McGill

Fall 2019
Midterm Examination

Algorithm Design

COMP 360 SEC 001

4:05-5:20 PM Oct 16, 2019

EXAMINER:	Hamed Hatami	ASSOC. EXAMINER:	

STUDENT NAME:		**McGILL ID:**									

INSTRUCTIONS

EXAM:	CLOSED BOOK ☒	OPEN BOOK ☐
	SINGLE-SIDED ☒	PRINTED ON BOTH SIDES OF THE PAGE ☐
	EXTRA BOOKLETS PERMITTED: YES ☒ NO ☐	ANSWER ON EXAM ☒
	SHOULD THE EXAM BE:	RETURNED ☒ KEPT BY STUDENT ☐
CRIB SHEETS:	NOT PERMITTED ☒	PERMITTED ☐ e.g. one 8 1/2X11 handwritten double-sided sheet Specifications:
DICTIONARIES:	TRANSLATION ONLY ☐	REGULAR ☒ NONE ☐
CALCULATORS:	NOT PERMITTED ☒	PERMITTED (Non-Programmable) ☐

ANY SPECIAL INSTRUCTIONS:

- Pick **three** problems and answer.
- **Do not answer all the four problems** (otherwise only the first three questions will be graded).
- Your grade is considered out of 20, but you can potentially acquire a higher grade by answering the last question. This will carry over towards your final grade in the course.

1	2	3	4	**Total**
/6	/6	/8	/10	**/20**

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1. A farmer can choose from three feeds for his milk cows. The nutritional facts and costs of these feeds are shown in the following table. The minimum daily requirements of nutrients A, B, and C are 65, 82, 70 units, respectively. Write a linear program to determine the mixture of feeds that will supply the minimum nutritional requirement at smallest possible cost. (You do not need to solve the linear program).

Feed	A (units/lb)	B (units/lb)	C (units/lb)	Cost /lb
Feed 1	4	7	3	0.10
Feed 2	2	3	4	0.07
Feed 3	5	5	3	0.06

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2. We are given the locations of n propaganda broadcast stations, the radius they cover, and the type of the propaganda that they broadcast (which is either Red or Blue). More precisely, we are given n quadruples (x_i, y_i, r_i, T_i), where x_i, y_i, r_i are integers, and T_i is either equal to B or to R. This means that there is a station at coordinate (x_i, y_i) broadcasting propaganda of type T_i and it covers the circle of radius r_i centered at (x_i, y_i).

   We want to remove the smallest number of stations so that the area that is covered by Blue propaganda is disjoint from the area that is covered by Red propaganda. Show that this problem can be solved efficiently.

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3. Consider the variant of the maximum flow problem where every node v also has an integer capacity $c_v \geq 0$. We are interested in finding the maximum flow as before, but now with the extra restriction that $f^{\text{in}}(v) \leq c_v$ for every node v. Solve this problem using the original maximum flow problem. (You just need to explain how to set up the flow network, and explain how its solution corresponds to the solution of this problem. You are not required to give a detailed proof).

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4. We are given a flow network G and our goal is to improve this network by increasing the capacities of some of the edges. More precisely, we want to find the smallest number of edges such that if we increase the capacities of all of them by 1 (simultaneously), then the value of the max-flow will increase (we do not care by how much). Design an efficient algorithm for this task based on FF.