1. (4 points) Either prove that the following problem is NP-complete or show that it belongs to P by giving a polynomial time algorithm:

- Input: A positive integer $k$, a set of positive integers $a_1, \ldots, a_k$, and a target number $M$ (all given in binary).
- Question: Are there $\epsilon_1, \ldots, \epsilon_k \in \{1, 3\}$ such that $\sum_{i=1}^k \epsilon_i a_i = M$?

2. (4 points) Either prove that the following problem is NP-complete or show that it belongs to P by giving a polynomial time algorithm:

- Input: A CNF $\phi$ with at most 100 clauses.
- Question: Is $\phi$ satisfiable?

3. (4 points) Either prove that the following problem is NP-complete or show that it belongs to P by giving a polynomial time algorithm:

- Input: A positive integer $k$, a set of positive integers $a_1, \ldots, a_k$, and a target number $M$ (all given in binary).
- Question: Is there a subset of indices $S \subseteq \{1, \ldots, k\}$ such that $\prod_{i \in S} a_i = M$?

4. (4 points) Consider the triangle elimination problem. We are given an undirected graph $G = (V, E)$, and want to find the smallest possible set of vertices $U \subseteq V$ such that deleting these vertices removes all the triangles (i.e. cycles of length 3) from the graph. For each one of the following algorithms, either show that it is a 3-factor approximation algorithm, or give an example to show that it is not.

**Algorithm I:**
• While there is still a triangle $C$ left in $G$
  • Delete all the three vertices of $C$ from $G$
  • EndWhile
  • Output the set of the deleted vertices

Algorithm II:

• While there is still a triangle $C$ left in $G$
  • Delete one arbitrarily chosen vertex of $C$ from $G$
  • EndWhile
  • Output the set of the deleted vertices

Algorithm III:

• While there is still a triangle left in $G$
  • Delete a vertex that is in the largest number of triangles
  • EndWhile
  • Output the set of the deleted vertices

Algorithm VI:

• Write an Integer Program for this problem with constraints $x_u + x_v + x_w \geq 1$ for every triangle $(u, v, w)$ in $G$ (and constraints $x_u \in \{0, 1\}$).
  • Solve the LP relaxation of this Integer Program (with constraints $0 \leq x_u \leq 1$).
  • EndWhile
  • Output the set of the vertices $u$ with $x_u \geq \frac{1}{3}$.

5. (4 points) We are given a graph $G$ together with an ordering of the vertices of $G$ such that every vertex $v$ has at most 5 neighbours that appear before $v$ in that order (but $v$ can have many neighbours appear later in the order).

• Show that the vertices of $G$ can be properly coloured using 6 colours.
• Next we want to colour the vertices of $G$ with 5 colours so as to maximize the number of edges that are properly coloured (that is they have different colours on their endpoints). Design a $\frac{14}{15}$-factor approximation algorithm for this problem.