COMP760, SUMMARY OF LECTURE 4.

HAMED HATAMI

- *Distributional communication complexity*: Let μ be a probability distribution on $\{0,1\}^n \times \{0,1\}^n$. The (μ, ϵ)-distributional computational complexity of f, denoted by $D_\epsilon^\mu(f)$ is the smallest communication among all deterministic protocols that satisfy
$$\Pr_{(x,y)\sim\mu}[P(x,y) \neq f(x,y)] \leq \epsilon.$$
- **Theorem:** $R_\epsilon^{pub}(f) = \max_\mu D_\epsilon^\mu(f)$.

  In particular every measure μ provides a lower-bound $R_\epsilon^{pub}(f) \geq D_\epsilon^\mu(f)$. The proof of this direction is very easy and the other direction is proved using von Neumann's min-max theorem (See [KN97, Theorem 3.20] for the proof). The main idea is that a public coin randomized protocol is just a probability distribution on deterministic protocols, and thus can be interpreted as a mixed strategy in the language of game theory.
- Product measures $\mu(x,y) = \mu_1(x)\mu_2(y)$ are easier to study as to pick a random input (x,y) according to such a distribution, one picks x and y *independently* (possibly with different distributions). This raises the following question: How does $\max_{\mu=\mu_1\times\mu_2} D_\epsilon^\mu(f)$ compare to $R_\epsilon(f)$ where μ_1 and μ_2 are probability measures on $\{0,1\}^n$?

  Recently Sherstov [She10] proved that there exists a function such that $R_\epsilon(f) = \theta(n)$ while $\max_{\mu=\mu_1\times\mu_2} D_\epsilon^\mu(f) = O(1)$. This shows - in the strongest possible sense - that in general to prove a nontrivial lower-bound for $R_\epsilon(\cdot)$ one cannot only look at product distributions.

References

[KN97] Eyal Kushilevitz and Noam Nisan, *Communication complexity*, Cambridge University Press, Cambridge, 1997. MR 1426129 (98c:68074)

[She10] Alexander A. Sherstov, *Communication complexity under product and nonproduct distributions*, Comput. Complexity **19** (2010), no. 1, 135–150. MR 2601197 (2011i:68045)

School of Computer Science, McGill University, Montréal, Canada
E-mail address: hatami@cs.mcgill.ca