COMP 360 - Fall 2019 - Sample Final Exam

1. We have a supply of m different kinds of material. Specifically, we have b_j units of raw material of kind j. They are n different kinds of products that we can make from these raw material. Each unit of product i takes a_{i1} unit of raw material 1, a_{i2} units of raw material 2 etc. to create, and can be sold at the price c_i dollars. It is possible to sell fractional amounts of any product. Our goal is to produce the most profitable set of products with our available material.

   (a) Formulate the above optimization problem as a linear program.

   (b) Write the dual of this program.

2. Formulate the following problem as a linear program: Let $G = (V, E)$ be an undirected graph. We want to assign a positive number to every vertex of G such that the total sum of these numbers is 1, and the largest load of a vertex in the graph is minimized. The load of a vertex is the sum of the number on that vertex and the numbers on its immediate neighbours.

3. Either prove that the following problem is NP-complete or prove that it belongs to P.

   - Input: A CNF ϕ with 10 clauses (and n variables).
   - Question: Is there a truth assignment that satisfies ϕ?

4. Either prove that the following problem is NP-complete or prove that it belongs to P.

   - Input: A CNF ϕ on $2n$ variables.
   - Question: Is there a truth assignment that satisfies ϕ and assigns True to exactly n variables?

5. Consider a graph $G = (V, E)$. The chromatic number of G is the minimum number of colors required to color the vertices of G properly. Let $\mathcal{I}$ be the set of all independent sets in G (Note that every element in $\mathcal{I}$ is a set).

   (a) Prove that the solution to the following linear program provides a lower-bound for the chromatic number of G.

$$\begin{array}{lll} \min & \sum_{I \in \mathcal{I}} x_I & \\ \text{s.t.} & \sum_{I : v \in I} x_I \geq 1 & \forall v \in V \\ & x_I \geq 0 & \forall I \in \mathcal{I} \end{array}$$

   (b) Write the dual of the above linear program.

   (c) Prove that every clique in G provides a solution to the dual linear program.

6. Show that if we strengthen linear programming by also allowing constraints of the form $\sum_{i,j=1}^{n} a_{ij} x_i x_j = b$ (for integers b and a_{ij}), then the problem becomes NP-complete.

7. Show that if we strengthen linear programming by also allowing constraints of the form $|\sum_{i=1}^{n} a_i x_i| \geq b$ (for integers b and a_i), then the problem becomes NP-complete.