Probabilistic Reasoning in AI - Assignment 1

Due Friday, January 23, 2004

1. [10 points] (From Russell & Norvig) Express the statement that X and Y are conditionally independent given Z as a constraint (or set of constraints) on the joint distribution entries for $P(X, Y, Z)$.

2. [10 points] Conditional probability warm-up.

   (a) [5 points] Prove the conditionalized version of the product rule:

   $$P(A, B|E) = P(A|B, E)P(B|E)$$

   (b) [5 points] Prove the conditionalized version of Bayes rule:

   $$P(A|B, C) = \frac{P(B|A, C)P(A|C)}{P(B|C)}$$

3. [40 points] This exercise investigates a few important properties of the conditional independence relation. Recall that the conditional independence property that $A \perp\!\!\!\perp B|C$ is a statement that the conditional probability distribution satisfies

   $$p(A|B, C) = p(A|C)$$

   (a) [5 points] Show that, if $A \perp\!\!\!\perp B|C$, then $p(A, B|C) = p(A|C)p(B|C)$

   (b) [5 points] Show that conditional independence is symmetric:

   $$A \perp\!\!\!\perp B|C \leftrightarrow B \perp\!\!\!\perp A|C$$

   (c) [5 points] Show the decomposition property:

   $$A \perp\!\!\!\perp (B \cup D)|C \rightarrow A \perp\!\!\!\perp B|C \text{ and } A \perp\!\!\!\perp D|C$$

   (d) [5 points] Show the weak union property:

   $$A \perp\!\!\!\perp (B \cup D)|C \rightarrow A \perp\!\!\!\perp B|(C \cup D) \text{ and } A \perp\!\!\!\perp D|(C \cup B)$$

   (e) [10 points] Show the contraction property:

   $$A \perp\!\!\!\perp B|(C \cup D) \text{ and } A \perp\!\!\!\perp D|C \rightarrow A \perp\!\!\!\perp (B \cup D)|C$$

   (f) [10 points] Show that, for strictly positive distributions (i.e., distributions in which there are no zero-probability events), we have the following intersection property:

   $$A \perp\!\!\!\perp B|(C \cup D) \text{ and } A \perp\!\!\!\perp C|(B \cup D) \rightarrow A \perp\!\!\!\perp (B \cup C)|D$$

4. [10 points] (from Russell & Norvig) This exercise investigates the way in which conditional independence relationships affect the amount of information needed for probabilistic calculations.

(a) Suppose we wish to compute $P(H|E_1, E_2)$ and we have no conditional independence information. Which of the following sets of numbers are sufficient for the calculation?

   i. $P(E_1, E_2), P(H), P(E_1|H), P(E_2|H)$

   ii. $P(E_1, E_2), P(H), P(E_1, E_2|H)$

   iii. $P(H), P(E_1|H), P(E_2|H)$

(b) Suppose that we know that $P(E_1|H, E_2) = P(E_1|H)$ for all values of H, E_1, E_2. Now which of the above sets is sufficient?

(c) Assuming H, E_1, E_2 are all Boolean, how many numbers are sufficient to represent the joint distribution in the two cases?

5. [25 points] Finally, a Bayes net question!

Data the android has been captured by an alien species and is trying to escape. He observes that 8 out of the 10 aliens with whom he came in contact are wearing some kind of weapon. He hasn't seen anything like it before, but it seems to him that there is a 0.6 chance that this weapon is a neurotransmitter disruptor, which would disable him for good. If the weapon is not a disruptor, then if the aliens fire it, there is only a 0.1 chance that he would be disabled. Data also thinks that he has a 0.5 chance of out-powering an alien. If the alien is armed and Data out-powers it, the alien does not get to fire its weapon. If Data out-powers the alien, he escapes from the planet.

(a) [10 points] Draw a Bayes net modeling this problem. Include all the conditional probability tables at the nodes.

(b) [5 points] What is the probability of Data being damaged, in the absence of other information?

(c) [5 points] What is the probability of Data being damaged, given that the alien has a weapon?

(d) [5 points] Suppose that your best friend watched the episode already and tells you that Data escapes unscathed. What is the probability that the alien was unarmed? What is the probability that the alien did have a weapon?