1. Either prove that the following problem is NP-complete or show that it belongs to P by giving a polynomial time algorithm:
   - Input: A bipartite graph $G$ where every edge $e$ has an integer weight $w_e$. A number $k$.
   - Question: Does $G$ have a matching with total weight $k$.

2. Write the dual of the following linear program. The variables are $x_1, x_2$.

   \[
   \begin{align*}
   \text{max} & \quad x_1 \\
   \text{s.t.} & \quad x_1 \leq 4 \\
   & \quad x_1 \leq 1 \\
   & \quad x_1 \leq 3 \\
   & \quad x_2 \geq 0
   \end{align*}
   \]

3. We are given a directed graph $G = (V, E)$, with two distinct nodes $s$ and $t$ marked as source and sink respectively. Furthermore to every edge $e$, a cost $\alpha_{e} \geq 0$ is assigned. The graph is considered as a flow network (but with no capacity conditions). What does the following linear program is trying to solve? Our variables are $f(e)$ for $e \in E$.

   \[
   \begin{align*}
   \text{min} & \quad \sum_{uv \in E} \alpha_{uv} f(uv) \\
   \text{s.t.} & \quad \sum_{uv \in E} f(vu_0) - \sum_{u_0w \in E} f(u_0w) = 0 \quad \forall u_0 \in V - \{s, t\} \\
   & \quad \sum_{vs \in E} f(vs) - \sum_{sw \in E} f(sw) = -1 \\
   & \quad \sum_{vt \in E} f(vt) - \sum_{tw \in E} f(tw) = 1 \\
   & \quad f(uv) \geq 0 \quad \forall uv \in E
   \end{align*}
   \]

4. Write the dual of the above linear program. What does the dual solve?