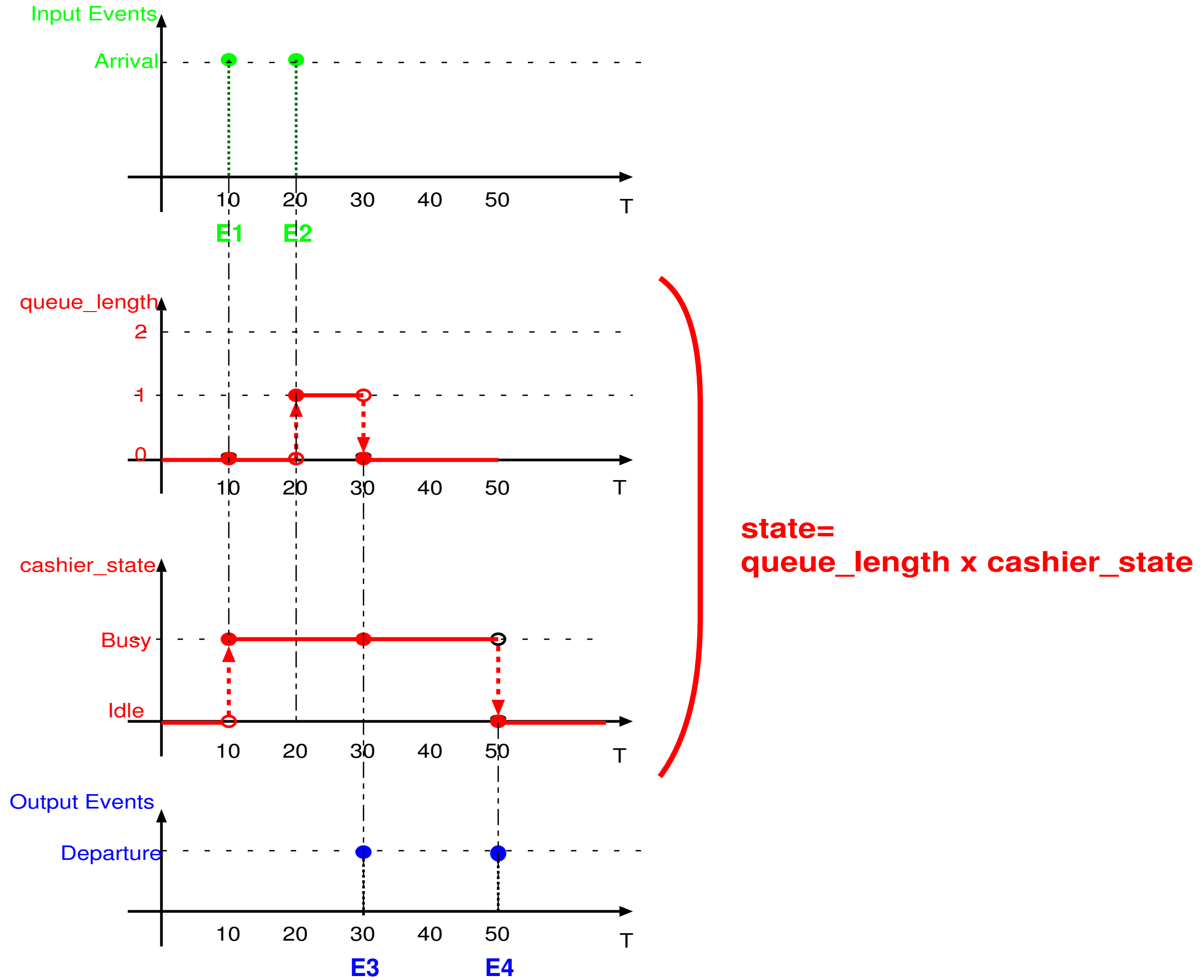


Physical View



Abstract View

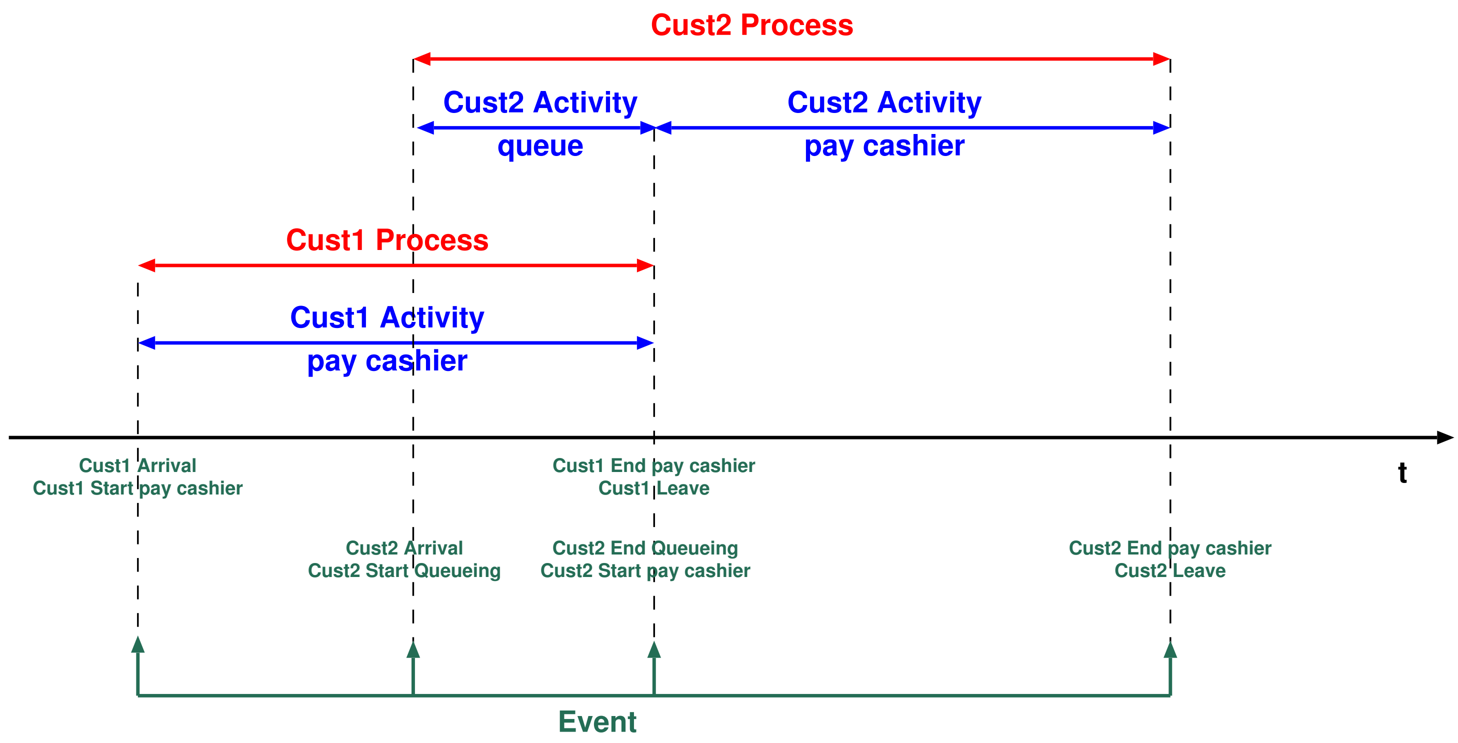
Queueing System State Trajectory



Time and State Relationships

- Activity: state of an object over an interval
- Event: change in object state, occurring at an instant.
  Initiates an activity
  - Determined: occurrence based on time (“time event”)
  - Contingent: based on system conditions (“state event”)
- Object activity: state of object between two events for that object
- Process: succession of states of object over a span

Event/Object Activity/Process



Event Scheduling

- Identify *objects* and their *attributes*
- Identify *system attributes* (global)
- Define what causes *changes* in attribute value as *event*
- Write *event routine* for each event:
  - *modify state* (attributes)
  - *schedule* event(s) at $t + \Delta t, \Delta t \geq 0$
- Priorities for *tie-breaking*
- Event scheduling logic

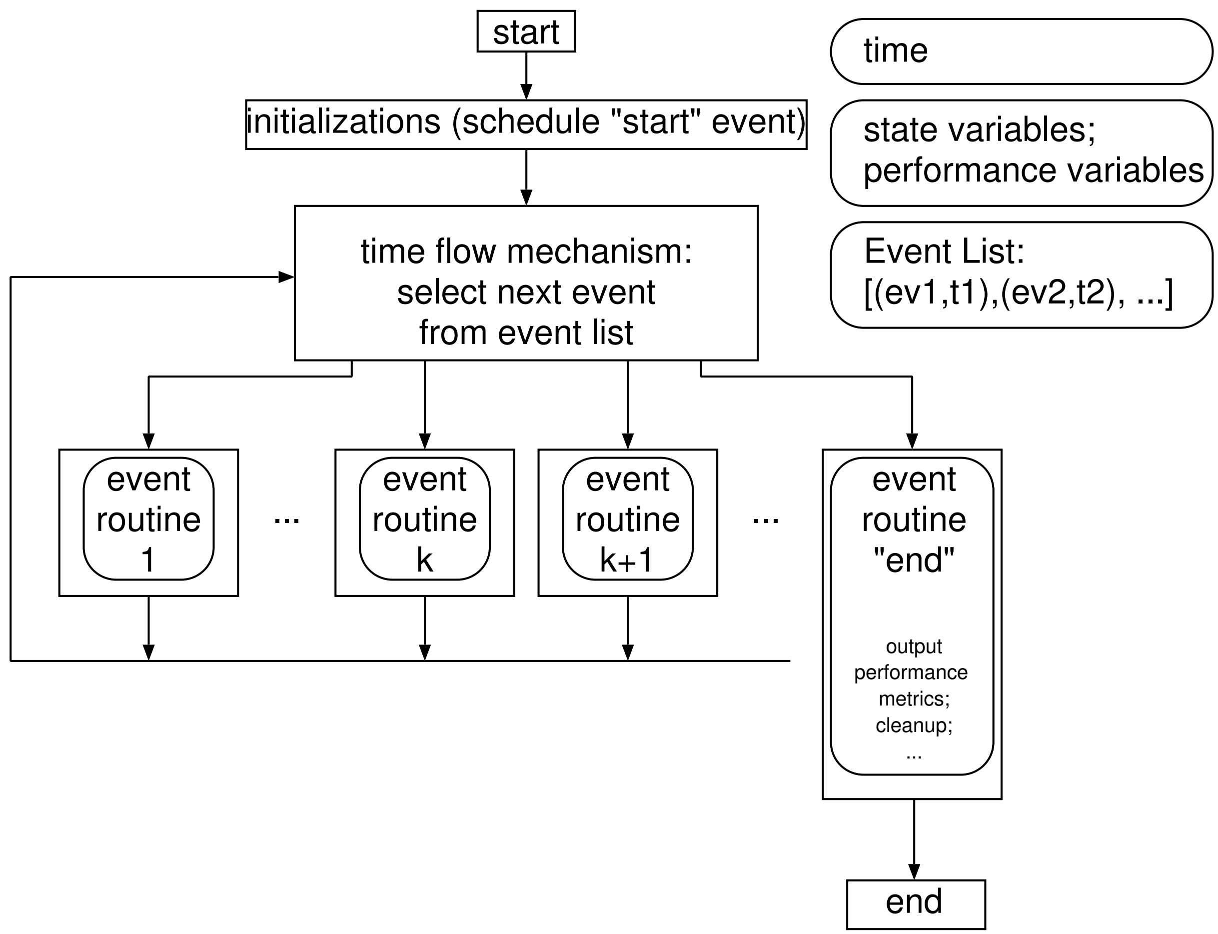
Cashier-queue Event Scheduling Model

```
declare variables:
 t              : Time
 queue_length   : PosInt
 cashier_state  : {Idle, Busy}

declare events:
 start, arrival, departure, end

define events:

 start event:
  /* scheduled first automatically by simulator */

  /* initializations */
  queue_length   = 0
  cashier_state  = Idle

  /* schedule end of simulation */
  schedule end absolute end_time
```

```
 /* schedule first arrival */
 schedule arrival relative 0

arrival event:
 schedule arrival relative Random(IATmean, IATspread)
 if (queue_length == 0)
  if (cashier_state == Idle)
   cashier_state = Busy
   schedule departure relative Random(SERVmean, SERVspread)
  else
   queue_length++
 else /* queue_length != 0 */
  queue_length++

departure event:
 if (queue_length == 0)
  cashier_state = Idle
 else /* queue_length != 0 */
  queue_length--
  schedule departure relative Random(SERVmean, SERVspread)
```

```
end event:
 /* terminates simulation */
 /* process/output performance metrics */
 print time, queue_length /* current */
 print average_queue_length
```

Event Scheduling Kernel

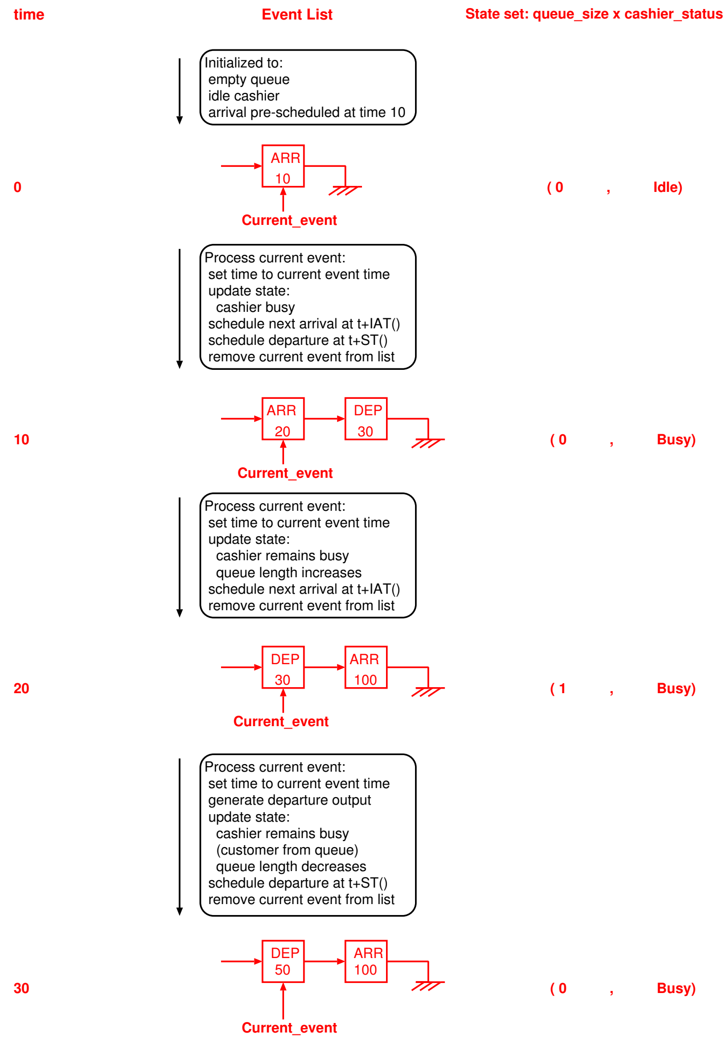


Input Generation

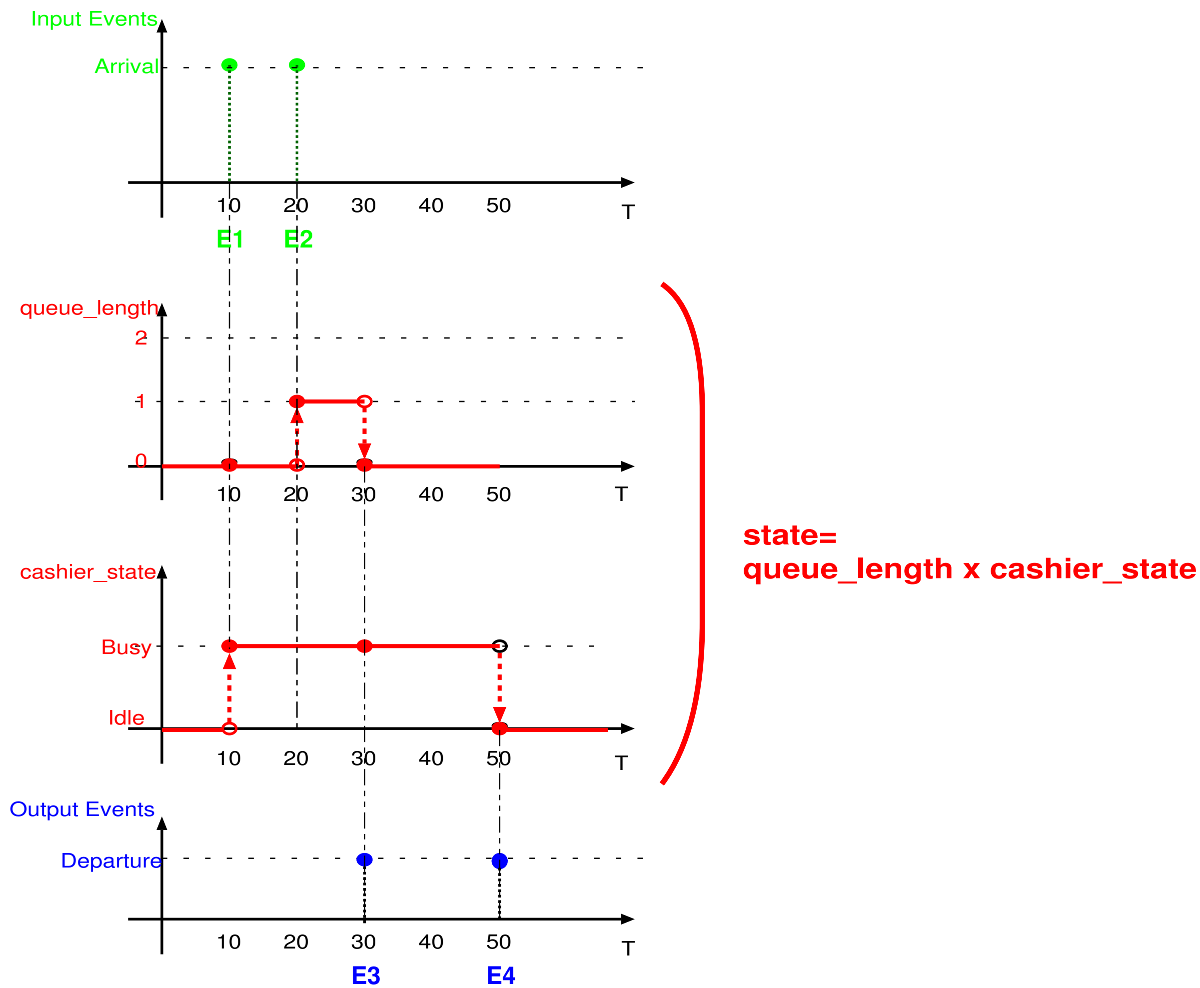
A “model” of input (sequence of Inter Arrival Times):

- Trace driven
- Auto generating (bootstrapping)

Cashier-queue Event List



Queueing System State Trajectory



Termination Conditions

- Empty Event List

  Need to stop generating arrivals after t_{end} when auto-generating arrivals

- Schedule Termination Event
  - process statistics
  - cleanup
  - stop
  - *caveat*: process *all* final events !
    * use reserved priority
    * re-schedule

- Similarly: schedule initialization/setup

Event Scheduling (dis)advantages

- advantage: run-time efficient
- disadvantage: hard to understand model

Activity Scanning (rule-based)

Activity:

- condition: must be satisfied for activity to take place.
  Becomes true *only* at event times.
- actions: operations performed when condition becomes true

Time-advance mechanism:

- fixed time-step

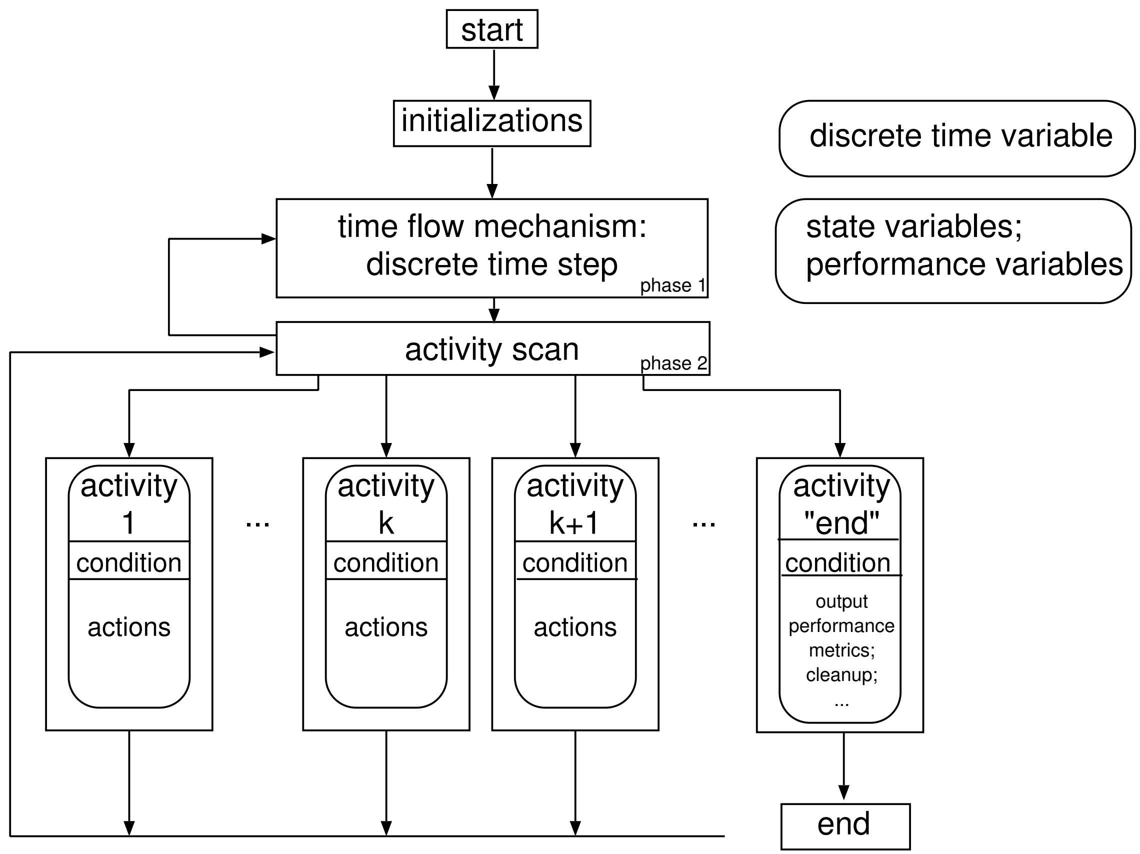
Also known as Two Phase Approach

Cashier-queue Activity Scanning Model

```
declare (and initialize) variables:
 t             : Time
 queue_length  : PosInt = 0
 cashier_state : {Idle, Busy} = Idle
 t_arrival     : Time = 0
 t_depart      : Time = plusInf

declare activities:
 queue_pay, depart, end

queue_pay activity
condition: t >= t_arrival
actions:
 if (queue_length == 0)
  if (cashier_state == Idle)
   keep queue_length == 0
   cashier_state = Busy
   t_depart = t + Random(SERVmean, SERVspread) /* service time */
  else
   queue_length++
```

```
 else /* queue_length != 0 */
  queue_length++, keep cashier_state == Busy
 t_arrival = t + Random(IATmean, IATspread)   /* inter arrival time */

depart activity
condition: t >= t_departure
actions:
 if (queue_length == 0)
  cashier_state = Idle
 else /* queue_length != 0 */
  queue_length--, keep cashier_state == Busy
  t_depart = t + Random(SERVmean, SERVspread) /* service time */

end activity
condition: t >= t_end
actions:
 print t, queue_length /* current */
 print avg_queue_length /* performance metric */
```

Activity Scanning



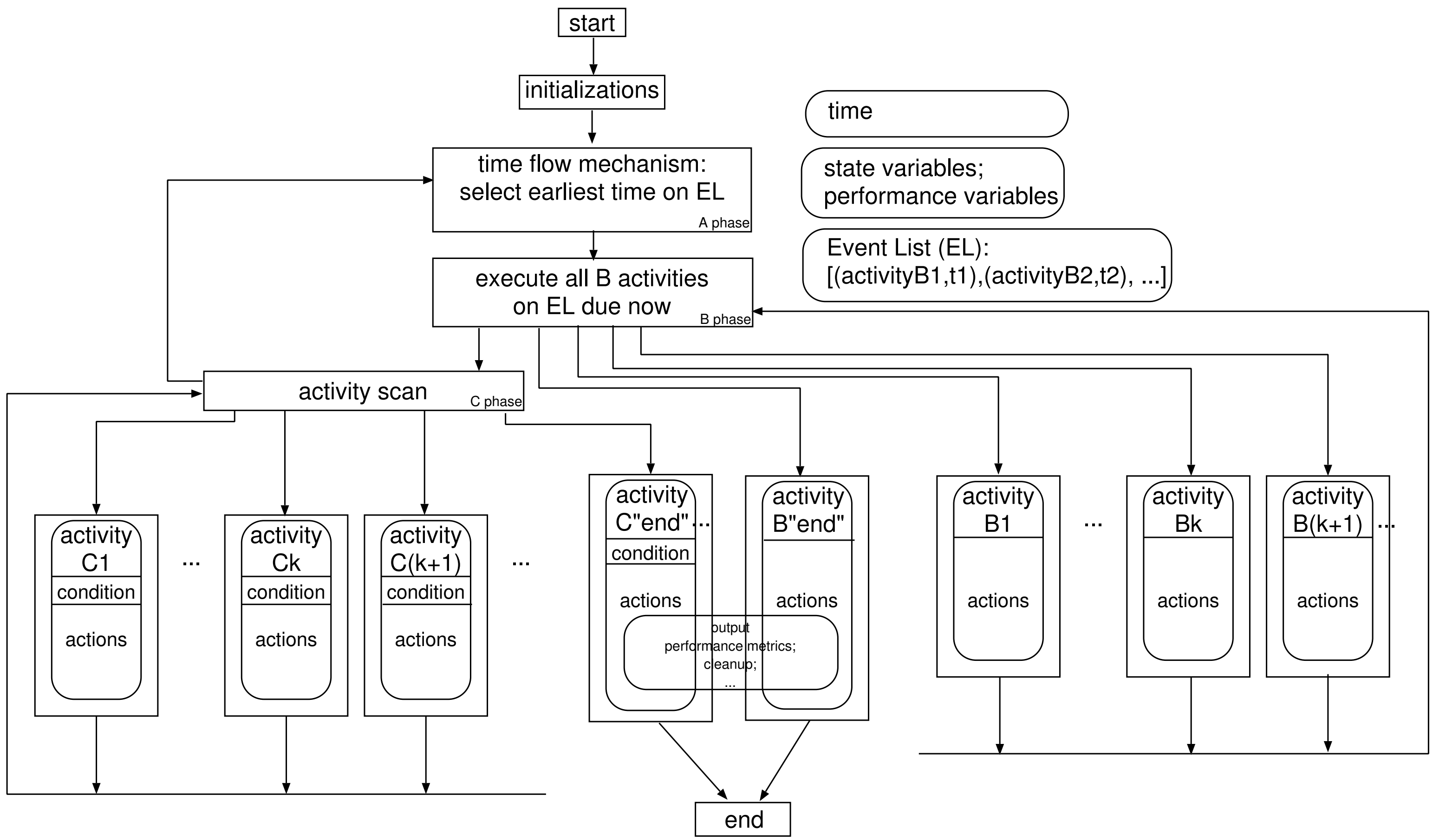
Activity Scanning (dis)advantages

- advantage: declarative model
- disadvantages:
  - inaccurate if changes occur in between time-steps
  - run-time inefficient (fixed time-step)

Three Phase Approach

- Bound to occur activities: unconditional state changes. Pre-scheduled.
- Conditional activites

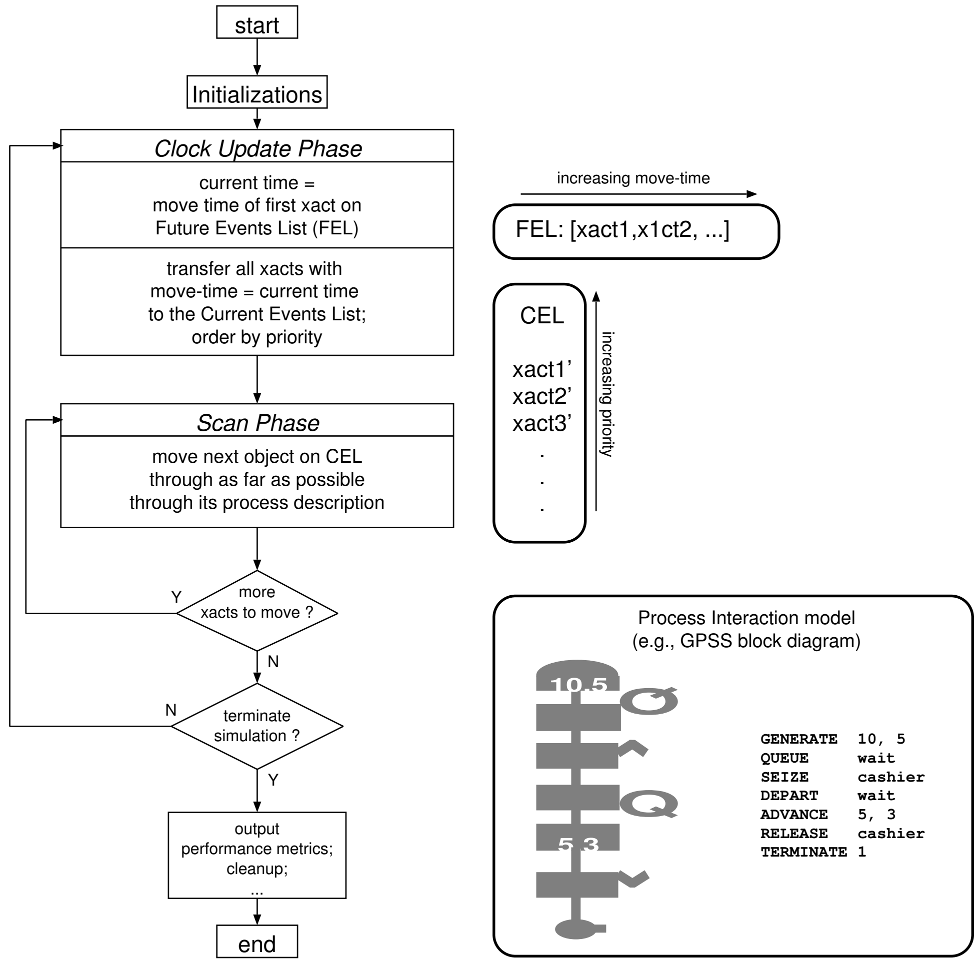
Three Phase Approach



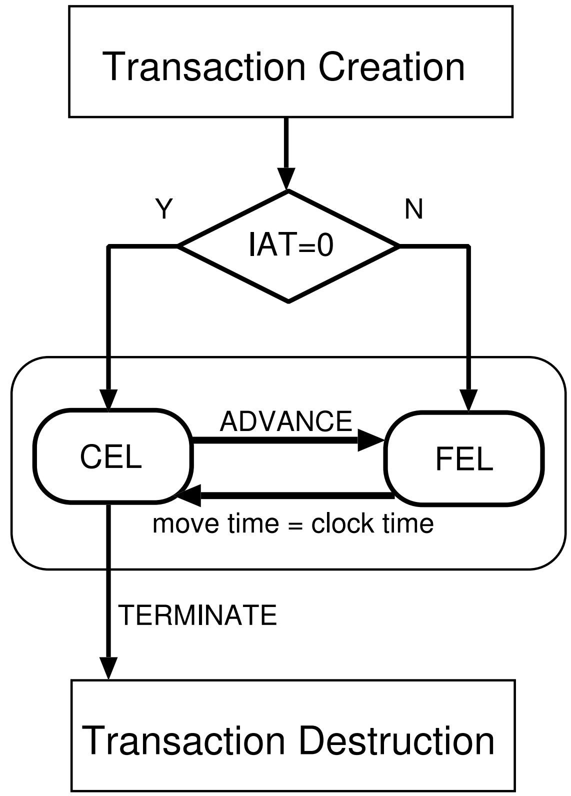
Three Phase Approach (dis)advantages

- advantage: performance added to Activity Scanning
- disadvantage: mixing two views

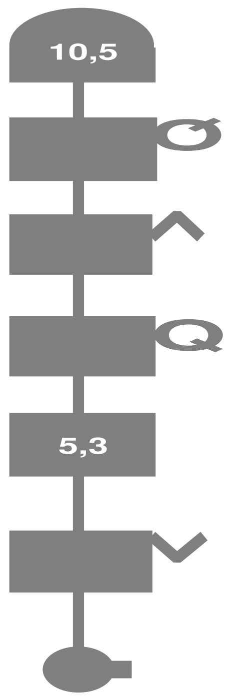
Process Interaction



Process Interaction: Transaction Life



Cashier-Queue: GPSS Process Interaction View




```
GENERATE  10, 5
QUEUE     wait
SEIZE     cashier
DEPART    wait
ADVANCE   5, 3
RELEASE   cashier
TERMINATE 1
```

Process Interaction (dis)advantages

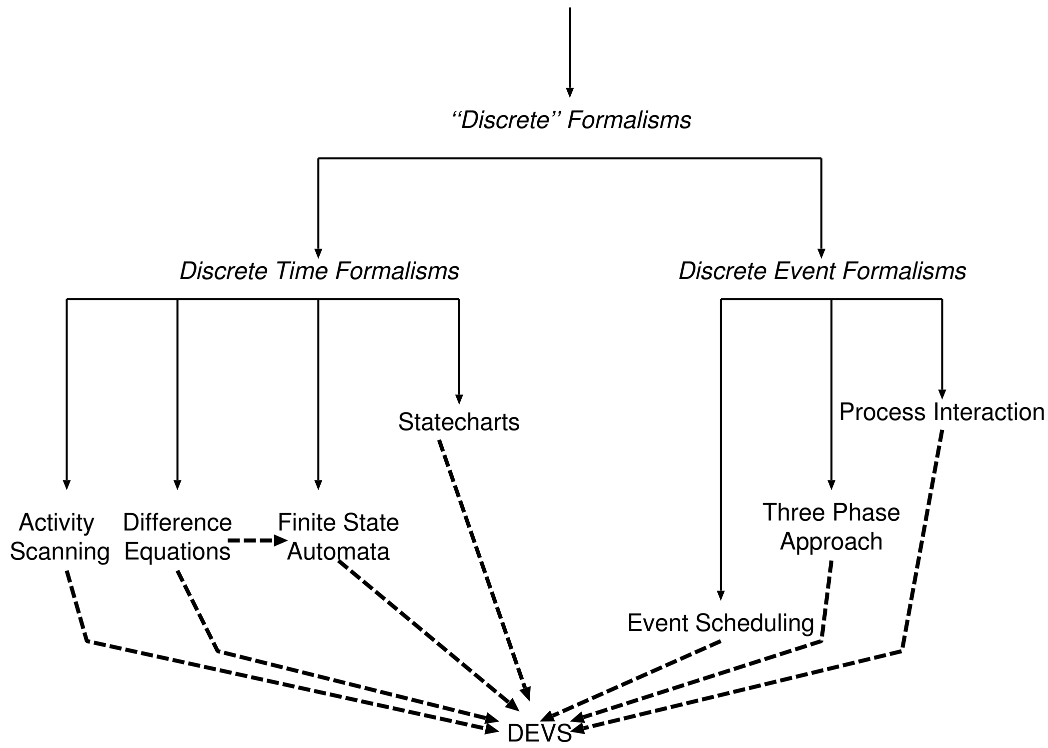
- advantage: declarative model, high-level "process view"
- disadvantage: rather inefficient

General disadvantages

- (here) not formally defined, is possible
- non-modular, is possible

$\Rightarrow$ DEVS formalism

World Views: Classification



(Pseudo-) Random-number Generators

- SYS model is deterministic + random constructs
- randomness $\equiv$ not enough detail known or don't care
- randomness: characterized by distribution
- In SYS: *draw* from distribution and
  Monte-Carlo run multiple deterministic simulations.
- Alternatives:
  - Transform to deterministic.
  - Markov Chains (analytical).

Probability Distributions

- Continuous vs. discrete
- Probability Density Function ($f(x)$)
- Cumulative Probability Function ($F(X)$)
- see probability course: Poisson, Erlang, ...

Pseudo-random

- Sample from distribution ($U(0,1)$)
- Reproducability/comparison of experiments !
  - science needs reproducable results
  - makes debugging easier
  - *identical* random numbers to compare *different* systems
- Quality of generator:
  - appear uniformly distributed
  - non-correlated
  - fast and doesn't need much storage
  - long period, dense (full) coverage
  - provision for *streams* (subsegments)

Linear Congruential Generators

$$Z_i = (aZ_{i-1} + c) mod\ m$$

m is modulus

a is multiplier

c is increment

Z_0 is seed

$c = 0$ is called *multiplicative* LCG

Generators ctd.

- Composite Generators
- Tausworthe generators (operate on bits)
- L'Ecuyer, Devroye (non-uniform)
- Testing RNG: empirical vs. theoretical
- References: Knuth, Law & Kelton

Marse and Roberts' portable RNG

$$Z[i] = (630360016 * Z[i-1]) mod (2^{31} - 1)$$

- Prime modulus multiplicative linear congruential generator.
- Based on Fortran UNIRAN code.
- Multiple (100) streams are supported with seeds spaced 100,000 apart.
- Include file: rand.h
- C file: rand.c
- Example use: randtest.c

Non-uniform continuously distributed RNG

Inverse Transformation Method

Gathering Statistics (report generation)

1. *counters*
2. *summary measures*
3. *utilization*
4. *occupancy*
5. *distributions* and *transit times*

Counters

In all previous examples: keep/update counters (as state vars) !

- numbers of entities of different types in the system
- number of times a particular event occurred
- basis for statistics (performance metrics)

Summary Measures

- minima and maxima:
  compare new values to current min and max, update when necessary
- mean of a set of N observations $x_i, i = 1, 2, \ldots, N$

$$m = \frac{1}{N} \sum_{i=1}^{N} x_i$$

Summary Measures (ctd.)

- standard deviation (from mean)

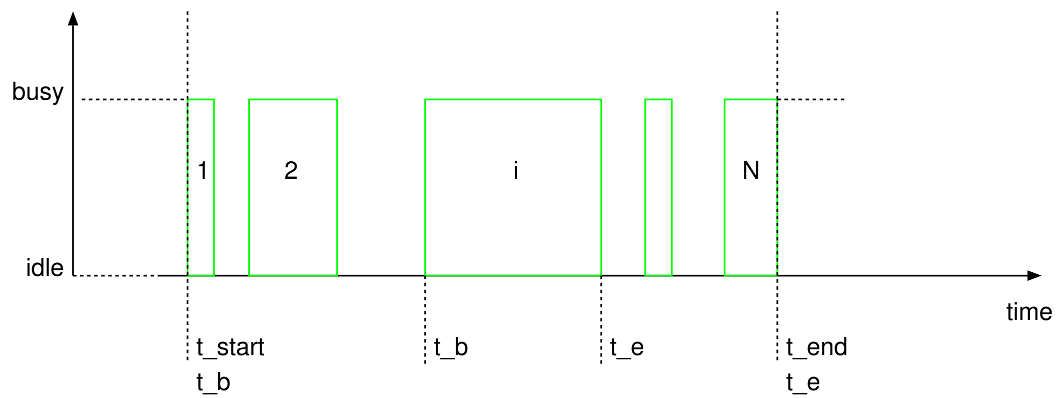
$$s = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(m - x_i)^2}$$

  - need to calculate m first $\rightarrow$ need to keep all observations
  - sum of squares may grow *very* large (accuracy $\downarrow$)

$$\sum_{i=1}^{N}(m - x_i)^2 = \sum_{i=1}^{N} x_i^2 - Nm^2$$

Utilization

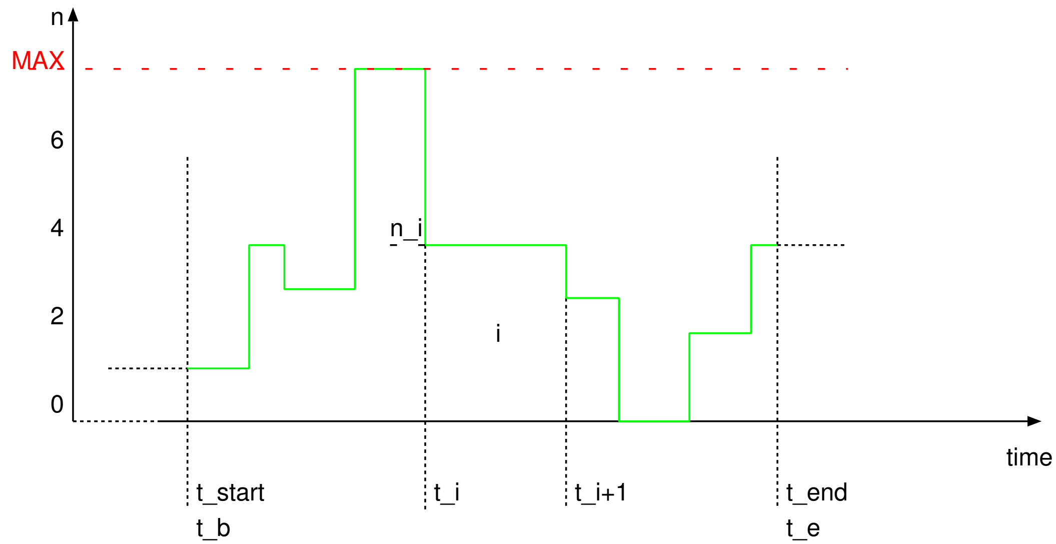
The fraction (or %) of time each *individual* entity is engaged



$$U = \frac{1}{t_{end} - t_{start}} \sum_{i=1}^{N} (t_e - t_b)_i$$

Average Use and Occupancy

for *groups* and *classes* of entities



Average Use and Occupancy (ctd.)

- Average use over time (t_i are times of change)

$$A = \frac{1}{t_{end} - t_{start}} \sum_{i=1}^{N} n_i (t_{i+1} - t_i)$$

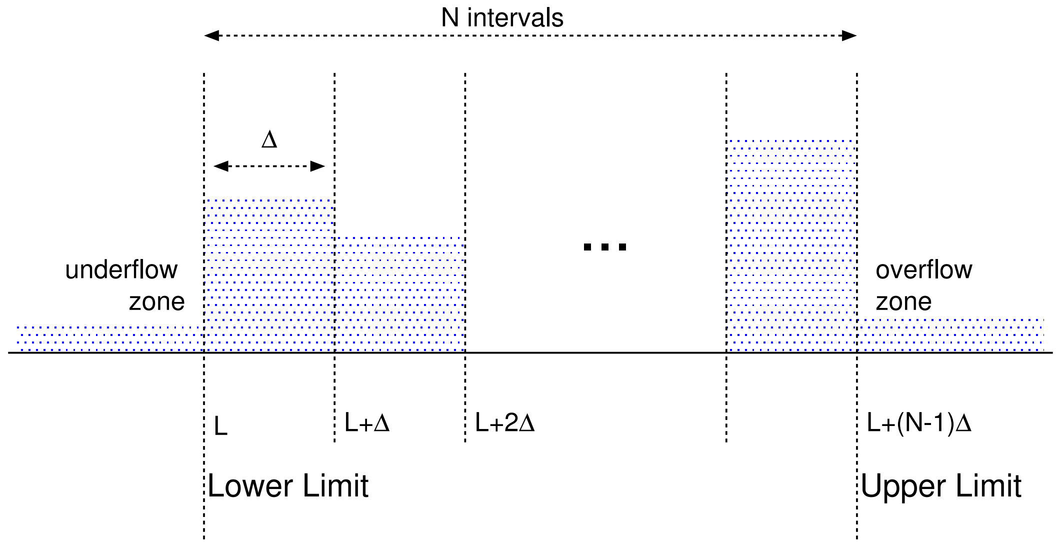
  Example use: average queue length.

- Occupancy: average number in use with respect to MAX

$$O = \frac{A}{MAX}$$

No bookkeeping of individual entity information required, only *total* use (n_i) and *when* change occurs. This, as opposed to for example average transit time computation where individual times must be kept.

Distributions and Transit Times



Number of intervals N, Uniform interval size Δ, Lower tabulation limit L.

Implementation: table of interval *counters*.

Global accumulation: number of entries, sum of entries, sum of squares.

Distributions and Transit Times (ctd.)

- Transit times: use clock as *time stamp*, enter in table at end of transit.
- Distribution of number of entitities: measure at uniform intervals of time.