COMP 760 - Assignment 2 - Due: Feb 26th.

You are allowed to collaborate in solving these questions, but each person should write and submit her own solution.

1. Show that for every function $f : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}$, we have $U^{pub}(f) = O(1)$.
2. Show that PP^{cc} is the class of problems with $\mathrm{disc}(f_n) \geq 2^{-\log^c(n)}$.
3. Show that $\mathrm{PP}^{cc} \supseteq \mathrm{NP}^{cc} \cup \mathrm{CoNP}^{cc}$.
4. Consider a problem $\{f_n\}$ in BPP^{cc}.
   (a) Show that there is a randomized protocol $P(x,y,r)$ which uses only $|r| = O(\log n)$ random bits, and still achieves
   $$\Pr[P(x,y,r) \neq f_n(x,y)] \leq \frac{1}{3}.$$
   (b) Show that there is a randomized protocol $Q(x,y,r)$ which uses only $m := |r| = O(\log^2 n)$ random bits, and achieves
   $$\Pr[Q(x,y,r) \neq f_n(x,y)] \leq \frac{1}{3m}.$$
   (c) Let m, and $Q(x,y,r)$ be as above, and we will interpret r as taking values in $\mathbb{F}_2^m$ according to some distribution. Use probabilistic method to prove the following statements:
   - If $f_n(x,y) = 1$ then there exists a choice of $a_1, \ldots, a_m \in \mathbb{F}_2^m$ such that for every $z \in \mathbb{F}_2^m$ there is at least one $i \in \{1, \ldots, m\}$ for which $Q(x,y,a_i + z) = 1$.
   - If $f_n(x,y) = 0$ then for every choice of $a_1, \ldots, a_m \in \mathbb{F}_2^m$, one can find a $z \in \mathbb{F}_2^m$ such that $Q(x,y,a_i + z) = 0$ for all $i \in \{0, \ldots, m\}$.

   (d) Conclude that $\mathrm{BPP}^{cc} \subseteq \Sigma_2^{cc} \cap \Pi_2^{cc}$.