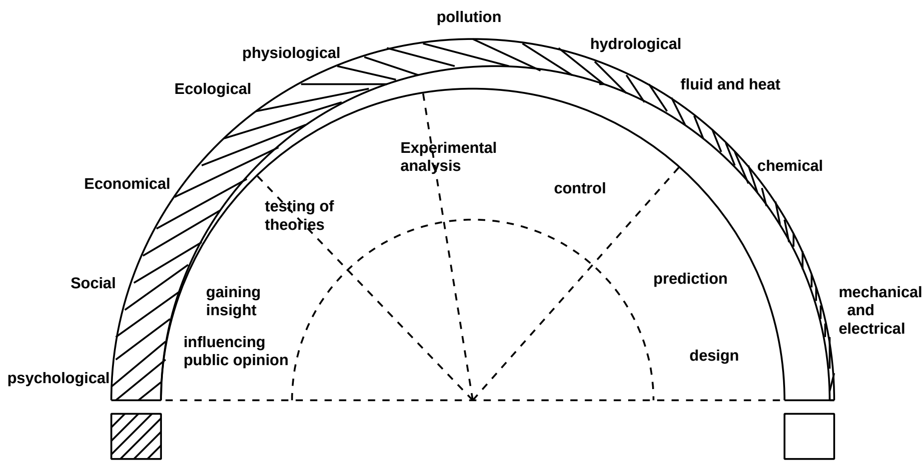
Overview

- Model Classifications
- Hierarchy of System Specification
  1. Time Base
  2. Segments and Trajectories
  3. I/O Observation Frame
  4. I/O Observation Relation
  5. I/O Function Observation
  6. I/O System, Classification re-visited
  7. Iterative Specification of Systems
  8. Structured Systems
  9. Coupled Systems

Different Model Type Classifications

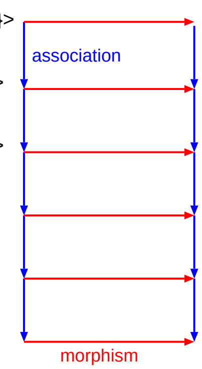
- well-defined (white box) vs. ill-defined (black box)
- continuous vs. discrete
- deterministic vs. stochastic
- graphical vs. textual
- . . .
- which formalism ?

Arch of Karplus



Hierarchy of system specification

5	N = <D, {S_i}, {I_i}, {Z_i}>	N'	**Coupled Model**
4	S = <T, X, Ω, Q, Y, δ, λ>	S'	Structured Model
3	S = <T, X, Ω, Q, Y, δ, λ>	S'	**I/O Model**
2	IOFO = <T, X, Ω, Y, F>	IOFO'	I/O Function Observation
1	IORO = <T, X, Ω, Y, R>	IORO'	I/O Relation Observation
0	O = <T, X, Y>	O'	**Observation Frame**



Set Theory for Abstraction

$$\{1, 2, \ldots, 9\}$$

$$\{a, b, \ldots, z\}$$

$$\mathbb{N}, \mathbb{N}^+, \mathbb{N}^+_\infty$$

$$\mathbb{R}, \mathbb{R}^+, \mathbb{R}^+_\infty$$

$$EV = \{ARRIVAL, DEPARTURE\}$$

$$EV^\phi = EV \cup \{\phi\}$$

$$A \times B = \{(a,b) | a \in A, b \in B\}$$

Time Base

$$time = \langle T, < \rangle$$

- Dynamic system: irreversible passage of *time*.
- Set T, *ordering* relation $<$ on elements of T.
  - transitive: $A < B \wedge B < C \Rightarrow A < C$
  - irreflexive: $A \not< A$
  - antisymmetric: $A < B \Rightarrow B \not< A$
- *Total* (linear) ordering
  $\forall t, t' \in T : t < t' \vee t' < t \vee t = t'$
- *Partial* ordering: uncertainty or multiplicity.

Past, Future, Intervals

- Past: $T_{t)} = \{\tau | \tau \in T, \tau < t\}$
- Future: $T_{(t} = \{\tau | \tau \in T, t < \tau\}$
- $\langle t$ means $(t$ or $[t$
- Interval $T_{\langle t_b, t_e \rangle}$
- Abelian group $(T, +)$ with zero 0 and inverse $-t$
- Order preserving $+$: $t_1 < t_2 \Rightarrow t_1 + t < t_2 + t$
- Lower bound, upper bound
- Time bases: $\mathbb{R}$: *continuous*, $\mathbb{N}$ or isomorphic: *discrete*

Time Bases

continuous, discrete, partial ordering

Segments and Trajectories

- With time base, describe *behaviour over time*
- Time function, *trajectory*, signal: $f : T \to A$
- Restriction to $T' \subseteq T$
  $f|T' : T' \to A, \forall t \in T' : f|T'(t) = f(t)$
- Past of f: $f|T_{t\rangle}$
- Future of f: $f|T_{\langle t}$
- Restriction to an interval: *segment*
  $\omega : \langle t_1, t_2 \rangle \to A$
- $\Omega = (A, T)$ set of all segments

Segments

- Length $l : \Omega \to T_0^+$
- Contiguous segments if domains are contiguous $\langle t_1, t_2 \rangle, \langle t_3, t_4 \rangle, t_2 = t_3$
- Concatenation of contiguous segments: $\omega_1 \bullet \omega_2$
  $\omega_1 \bullet \omega_2(t) = \omega_1(t), \forall t \in dom(\omega_1)$
  $\omega_1 \bullet \omega_2(t) = \omega_2(t), \forall t \in dom(\omega_2)$
- Must remain *function*: unique values !
- Ω closed under concatenation
- Left and right segments:
  $\omega_{t\rangle} \bullet \omega_{\langle t} = \omega$

Types of Segments

- Continous: $\omega : \langle t_1, t_2 \rangle \rightarrow \mathbb{R}^n$
- Piecewise continuous
- Piecewise constant
- Event segments: $\omega : \langle t_1, t_2 \rangle \rightarrow A \cup \{\phi\}$
- Correspondence between
  piecewise constant and event segments (later, state trajectory)

Observation Frame

$$O = \langle T, X, Y \rangle$$

- T is *time-base*: $\mathbb{N}$ (discrete-time), $\mathbb{R}$ (continuous-time)
- X input value set: $\mathbb{R}^n, EV^{\phi}$
- Y output value set: system response

I/O Relation Observation

$$IORO = \langle T, X, \Omega, Q, Y, R \rangle$$

- $\langle T, X, Y \rangle$ is Observation Frame
- Ω is the set of all possible input segments
- R is the *I/O relation*
  $\Omega \subseteq (X, T)$, $R \subseteq \Omega \times (Y, T)$
  $(\omega, \rho) \in R \Rightarrow dom(\omega) = dom(\rho)$
- $\omega : \langle t_i, t_f \rangle \rightarrow X$ input *segment*
- $\rho : \langle t_i, t_f \rangle \rightarrow Y$ output *segment*
- note: not really necessary to observe over same time domain

I/O Function Observation

$$SYS = \langle T, X, \Omega, Q, Y, F \rangle$$

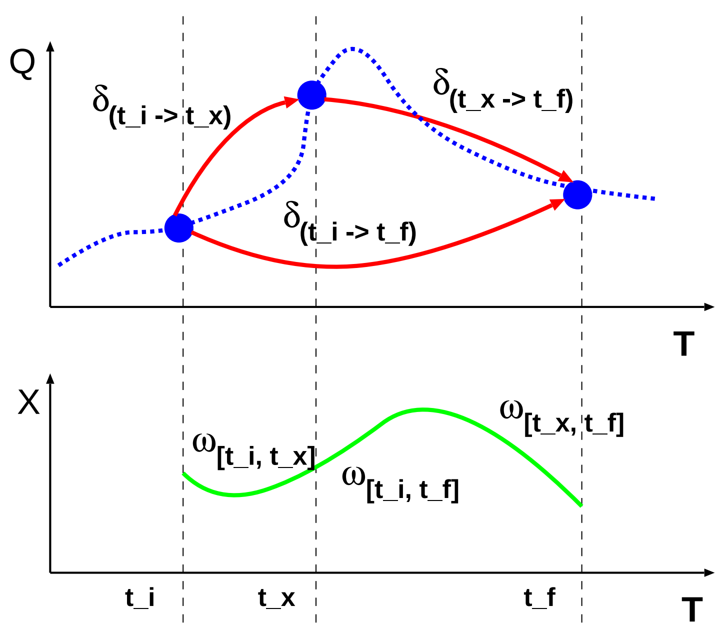
- $\langle T, X, Y \rangle$ is Observation Relation Observation
- Ω is the set of all possible input segments
- F is the *set of I/O functions*
  $f \in F \rightarrow f \subset \Omega \times (Y, T)$
  f is a function such that $dom(f(\omega)) = dom(\omega)$
- f = *initial state*: unique response to ω
- $R = \bigcup_{f \in F} f$

$$SYS = \langle T, X, \Omega, Q, \delta, Y, \lambda \rangle$$

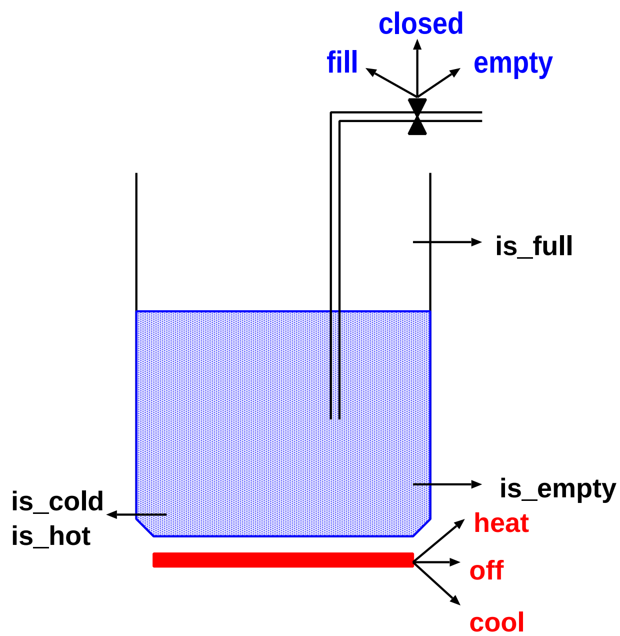
T	time base
X	input set
$\omega : T \to X$	input segment
Q	state set
$\delta : T \times \Omega \times Q \to Q$	transition function
Y	output set
$\lambda : Q \to Y$	output function

$$\forall t_x \in [t_i, t_f] : \delta(t_i, \omega_{[t_i,t_f]}, q_i) = \delta(t_x, \omega_{[t_x,t_f]}, \delta(t_i, \omega_{[t_i,t_x]}, q_i))$$

Closure: Ω closed under concatenation and left segmentation.



System under study: T, h controlled liquid



Detailed (continuous) view, ALG + ODE formalism

Inputs (discontinuous $\rightarrow$ hybrid model):

- Emptying, filling flow rate ϕ
- Rate of adding/removing heat W

Parameters:

- Cross-section surface of vessel A
- Specific heat of liquid c
- Density of liquid ρ

State variables:

- Temperature T
- Level of liquid l

Outputs (sensors):

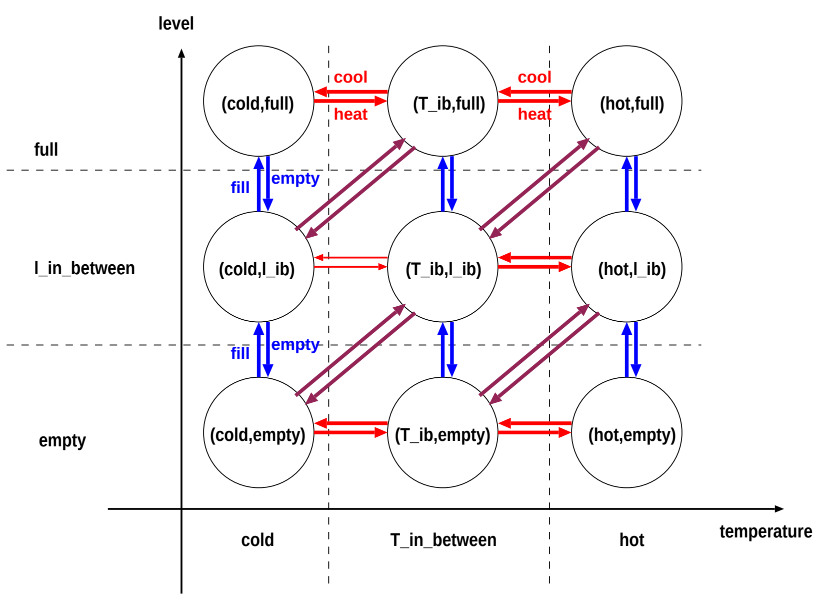
- $is\_low, is\_high, is\_cold, is\_hot$

$$\begin{cases} \frac{dT}{dt} &= \frac{1}{l}\left[\frac{W}{c\rho A} - \phi T\right] \\ \frac{dl}{dt} &= \phi \\ is\_low &= (l < l_{low}) \\ is\_high &= (l > l_{high}) \\ is\_cold &= (T < T_{cold}) \\ is\_hot &= (T > T_{hot}) \end{cases}$$

$$SYS^{ODE}_{VESSEL} = \langle \mathcal{T}, X, \Omega, Q, \delta, Y, \lambda \rangle$$

$$\mathcal{T} = \mathbb{R}$$
$$X = \mathbb{R} \times \mathbb{R} = \{(W, \phi)\}$$
$$\omega : \mathcal{T} \rightarrow X$$
$$Q = \mathbb{R}^+ \times \mathbb{R}^+ = \{(T, l)\}$$
$$\delta : \Omega \times Q \rightarrow Q$$
$$\delta(\omega_{[t_i,t_f]}, (T(t_i), l(t_i))) =$$
$$(T(t_i) + \int_{t_i}^{t_f} \frac{1}{l(\alpha)}[\frac{W(\alpha)}{c\rho A} - \phi(\alpha)T(\alpha)]d\alpha,\ l(t_i) + \int_{t_i}^{t_f} \phi(\alpha)d\alpha)$$
$$Y = \mathbb{B} \times \mathbb{B} \times \mathbb{B} \times \mathbb{B} = \{(is\_low, is\_high, is\_cold, is\_hot)\}$$
$$\lambda : Q \rightarrow Y$$
$$\lambda(T, l) = ((l < l_{low}), (l > l_{high}), (T < T_{cold}), (T > T_{hot}))$$

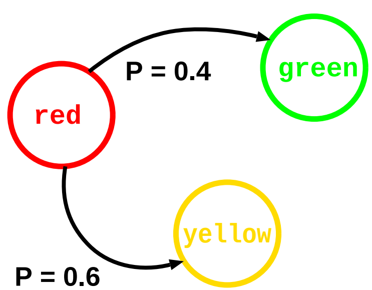
High-level (discrete) view, FSA formalism



$$SYS^{FSA}_{VESSEL} = \langle \mathcal{T}, X, \Omega, Q, \delta, Y, \lambda \rangle$$

$$\mathcal{T} = \mathbb{N}$$
$$X = \{heat, cool, off\} \times \{fill, empty, closed\}$$
$$\omega : \mathcal{T} \to X$$
$$Q = \{cold, T_{between}, hot\} \times \{empty, l_{between}, full\}$$
$$\delta : \Omega \times Q \to Q$$
$$\delta((off, fill), (cold, empty)) = (cold, l_{between})$$
$$\delta((off, fill), (cold, l_{between})) = (cold, full))$$
$$\delta((off, fill), (cold, full))) = (cold, full))$$
$$\vdots$$
$$\delta((heat, fill), (hot, full))) = (hot, full))$$
$$Y = \mathbb{B} \times \mathbb{B} \times \mathbb{B} \times \mathbb{B}$$
$$\lambda : Q \to Y$$
$$\lambda(T, l) = ((l = low), (l = high), (T = cold), (T = hot))$$

In SYS: δ is deterministic, but . . .



non-determinism:
sample from distribution

pseudo-random !!

Discrete-event models ($T = \mathbb{R}$, finite non-ϕ)

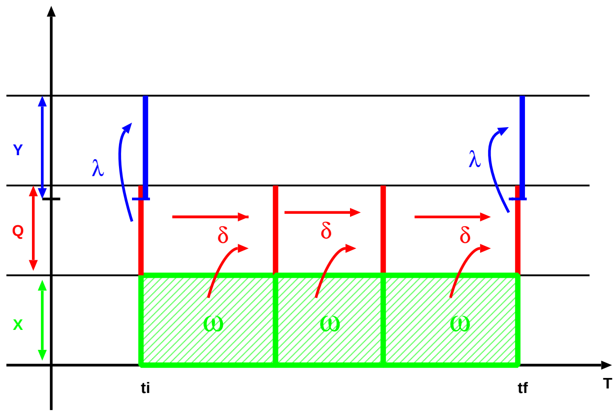
- Specification and analysis of behaviour
  - physical systems (time-scale, parameter abstraction)
    queueing systems
  - non-physical systems (software)
- Traditionally: World Views
  1. Event Scheduling
  2. Activity Scanning
  3. Three Phase Approach
  4. Process Interaction
- Emulate non-determinism by deterministic + pseudo RNG
- All can be *represented* as DEVS

Formalism classification
based on general system model

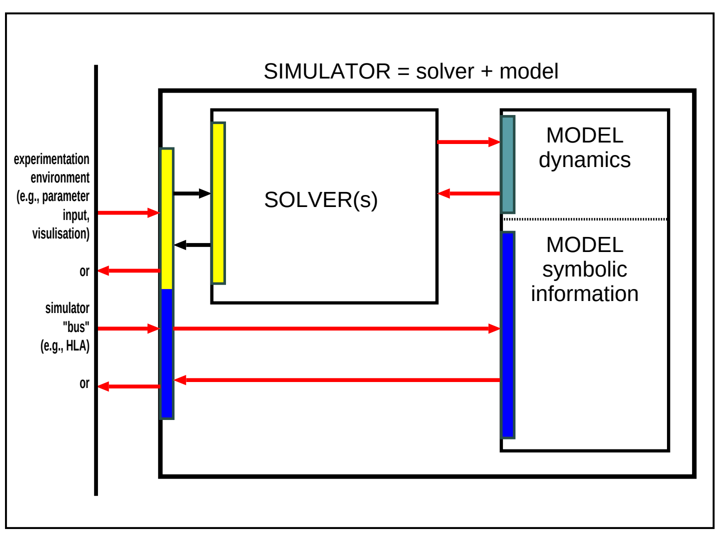
	T: **Continuous**	T: **Discrete**	T: $\{$**NOW**$\}$
Q: **Continuous**	DAE	Difference Eqns.	Algebraic Eqns.
Q: **Discrete**	Discrete-event	Finite State Automata	Integer Eqns.
	Naive Physics	Petri Nets	

Basis for **general, standard software architecture of simulators**
Other classifications based on **structure of formalisms**

Simulation Kernel Operation



Model-Solver Architecture



Difference Equations (solving may be symbolic)

$$\begin{cases} x_1 &= 1 \\ x_{i+1} &= a x_i + 1 \end{cases}$$

$$\begin{aligned} x_n &= 1 + a + a^2 + \ldots + a^{n-1} \\ a x_n &= a + a^2 + \ldots + a^{n-1} + a^n \end{aligned}$$

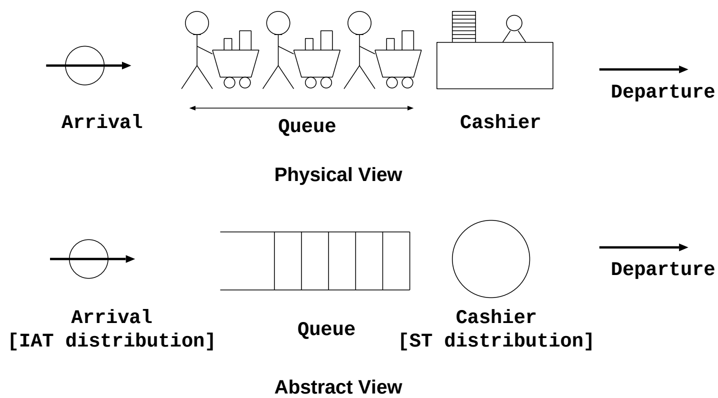
$$\Longrightarrow x_n(1-a) = 1 - a^n$$

$$\Longrightarrow x_n = \frac{1-a^n}{a-a}$$

Iterative Specification of Systems

- Iterative Application of Generator Segments
- Elementary Segments, generate any input segment
- Generator State Transition function, apply iteratively

State set can be product set



Adding Structure

- no *structure* is imposed on sets upto now
- additional information: construct sets from primitives
- cross-product $\times$
- building *concrete* systems from building blocks
- system $\rightarrow$ structured system
- structured sets and functions $\sim$ *variables*

Multivariable Sets

Variables, coordinates, ports v_i

$$V = (v_1, v_2, \ldots, v_n)$$

$$S_1, S_2, \ldots S_n$$

$$S = (V, S_1 \times S_2 \times \ldots S_n)$$

Projection operator

$$. : S \times V \to \bigcup_{j=1}^{n} S_j, S.v_i = s_i$$

$$. : S \times 2^V \to \bigcup_{v \in 2^V} \times_{j \in V} S_j, S.(v_i, v_j, \ldots) = s.v_i, s.v_j, \ldots$$

Examples

Ports

$$X_1 = ((heatFlow, liquidFlow), \mathbb{R} \times \mathbb{R})$$

$$x \in X_1, x.heatFlow$$

Variables

$$S_1 = ((temperature, level),]0.0, 100.0[\times [0, H])$$

$$S_2 = ((qLength, cashStatus), \mathbb{N} \times \{Idle, Busy\})$$

$$s \in S_2, s.qLength$$

Structured Functions

$$f : A \to B$$

with A and B structured sets

Projection

$$f.b_i : A \to ((b_i), B_i),$$

$$f.b_i(a) = f(a).b_i$$

$$f.(b_i, b_j, \ldots) : A \to ((b_i, b_j, \ldots), B_i \times B_j \times \ldots)$$

$$f.(b_i, b_j, \ldots)(a) = f(a).(b_i, b_j, \ldots)$$

Structured System

$$S = < T, X, \Omega, Q, \delta, Y, \lambda >$$

$$X, Q, \delta, Y, \lambda$$

are structured sets/functions

similarly, structured IOR, structured IOF, ...

Non-modular Multicomponent Specification

- Collections of *interacting* components
- *Compositional* modelling
- *Modular* (through ports) *vs. non-modular* (direct interaction)
- Direct interaction through transition function

Multicomponent System

$$MC = \langle T, X, \Omega, Y, D, \{M_d | d \in D\} \rangle$$

$$M_d = \langle Q_d, E_d, I_d, \delta_d, \lambda_d \rangle, \forall d \in D$$

- D is a set of component *references/names*
- Q_d is the *state set* of component d
- $I_d \subseteq D$ is the set of *influencers* of d
- $E_d \subseteq D$ is the set of *influencees* of d
- δ_d is the *state transition function* of d
  $\delta_d : \times_{i \in I_d} Q_i \times \Omega \rightarrow \times_{j \in E_d} Q_j$
- λ_d is the *output function* of d
  $\lambda_d : \times_{i \in I_d} Q_i \times \Omega \rightarrow Y$

Constraints to keep DTSS and ODE meaningful

- $E_d = \{d\}$
- $Y = \times_{d \in D} Y_d$
- $Q = \times_{d \in D} Q_d$
- $\delta(q, \omega).d = \delta_d(\times_{i \in I_d} q_i, \omega)$
- $\lambda(q, \omega).d = \lambda_d(\times_{i \in I_d} q_i, \omega)$
- Less constrained for Discrete Event

Example: GPSS model

Coupled or Network System

$$N = \langle T, X_N, Y_N, D, \{M_d | d \in D\}, \{I_d | d \in D \cup \{N\}\}, \{Z_d | d \in D \cup \{N\}\} \rangle$$

- X_N and Y_N are external network inputs and outputs
- D is a set of component *references* or *names*
- M_d is an I/O system, for all components d
- $I_d \subseteq D \cup \{N\}$ is the set of *influencers* of d
- $Z_d : \times_{i \in I_d} YX_i \rightarrow XY_d$ is the *interface map* for d
  $YX_i = X_i$ if $i = N$, $YX_i = Y_i$ if $i \neq N$
  $XY_d = Y_d$ if $d = N$, $YX_d = Y_d$ if $d \neq N$

Example

Flattening/Closure under coupling

$$\langle T, X_N, Y_N, D, \{M_d | d \in D\}, \{I_d | d \in D \cup \{N\}\}, \{Z_d | d \in D \cup \{N\}\}\rangle$$

$$\rightarrow \langle T, X, \Omega, Q, \delta, Y, \lambda \rangle$$

- Discrete Event (later, DEVS)
- Continuous
  - unique names (scope resolution)
  - `connect(M1.o,M2.i)` $\equiv M2.i := M1.o \rightarrow \#I_d \leq 1$
  - *closure* of the ALG+ODE formalism
- Allows for *hierarchy*

Structured Coupled System Specification

- Structured interfaces (ports)
- External Input Coupling (EIC)
- External Output Coupling (EOC)
- Internal Coupling (IC)
- $N = \langle T, X_N, Y_N, D, \{M_d | d \in D\}, EIC, EOC, IC \rangle$
- range constraints
- special case of Coupled System
  - Influencers
  - Interface Map

Transforming Non-modular into Modular Specification

Hierarchy of system specification

5	$N = \langle D, \{S\_i\}, \{I\_i\}, \{Z\_i\} \rangle$		N’	**Coupled Model**
4	$S = \langle T, X, \Omega, Q, Y, \delta, \lambda \rangle$		S’	Structured Model
3	$S = \langle T, X, \Omega, Q, Y, \delta, \lambda \rangle$		S’	**I/O Model**
2	$IOFO = \langle T, X, \Omega, Y, F \rangle$		IOFO’	I/O Function Observation
1	$IORO = \langle T, X, \Omega, Y, R \rangle$		IORO’	I/O Relation Observation
0	$O = \langle T, X, Y \rangle$		O’	**Observation Frame**