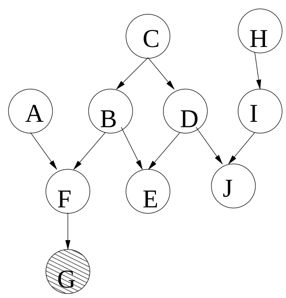
Probabilistic Reasoning in AI - Assignment 2

Due Friday, February 6, 2004

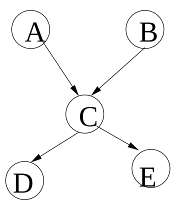
1. [10 points] **Bayes ball**

   For the graph below, which nodes are reachable from A, given the evidence (represented by the shaded node)?



2. [12 points] **Variable elimination**

   Consider the network showed below. Suppose that we want to compute $p(D)$ and we use the elimination order A, B, E, C, D.

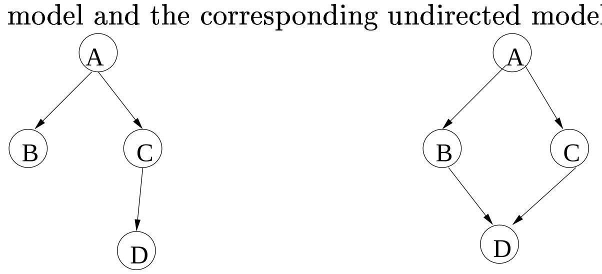


   (a) [10 points] Trace the variable elimination algorithm. For each of the intermediate factors created, explain what probabilistic function it represents

   (b) [2 points] Is variable E relevant in the computation of $p(D)$? Why or why not?

3. [10 points] **Directed vs. undirected models**

   For each of the models below, list all independencies implied by the model. If there is an unconditional model that captures the same independencies, show it. If not, remove the *fewest possible edges* from the directed model until there is an undirected model capturing the same independencies. Show the new directed model and the corresponding undirected model.



(a) (b)

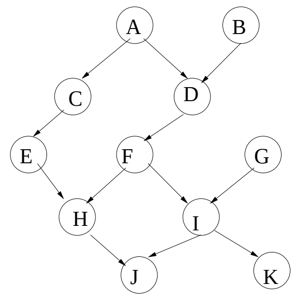
4. [10 points] **Undirected models**

(a) [5 points] Draw an undirected graphical model on 4 variables which can capture *any* joint distribution. List all the maximal cliques

(b) [5 points] If you have n variables, how many edges do you need in order to capture any joint distribution? Justify your answer.

5. [23 points] **Junction tree**

Consider the Bayesian network described below:



(a) [3 points] Draw the moral graph

(b) [5 points] Triangulate the moral graph

(c) [5 points] Construct the junction tree

(d) [10 points] Show the message passing computation assuming that H and K are observed.

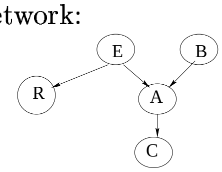
6. [10 points] **Clique trees**

Assume that we have constructed a clique tree for a Bayesian network, and each clique has at most k nodes. Suppose we add an arc between two nodes in the original network. Give an upper bound on the maximum clique size in the clique tree for the new network. Justify your answer.

7. [25 points] **I-maps**

Usually we think of Bayes nets as representing causal influences, but of course this need not be the case. This question involves transforming a Bayes net G which contains the arc $X \rightarrow Y$ into another Bayes net G' which contains the reversed arc, $Y \rightarrow X$. When we do this transformation, we want G' to represent the same distribution as G. Therefore, G' will need to be an I-map of the original distribution.

(a) [5 points] Consider the alarm network:



Suppose we want to reverse the arc $B \rightarrow A$. What additional minimal modifications to the structure of the network are necessary to ensure that the new network is an I-map of the original one? You do not need to prove that the modifications are minimal.

(b) [15 points] Now consider a general Bayes net G. Assume for simplicity that the arc $X \rightarrow Y$ is the only directed path form X to Y. If you reverse the arc, what additional minimal modifications to G are needed in order to ensure that the new network G' is an I-map of the original distribution? Hint: Consider the algorithm for constructing a minimal I-map that we discussed in class.

(c) [5 points] Suppose that you use the above method to transform G into G', which has an arc $Y \rightarrow X$. Now suppose you want to reverse the arc back to its original direction, $X \rightarrow Y$, by using your method to transform G' into G''. Are you guaranteed that the final network structure is equivalent to the original network structure ($G = G''$)?