COMP 760 - Assignment 3 - Due: April 30th.

You are allowed to collaborate in solving these questions, but each person should write and submit her own solution.

1. Show that if μ is a product distribution (i.e X and Y are indpendent when $(X, Y) \sim \mu$), then $\mathrm{IC}_\mu^{\mathrm{ext}}(\pi) = \mathrm{IC}_\mu(\pi)$.

2. Show that $\mathrm{IC}_\mu(f, \mu, 0)$ is not necessarily continuous with respect to μ.

3. We say that a probability measure μ on $\mathcal{X} \times \mathcal{Y}$ is *internal-trivial for* f if there is a communication protocol π that computes f correctly on all inputs and has internal information cost zero: $\mathrm{IC}_\mu(\pi) = 0$. Prove that μ is *internal-trivial* for f if and only if there exist partitions $S_A = \bigcup_{i \in I} \mathcal{X}_i$ and $S_B = \bigcup_{i \in I} \mathcal{Y}_i$ such that the value of f is constant on each $\mathcal{X}_i \times \mathcal{Y}_i$ and $\mathrm{supp}(\mu) \subseteq \cup_{i \in I} \mathcal{X}_i \times \mathcal{Y}_i$.

4. Suppose that $f : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}$ satisfies $f(x, y) = f(y, x)$ for all x and y. Show that the measure μ that achieves the maximum of $\mathrm{IC}_\mu(f)$ also satisfies $\mu(x, y) = \mu(y, x)$.

5. Adopt the Braverman-Rao compression to the setting of the external information cost. However, in this case your compression must not introduce any error.

6. Use the previous question to show that

$$\lim_{n \to \infty} \frac{\mathrm{CC}_{\mu^n}(f^n)}{n} \leq I_\mu^{\mathrm{ext}}(f).$$