COMP 360 - Fall 2019 - Sample Final Exam

1. Prove that the following problem belongs to P: Given a graph G, we want to know whether G has an independent set of size 100.
2. Give an example of a linear program that is infeasible and its dual is also infeasible.
3. Prove that the following problem belongs to PSPACE: Given a graph G and an integer k, we want to know whether the number of independent sets in G is equal to k.
4. Show that the following problem is NP-complete:
   - Input: An undirected graph G and an edge e.
   - Question: Does G have a Hamiltonian cycle that passes through the edge e.
5. Prove that the following algorithm is a 2-factor approximation algorithm for the minimum vertex cover problem:
   - While there is still an edge e left in G:
   - Delete all the two endpoints of e from G
   - EndWhile
   - Output the set of the deleted vertices
6. Prove that the following algorithm is a $\frac{1}{2}$-factor approximation algorithm for the MAX-SAT problem, in which the goal is to maximize the number of satisfied clauses: Given a CNF ϕ on n variables $x_1, \ldots, x_n$:
   - For $i = 1, \ldots, n$ do
   - IF x_i appears in more clauses than $\overline{x_i}$ THEN
   - Set $x_i = T$
   - Else
   - Set $x_i = F$
   - Remove all True clauses from ϕ and remove x_i and $\overline{x_i}$ from all the other clauses
   - EndFor
7. A *kite* is a graph on an even number of vertices, say $2k$, in which k of the vertices form a clique and the remaining k vertices are connected in a tail that consists of a path joined to one of the vertices of the clique. Prove that KITE problem defined as in the following is NP-complete.
   - Input: An undirected graph G, and a positive integer k.
   - Question: Does G contain a kite on $2k$ vertices as a subgraph?