COMP760, SUMMARY OF LECTURE 1.

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- The definition of the basic deterministic two party model (Yao 1979): Alice and Bob want to collaboratively compute $f(x,y)$ when Alice only knows x and Bob knows y. They both know $f : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}$ and have agreed on a communication protocol beforehand.
- The *cost* of a protocol is the number of bits communicated in the worst-case input (x,y).
- The *deterministic communication complexity* $D(f)$ is the minimum cost of a (deterministic) protocol that computes f.
- $D(f) \leq n+1$ for every f as Alice can send all of her input to Bob.
- The tree representation of a protocol. The cost of a protocol is the height of the corresponding tree.
- A *combinatorial rectangle* is a set $S \times T \subseteq \{0,1\}^n \times \{0,1\}^n$ where $S \subseteq \{0,1\}^n$ and $T \subseteq \{0,1\}^n$. Equivalently $A \subseteq \{0,1\}^n \times \{0,1\}^n$ is a combinatorial rectangle if and only if
$$(x_1,y_1),(x_2,y_2) \in A \Rightarrow (x_1,y_2),(x_2,y_1) \in A.$$
- For every node v of the protocol tree, the set $R_v \subseteq \{0,1\}^n \times \{0,1\}^n$ of the inputs that reach the node v form a combinatorial rectangle. See [KN97, Proposition 1.14].
- $D(f) \geq \log_2 C^D(f)$ where $C^D(f)$ is the smallest number of monochromatic rectangles in a *partition* of $\{0,1\}^n \times \{0,1\}^n$. See [KN97, Corollary 1.17].
- $D(\mathrm{EQ}) = n+1$ using fooling sets. See [KN97, Definition 1.19 and Example 1.21]. Here $\mathrm{EQ} : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}$ is the equality function defined as $\mathrm{EQ}(x,y) = 1$ if and only if $x = y$.

References

[KN97] Eyal Kushilevitz and Noam Nisan, *Communication complexity*, Cambridge University Press, Cambridge, 1997. MR 1426129 (98c:68074)

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