COMP 360 - Sample Midterm

1. Chemco produces two chemicals: A and B. These chemicals are produced via two manufacturing processes. Process 1 requires 2 hours of labor and 1kg of raw material to produce 1g of A and 1g of B. Process 2 requires 3 hours of labor and 2kg of raw material to produce 3g of A and 2g of B. Sixty hours of labor and 40kg of raw material are available. Chemical A sells for $16 per gram and B sells for $14 per gram. Formulate a linear program that maximizes Chemco's revenue.

2. There are n athletes, numbered $1, \ldots, n$, and m races are scheduled between them. Every race is between five athletes and has one winner. More formally we are given m sets $R_1, \ldots, R_m \subseteq \{1, \ldots, n\}$ each of size exactly 5, where R_i is the list of the athletes in the i-th race. We are also given positive integers $p_1, \ldots, p_n$. We want to see if it is possible for the races to finish in such a way that the i-th athlete wins at most p_i races (for all $1 \leq i \leq n$). Show that this problem can be modeled as a max flow problem and solved using the Ford-Fulkerson algorithm.

3. Prove that for every flow network $(G, s, t, \{c_e\})$, there is a maximum flow f such that the edges with non-zero flow on them form a graph that does not contain any directed cycles.

4. We are given two positive numbers n and d and a function $f : \{0, 1, \ldots, n\} \to \mathbb{R}$. Our goal is to find the best approximation of f with a polynomial of degree d. More precisely we want to find a polynomial of degree d that minimizes $\max_{x \in \{0,\ldots,n\}} |f(x) - p(x)|$.

   (a) Formulate this problem as a linear program.

   (b) Write the dual of your linear program.