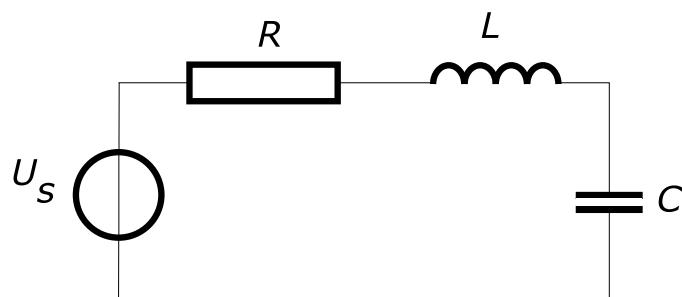


Bond Graphs (Paynter)

- *domain-independent, graphical* description
multi-domain: electric, mechanical (translational, rotational), hydraulic, thermodynamic
- *dynamic* behaviour of *physical* systems
- basis of *power bond*: energy and *energy exchange, analogy*
- *non-causal* → *re-use* of Bond Graph models
- *hierarchy*, shows physical structure
- *causality* assignment → equations
- systematic *procedure*: physical system → equations

Electrical Example: RLC circuit



Equations

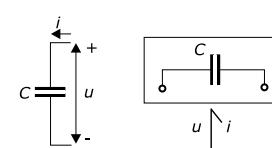
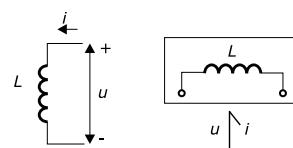
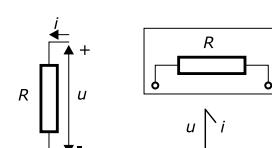
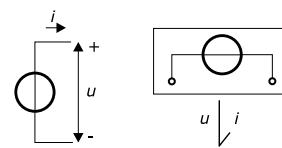
$$P = ui$$

$$u_R = iR$$

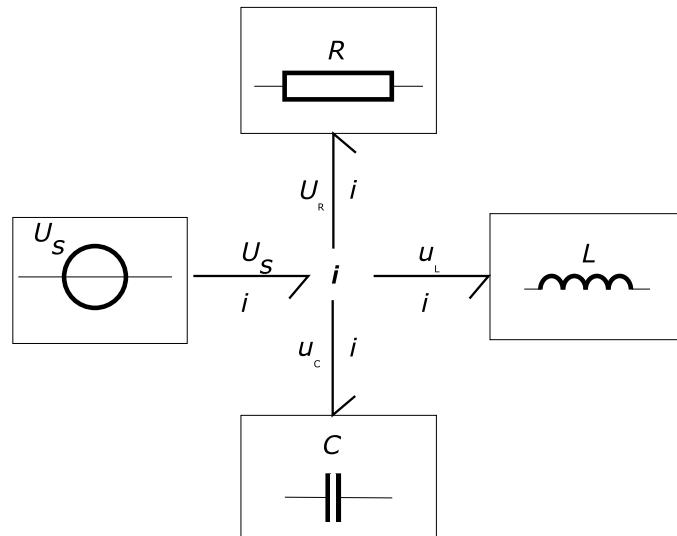
$$u_C = \frac{1}{C} \int idt$$

$$u_L = L \frac{di}{dt} \text{ or } i_L = \frac{1}{L} \int u_L dt$$

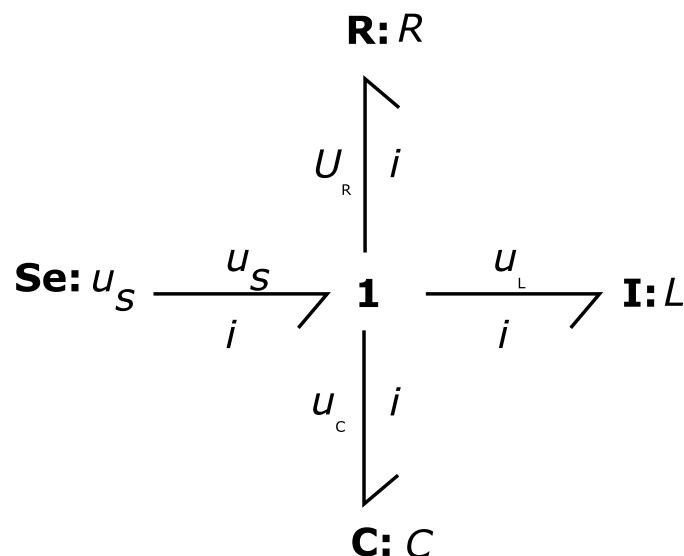
Power Ports



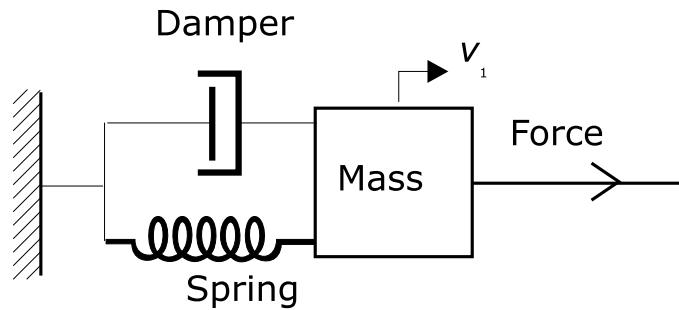
Bond Graph, electrical symbols



Bond Graph, standard



Mechanical Example: Mass Spring



Equations

$$P = Fv \quad (P = T\omega)$$

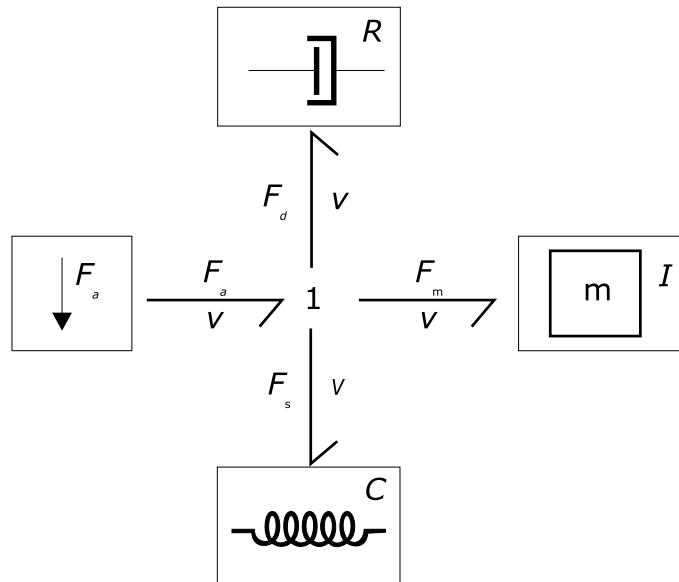
$$F_d = av$$

$$F_S = K_S \int v dt = \frac{1}{C_S} \int v dt$$

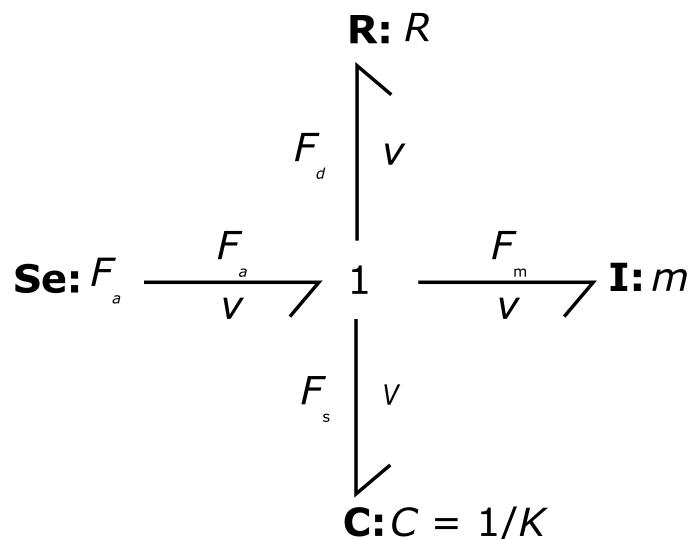
$$F_m = m \frac{dv}{dt} \text{ or } v = \frac{1}{m} \int F_m dt$$

$$F_a = force$$

Bond Graph, mechanical symbols



Bond Graph, standard



Power (e, f) and Energy (p, q) variables

	Effort	Flow	Momentum	Displacement
	e	f	p	q
Electrical	Voltage	Current	Flux	Charge
	u [V]	i [A]	Φ [V s]	q [A s]
Translational	Force	Velocity	Momentum	Displacement
	F [N]	v [$m\ s^{-1}$]	\mathcal{I} [N s]	x [m]
Rotational	Torque	Angular Velocity	Twist	Angle
	T [N m]	ω [$rad\ s^{-1}$]	\mathcal{T} [N m s]	ϕ [rad]
Hydraulic	Pressure	Volume Flow	Pressure Momentum	Volume
	p [$N\ m^{-2}$]	q [$m^3\ s^{-1}$]	Γ [$N\ m^{-2}\ s$]	V [m^3]
Thermodyn.	Temperature	Entropy Flow	—	Entropy
	T [K]	$\frac{dS}{dt}$ [$W\ K^{-1}$]		S [$J\ K^{-1}$]

Physical Analogy



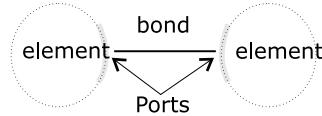
dissipation	reservoir of potential	reservoir of flux
$\frac{\partial P}{\partial x} = -D f$	$\frac{\partial P}{\partial t} = - \frac{1}{\epsilon_p} \frac{\partial f}{\partial x}$	$\frac{\partial P}{\partial x} = -E f \frac{\partial f}{\partial t}$

Bond Graph Foundations

Network-like description of physical systems:

- *conservation of energy*
- *lumped*, idealized physical concepts

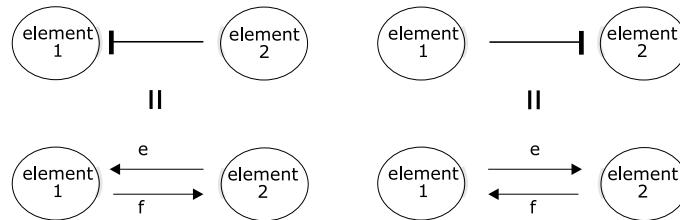
Bonds and Ports



Two interpretations:

1. Interaction of Energy. (for physical reasoning)
2. Bilateral signal flow. Signals: effort and flow.
(to derive a simulation model)

Signal Direction (causality)

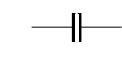


`element1.e := element2.e
element2.f := element1.f`

`element2.e := element1.e
element1.f := element2.f`

C-type Storage Elements

Domain specific symbols



Capacitor



Translational spring



Rotational spring

Bond-graph element

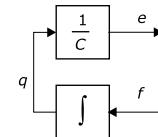
$$C : C \begin{array}{c} e \\ f \end{array}$$

$$e = \frac{1}{C} q$$

$$q = \int f dt + q(0)$$

Equations

Block diagram expansion



$$\frac{dq}{dt} = f, e = e(q)$$

$$\frac{dq}{dt} = i, \quad u = \frac{1}{C} q$$

$$\frac{dx}{dt} = v, \quad F = Kx = \frac{1}{C} x$$

I-type Storage Elements

Domain specific symbols Bond-graph element



Inductor



Mass



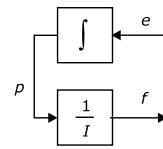
Inertance

Equations

$$f = \frac{1}{I} p$$

$$p = \int e dt + p(0)$$

Block diagram expansion



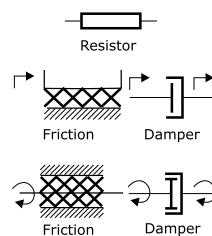
$$\frac{dp}{dt} = e, f = f(p)$$

$$\frac{dI}{dt} = u, i = \frac{1}{L} I$$

$$\frac{dp}{dt} = F, v = \frac{1}{m} p$$

Resistors

Domain-specific symbols



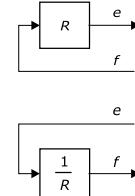
Bond-graph element

$$R : R \triangleleft \frac{e}{f}$$

Equations

$$e = Rf$$

Block diagram expansion

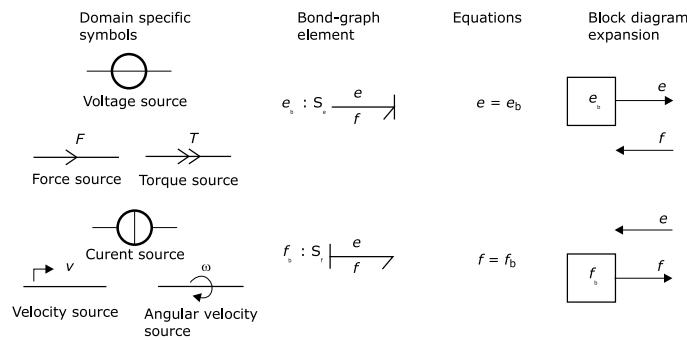


$$e = r(f)$$

$$u = Ri$$

$$F = Rv$$

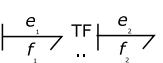
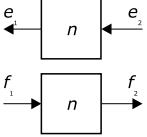
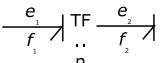
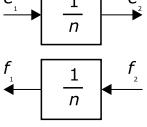
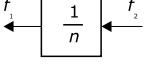
Sources



Signals



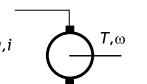
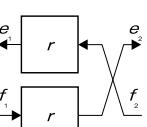
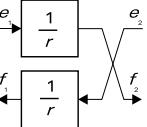
Transformers

Domain-specific symbols	Bond-graph element	Equations	Block-diagram expansion
Transformer		$e_2 = nf_1$ $e_1 = ne_2$	
Cantilever		$f_2 = f_1/n$ $e_2 = e_1/n$	
Mechanical gear		$e_1 = ne_2$ $f_2 = nf_1$	

$$e_1 = ne_2$$

$$f_2 = nf_1$$

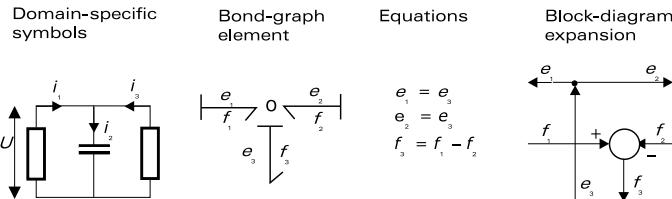
Gyrators

Domain-specific symbols	Bond-graph element	Equations	Block-diagram expansion
Motor Generator		$e_2 = rf_1$ $e_1 = rf_2$	
Pump Turbine		$f_2 = e_1/r$ $f_1 = e_2/r$	

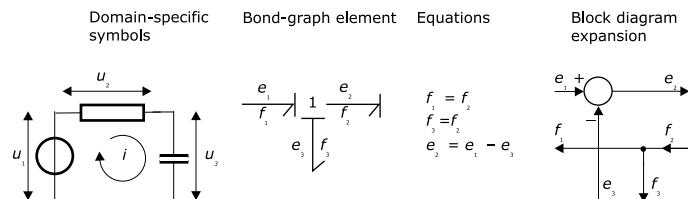
$$e_1 = rf_2$$

$$e_2 = rf_1$$

Junctions: 0



Junctions: 1



Simplifications

$$\begin{aligned}
 a & \quad \frac{e}{f_i} \nearrow 0 \frac{e}{f_{i+1}} \nearrow = \frac{e}{f} \nearrow \\
 b & \quad \frac{e}{f_i} \nearrow 1 \frac{e}{f_{i+1}} \nearrow = \frac{e}{f} \nearrow \\
 c & \quad \frac{e}{f_i} \nearrow 0 \frac{e}{f_i = f_i} \nearrow 0 \frac{e}{f_i} \nearrow = \frac{e}{f_i} \nearrow 0 \frac{e}{f_i} \nearrow \\
 d & \quad \frac{e}{f_i} \nearrow 1 \frac{e}{f_i = f_i} \nearrow 1 \frac{e}{f_i} \nearrow = \frac{e}{f_i} \nearrow 1 \frac{e}{f_i} \nearrow \\
 e & \quad \frac{e}{f_i} \nearrow 0 \frac{e}{f_i} \nearrow 1 \frac{e}{f_i} \nearrow 0 \frac{e}{f_i} \nearrow = \frac{e}{f_i} \nearrow 0 \frac{e}{f_i} \nearrow \\
 f & \quad \frac{e}{f_i} \nearrow 1 \frac{e}{f_i} \nearrow 0 \frac{e}{f_i} \nearrow 1 \frac{e}{f_i} \nearrow = \frac{e}{f_i} \nearrow 1 \frac{e}{f_i} \nearrow
 \end{aligned}$$

Remarks

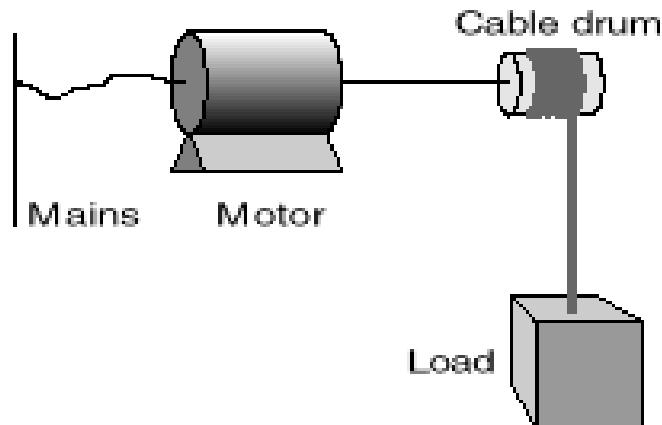
- Positive orientation (energy flow)
→ positive (physical) parameters
- Duality (potential/kinetic, electrical/magnetic)

Systematic Procedure

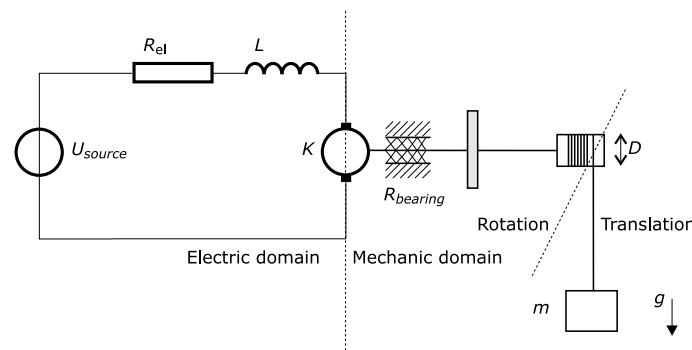
- 1.
- 2.
- 3.
- 4.
- 5.
- 6.
- 7.

$$\begin{array}{c} 0 \quad e_1 - e_2 = e_{12} \\ \nearrow \\ 0 \longrightarrow 1 \longrightarrow 0 \\ e_1 \qquad \qquad \qquad e_2 \end{array} \quad \left\{ \quad \begin{array}{c} 1 \quad v_1 - v_2 = v_{12} \\ \nearrow \\ 1 \longrightarrow 0 \longrightarrow 1 \\ v_1 \qquad \qquad \qquad v_2 \end{array} \right\}$$

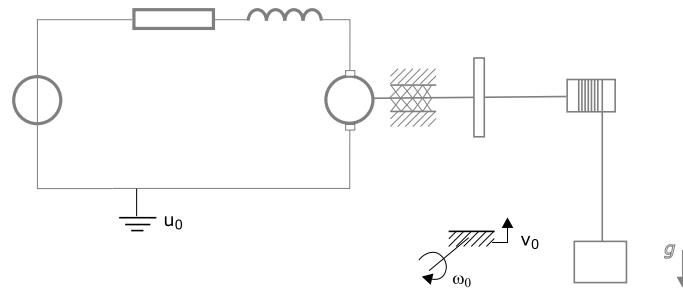
Multi-domain example



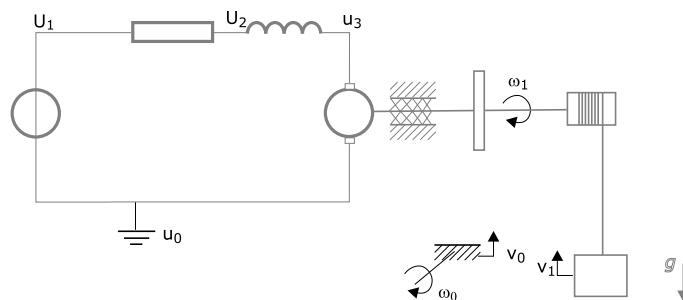
Idealized Physical Model (IPM)



IPM with references



IPM with voltages, velocities, . . .



O-junctions

0_{u_1}

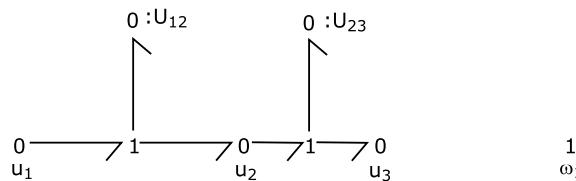
0_{u_2}

0_{u_3}

1_{ω_1}

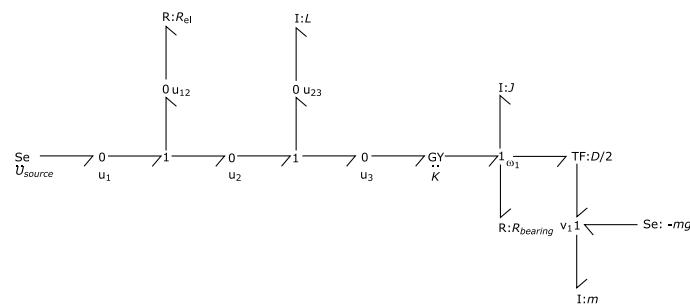
1_{v_1}

1-junctions (differences)

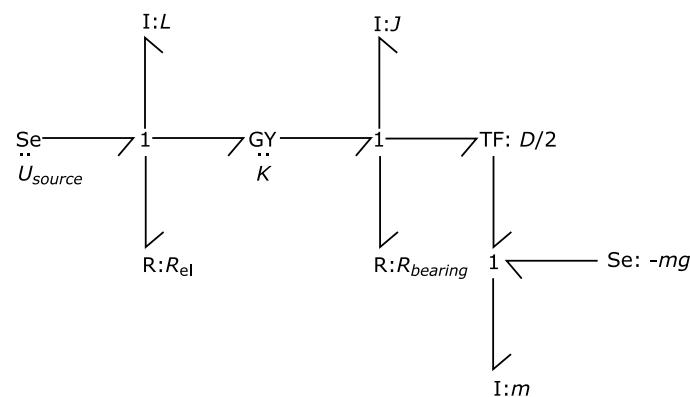


1_{v_1}

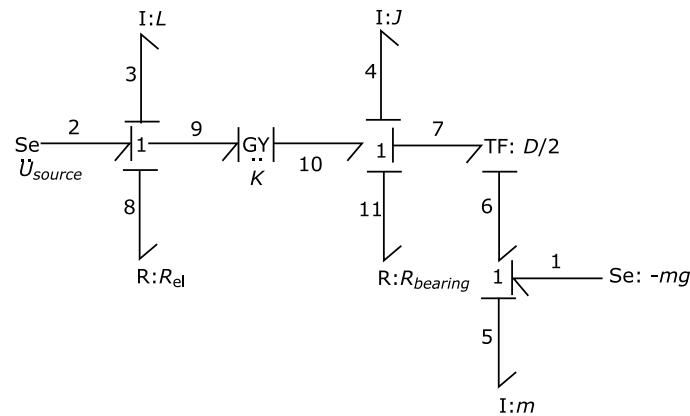
Complete Bond Graph



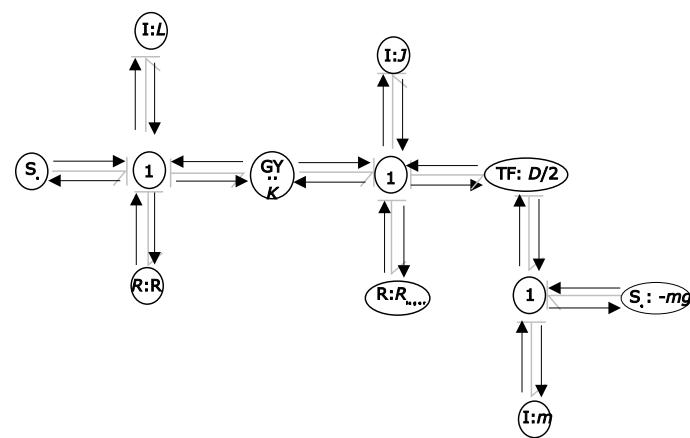
Simplified Bond Graph



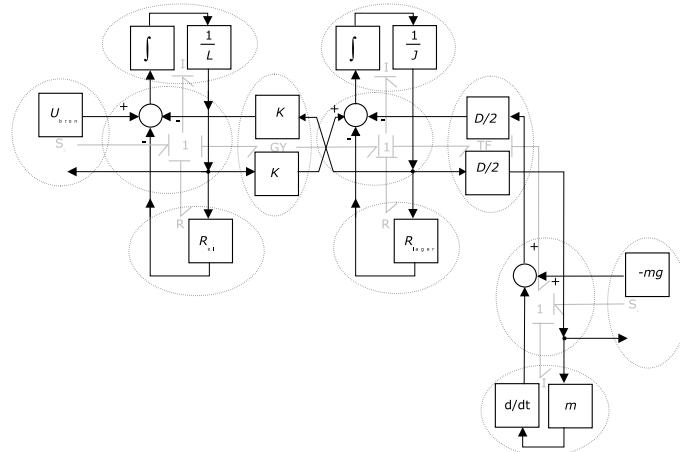
Causal Bond Graph



Block Diagram



Block Diagram



Block Diagram

