

## COMP 360, FINDING THE DUAL OF A LINEAR PROGRAMS

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Consider the following maximization linear form that is not necessarily in standard form.

$$\begin{array}{ll}
 \max & x_1 + 2x_2 + 3x_3 \\
 \text{s.t.} & x_1 + 2x_2 \leq 1 \\
 & x_1 - 4x_3 \geq 5 \\
 & x_1 + x_2 - x_3 = 6 \\
 & x_1 \geq 0 \\
 & x_2 \leq 0 \\
 & x_3 \text{ free}
 \end{array}$$

Recall that the dual comes from multiplying the constraints by new variables and adding them up. We want to be able to conclude the following:

$$\begin{array}{rcl}
 y_1 \times & & (x_1 + 2x_2 \leq 1) \\
 y_2 \times & & (x_1 - 4x_3 \geq 5) \\
 + y_3 \times & & (x_1 + x_2 - x_3 = 6) \\
 \hline
 & (y_1 + y_2 + y_3)x_1 + (2y_1 + y_3)x_2 + (-4y_2 - y_3)x_3 & \leq y_1 + 5y_2 + 6y_3
 \end{array}$$

For this to be true, we need  $y_1 \geq 0$ ,  $y_2 \leq 0$ , and  $y_3$  can be either positive or negative.

Next we want to be able to say that the left hand side of the above inequality is an upper-bound on the original objective function:

$$(y_1 + y_2 + y_3)x_1 + (2y_1 + y_3)x_2 + (-4y_2 - y_3)x_3 \geq x_1 + 2x_2 + 3x_3.$$

Since  $x_1 \geq 0$ ,  $x_2 \leq 0$  and  $x_3 \in (-\infty, +\infty)$ , in order to guarantee this, we need  $y_1 + y_2 + y_3 \geq 1$ ,  $2y_1 + y_3 \leq 2$ , and  $-4y_2 - y_3 = 3$ . Hence the dual linear program will be the following:

$$\begin{array}{ll}
 \min & y_1 + 5y_2 + 6y_3 \\
 \text{s.t.} & y_1 + y_2 + y_3 \geq 1 \quad (\text{since } x_1 \geq 0) \\
 & 2y_1 + y_3 \leq 2 \quad (\text{since } x_2 \leq 0) \\
 & -4y_2 - y_3 = 3 \quad (\text{since } x_3 \text{ is free}) \\
 & y_1 \geq 0 \quad (\text{since the 1st constraint in LP is standard}) \\
 & y_2 \leq 0 \quad (\text{since the 2nd constraint in LP is the opposite of standard}) \\
 & y_3 \text{ free} \quad (\text{since the 3rd constraint in LP is equality})
 \end{array}$$

### REFERENCES

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