April 2018
Final Examination

Algorithm Design

COMP 360 SEC 001

2:00 PM April 27, 2018

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STUDENT NAME:		**McGILL ID:**									

INSTRUCTIONS

EXAM:	CLOSED BOOK ☒	OPEN BOOK ☐
	SINGLE-SIDED ☒	PRINTED ON BOTH SIDES OF THE PAGE ☐
	MULTIPLE CHOICE ☐ Note: The Examination Security Monitor Program detects pairs of students with unusually similar answer patterns on multiple-choice exams. Data generated by this program can be used as admissible evidence, either to initiate or corroborate an investigation or a charge of cheating under Section 16 of the Code of Student Conduct and Disciplinary Procedures. ANSWER IN BOOKLET ☐ ANSWER ON EXAM ☒	EXTRA BOOKLETS PERMITTED: YES ☒ NO ☐
	SHOULD THE EXAM BE:	RETURNED ☒ KEPT BY STUDENT ☐
CRIB SHEETS:	NOT PERMITTED ☐	PERMITTED ☒ e.g. one 8 1/2X11 handwritten double-sided sheet Specifications: One handwritten double-sided letter size sheet
DICTIONARIES:	TRANSLATION ONLY ☐	REGULAR ☒ NONE ☐
CALCULATORS:	NOT PERMITTED ☒	PERMITTED (Non-Programmable) ☐
ANY SPECIAL INSTRUCTIONS:		

1	2	3	4	5	6	7	**Total**
/15	/15	/15	/15	/10	/15	/15	**/100**

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1. Write a maximization linear program in two variables such that the boundary of its feasible region is the square with vertices $(-1,0),(0,1),(1,0),(0,-1)$, and whose optimal solution has value 5 and is given only by the point $(0,-1)$.

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2. Either prove that the following problem is NP-complete or show that it belongs to P by giving a polynomial time algorithm:

   - Input: An undirected graph G, two nodes s, t and a positive integer k.
   - Question: Is there a path of length at least k between s and t?

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3. Either prove that the following problem is NP-complete or show that it belongs to P by giving a polynomial time algorithm:

   - Input: A flow network and a positive integer k.
   - Question: Is there a maximum flow f such that the total sum of the flow on all of the edges is at most k? That is $\sum_{e \in E} f(e) \leq k$, and f is a maximum flow?

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4. Either prove that the following problem is NP-complete or show that it belongs to P by giving a polynomial time algorithm:

   - Input: An undirected graph G.
   - Question: Does G have a proper colouring with three colours R, G, B that assigns the colour B to exactly one vertex?

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5. Given a set P of n points on the plane, consider the problem of finding the smallest circle containing all the points in P. Show that the following is a 2-factor approximation algorithm for this problem. Pick a point x in P, and set r to be the distance of the farthest point in P from x. Output the circle centered at x with radius r.

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6. Consider the triangle elimination problem. We are given an undirected graph $G = (V, E)$, and want to find the smallest possible set of vertices $U \subseteq V$ such that deleting these vertices removes all the triangles (i.e. cycles of length 3) from the graph. Prove that one of the following algorithms is a 3-factor approximation algorithm, and the other one is not.

   Algorithm I:

   - While there is still a triangle C left in G:
   - Delete all the three vertices of C from G
   - EndWhile
   - Output the set of the deleted vertices

   Algorithm II:

   - While there is still a triangle C left in G:
   - Delete one arbitrarily chosen vertex of C from G
   - EndWhile
   - Output the set of the deleted vertices

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7. In the MAX-CUT problem, given an undirected graph G we want to partition the vertices of G into two parts (A, B) such that the number of edges between A and B is maximized. Prove that the following is a $\frac{1}{2}$-factor approximation algorithm for this problem:

   - Let $v_1, \ldots, v_n$ be all the vertices of G.
   - Initially set $A = B = \emptyset$.
   - For $i = 1, \ldots, n$ do
   -     If v_i has more neighbours in A than in B then add v_i to B, otherwise add v_i to A.
   - EndFor